Marking Scheme **Strictly Confidential** **(For Internal and Restricted use only)** **Senior School Certificate Examination, 2024** **MATHEMATICS PAPER CODE - 65/1/1**

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓)while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out `with a note "Extra Question".**

10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks __________(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME

MATHEMATICS (Subject Code–041)

(PAPER CODE: 65/1/1)

Section A

Q.No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each.**	
1.	A function $f : R_+ \to R$ (where R_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is : (A) one-one but not onto (B) onto but not one-one (C) both one-one and onto (D) neither one-one nor onto	
Sol.	(A) one-one but not onto	1
2.	If a matrix has 36 elements, the number of possible orders it can have, is : (A) 13 (B) 3 (C) 5 (D) 9	
Sol.	(D) 9	1
3.	Which of the following statements is true for the function $f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$? (A) $f(x)$ is continuous and differentiable $\forall x \in \mathbb{R}$ (B) $f(x)$ is continuous $\forall x \in \mathbb{R}$ (C) $f(x)$ is continuous and differentiable $\forall x \in \mathbb{R} - \{0\}$ (D) $f(x)$ is discontinuous at infinitely many points	
Sol.	(C) $f(x)$ is continuous and differentiable $\forall\, x \in R - \{0\}$	1

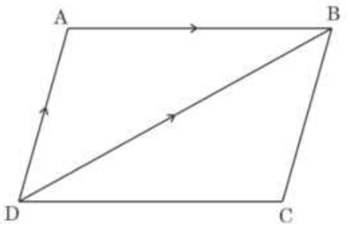
4.	Let f(x) be a continuous function on [a, b] and differentiable on (a, b). Then, this function f(x) is strictly increasing in (a, b) if (A) $f'(x) < 0, \forall x \in (a, b)$ (B) $f'(x) > 0, \forall x \in (a, b)$ (C) $f'(x) = 0, \forall x \in (a, b)$ (D) $f(x) > 0, \forall x \in (a, b)$	
Sol.	(B) $f'(x) > 0, \forall x \in (a, b)$	1
5.	If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is : (A) 7 (B) 6 (C) 8 (D) 18	
Sol.	(D) 18	1
6.	$\int_a^b f(x)\,dx$ is equal to : (A) $\int_a^b f(a-x)\,dx$ (B) $\int_a^b f(a+b-x)\,dx$ (C) $\int_a^b f(x-(a+b))\,dx$ (D) $\int_a^b f((a-x)+(b-x))\,dx$	
Sol.	(B) $\int_a^b f(a+b-x)dx$	1
7.	Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin\theta = \frac{3}{5}$ Then, $\hat{a} \cdot \hat{b}$ is equal to : (A) $\pm\frac{3}{5}$ (B) $\pm\frac{3}{4}$ (C) $\pm\frac{4}{5}$ (D) $\pm\frac{4}{3}$	
Sol.	(C) $\pm\frac{4}{5}$	1
8.	The integrating factor of the differential equation $(1-x^2)\frac{dy}{dx} + xy = ax$, $-1 < x < 1$, is : (A) $\frac{1}{x^2-1}$ (B) $\frac{1}{\sqrt{x^2-1}}$ (C) $\frac{1}{1-x^2}$ (D) $\frac{1}{\sqrt{1-x^2}}$	
Sol.	(D) $\frac{1}{\sqrt{1-x^2}}$	1

9.	If the direction cosines of a line are $\sqrt{3}\,k$, $\sqrt{3}\,k$, $\sqrt{3}\,k$, then the value of k is : (A) ± 1 (B) $\pm\sqrt{3}$ (C) ± 3 (D) $\pm\frac{1}{3}$	
Sol.	(D) $\pm\frac{1}{3}$	1
10.	A linear programming problem deals with the optimization of a/an : (A) logarithmic function (B) linear function (C) quadratic function (D) exponential function	
Sol.	(B) linear function	1
11.	If $P(A \mid B) = P(A' \mid B)$, then which of the following statements is true ? (A) $P(A) = P(A')$ (B) $P(A) = 2\,P(B)$ (C) $P(A \cap B) = \frac{1}{2}\,P(B)$ (D) $P(A \cap B) = 2\,P(B)$	
Sol.	(C) $P(A \cap \boldsymbol{B}) = \frac{1}{2}\,\boldsymbol{P}\,(\boldsymbol{B})$	1
12.	$\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix}$ is equal to : (A) $2x^3$ (B) 2 (C) 0 (D) $2x^3 - 2$	
Sol.	(B) 2	1
13.	The derivative of $\sin(x^2)$ w.r.t. x, at $x = \sqrt{\pi}$ is : (A) 1 (B) -1 (C) $-2\sqrt{\pi}$ (D) $2\sqrt{\pi}$	
Sol.	(C) $-2\sqrt{\pi}$	1
14.	The order and degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ respectively are : (A) 1, 2 (B) 2, 3 (C) 2, 1 (D) 2, 6	
Sol.	(C) 2, 1	1

15.	The vector with terminal point A (2, – 3, 5) and initial point B (3, – 4, 7) is : (A) $\hat{i} - \hat{j} + 2\hat{k}$ (B) $\hat{i} + \hat{j} + 2\hat{k}$ (C) $-\hat{i} - \hat{j} - 2\hat{k}$ (D) $-\hat{i} + \hat{j} - 2\hat{k}$	
Sol.	(D) $-\hat{\imath} + \hat{\jmath} - 2\hat{k}$	1
16.	The distance of point P(a, b, c) from y-axis is : (A) b (B) b^2 (C) $\sqrt{a^2 + c^2}$ (D) $a^2 + c^2$	
Sol.	(C) $\sqrt{a^2 + c^2}$	1
17.	The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is : (A) 0 (B) 1 (C) 2 (D) 3	
Sol.	(C) 2	1
18.	If A and B are two non-zero square matrices of same order such that $(A + B)^2 = A^2 + B^2$, then : (A) AB = O (B) AB = – BA (C) BA = O (D) AB = BA	
Sol.	(B) AB = – BA	1
	Questions No. **19** & **20**, are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below : (A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A). (B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* For matrix $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$, $\lvert A \rvert \in [2, 4]$. *Reason (R) :* $\cos\theta \in [-1, 1], \forall \theta \in [0, 2\pi]$.	
Sol.	(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).	1

20.	*Assertion (A) :* A line in space cannot be drawn perpendicular to x, y and z axes simultaneously. *Reason (R) :* For any line making angles, α, β, γ with the positive directions of x, y and z axes respectively, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.	
Sol.	(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).	1
	SECTION B In this section there are 5 very short answer type questions of 2 marks each.	
21(a).	Check whether the function $f(x) = x^2\ \lvert x \rvert$ is differentiable at $x = 0$ or not.	
Sol.	$f(x) = \begin{cases} x^3, x \geq 0 \\ -x^3,\ x \leq 0 \end{cases}$ $\text{RHD} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} h^2 = 0$ $\text{LHD} = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0}(-h^2) = 0$ $\because RHD = LHD = 0,\quad So\ f(x)\ is\ differentiable\ at\ x = 0$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	OR	
21(b).	If $y = \sqrt{\tan\sqrt{x}}$, prove that $\sqrt{x}\ \frac{dy}{dx} = \frac{1+y^4}{4y}$.	
Sol.	$y = \sqrt{tan\sqrt{x}}$ $\frac{dy}{dx} = \frac{sec^2\sqrt{x}}{2\sqrt{tan\sqrt{x}}} \times \frac{1}{2\sqrt{x}}$ $\sqrt{x}\frac{dy}{dx} = \frac{sec^2\sqrt{x}}{4\sqrt{tan\sqrt{x}}}$ $= \frac{1+(tan\sqrt{x})^2}{4\sqrt{tan\sqrt{x}}} = \frac{1+y^4}{4y}$	1 1
22.	Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.	
Sol.	$f'(x) = 12x^2 - 36x + 27$ $= 3(2x-3)^2 \geq 0$ for all $x \in R$ $\therefore$ f is increasing on R . Hence f(x) does not have maxima or minima.	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

23(a).	Find : $\int x\sqrt{1+2x}\,dx$	
Sol.	$1+2x=t^2$ $2\,dx=2t\,dt$ $\frac{1}{2}\int(t^4-t^2)dt=\frac{1}{2}\left[\frac{t^5}{5}-\frac{t^3}{3}\right]+C$ $=\frac{(1+2x)^{\frac{5}{2}}}{10}-\frac{(1+2x)^{\frac{3}{2}}}{6}+C$	$\frac{1}{2}$ 1 $\frac{1}{2}$
	OR	
23(b).	Evaluate : $\int_0^{\frac{\pi^2}{4}}\frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$	
Sol.	$\int_0^{\frac{\pi^2}{4}}\frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ Put $\sqrt{x}=t \Rightarrow dx=2t\,dt$ $2\int_0^{\frac{\pi}{2}}\sin t\,dt=2\,[-\cos t]_0^{\frac{\pi}{2}}$ $=2$	$\frac{1}{2}$ 1 $\frac{1}{2}$
24.	If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a}+\vec{b})\perp\vec{a}$ and $(2\vec{a}+\vec{b})\perp\vec{b}$, then prove that $\lvert\vec{b}\rvert=\sqrt{2}\,\lvert\vec{a}\rvert$.	
Sol.	$(\vec{a}+\vec{b}).\vec{a}=0 \Rightarrow \lvert\vec{a}\rvert^2+\vec{b}.\vec{a}=0$ -----------------(1) $(2\vec{a}+\vec{b}).\vec{b}=0 \Rightarrow 2\vec{a}.\vec{b}+\lvert\vec{b}\rvert^2=0$ -------------(2) $2(-\lvert\vec{a}\rvert^2)+\lvert\vec{b}\rvert^2=0$ {Using (1) and (2)} $\lvert\vec{b}\rvert^2=2\lvert\vec{a}\rvert^2 \Rightarrow \lvert\vec{b}\rvert=\sqrt{2}\lvert\vec{a}\rvert$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

25.	In the given figure, ABCD is a parallelogram. If $\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\overrightarrow{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find $\overrightarrow{AD}$ and hence find the area of parallelogram ABCD. 	
Sol.	$\overrightarrow{AD} + \overrightarrow{DB} = \overrightarrow{AB}$ $\overrightarrow{AD} = (2\hat{i} - 4\hat{j} + 5\hat{k}) - (3\hat{i} - 6\hat{j} + 2\hat{k})$ $= -\hat{i} + 2\hat{j} + 3\hat{k}$ $\overrightarrow{AD} \times \overrightarrow{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ 2 & -4 & 5 \end{vmatrix} = 22\hat{i} + 11\hat{j}$ Area $= \lvert\overrightarrow{AD} \times \overrightarrow{AB}\rvert = \lvert 22\hat{i} + 11\hat{j}\rvert$ $= \sqrt{605}$ *or* $11\sqrt{5}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
	SECTION C In this section there are 6 short answer type questions of 3 marks each.	
26(a).	A relation R on set A = {1, 2, 3, 4, 5} is defined as $R = \{(x, y) : \lvert x^2 - y^2\rvert < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.	
Sol.	(a) Reflexive: $\because \lvert x^2 - x^2\rvert < 8\ \forall\ x \in A \Rightarrow (x, x) \in R\ \therefore R$ is reflexive . (b) Symmetric: Let $(x,y) \in R$ for some $x,y \in A$ $\therefore \lvert x^2 - y^2\rvert < 8 \Rightarrow \lvert y^2 - x^2\rvert < 8 \Rightarrow (y, x) \in R$ Hence R is symmetric. (c) Transitive: $(1,2)$, $(2,3) \in R$ as $\lvert 1^2 - 2^2\rvert < 8$, $\lvert 2^2 - 3^2\rvert < 8$ respectively But $\lvert 1^2 - 3^2\rvert \nless 8 \Rightarrow (1, 3) \notin R$ Hence R is not transitive.	$\frac{1}{2}$ 1 $1\frac{1}{2}$
	OR	
26(b).	A function f is defined from $R \to R$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.	

Sol.	$f(x) = ax + b$ Solving $a + b = 1$ and $2a + b = 3$ to get $a=2$, $b = -1$ $f(x) = 2x - 1$ Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in \boldsymbol{R}$ $2x_1 - 1 = 2x_2 - 1 \Rightarrow x_1 = x_2$ Hence f is one – one. Let $y = 2x - 1$, $y \in \boldsymbol{R}$ (Codomain) $\Rightarrow x = \frac{y+1}{2} \in \boldsymbol{R}$ (domain) Also, $f(x) = f\left(\frac{y+1}{2}\right) = y$ $\therefore$ f is onto.	 1 1 1
27(a).	If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.	
Sol.	$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ Put $x = \sin\theta$, $y = \sin\phi$ $\Rightarrow \cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$ $\Rightarrow 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right) = 2a\sin\left(\frac{\theta-\phi}{2}\right)\cos\left(\frac{\theta+\phi}{2}\right)$ $\Rightarrow \cot\left(\frac{\theta-\phi}{2}\right) = a$ $\Rightarrow \theta - \phi = 2\cot^{-1} a$ $\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$ $\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$	 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$
	OR	
27(b).	If $y = (\tan x)^x$, then find $\frac{dy}{dx}$.	
Sol.	$y = (\tan x)^x$ $\log y = x \log(\tan x)$ $\frac{1}{y}\frac{dy}{dx} = x\left(\frac{\sec^2 x}{\tan x}\right) + \log(\tan x)$ $\frac{dy}{dx} = (\tan x)^x\left[\left(\frac{x\sec^2 x}{\tan x}\right) + \log(\tan x)\right]$	 $\frac{1}{2}$ 2 $\frac{1}{2}$
28(a).	Find : $\int \frac{x^2}{(x^2+4)(x^2+9)}\,dx$	

Sol.	Let $I = \int \frac{x^2}{(x^2+4)(x^2+9)} dx$ Put $x^2 = t$ $\frac{t}{(t+4)(t+9)} = \frac{A}{t+4} + \frac{B}{t+9} \Rightarrow A = \frac{-4}{5},\ B = \frac{9}{5}$ $I = \frac{-4}{5}\int \frac{1}{2^2+x^2} dx + \frac{9}{5}\int \frac{1}{3^2+x^2} dx$ $= \frac{-2}{5}\tan^{-1}\left(\frac{x}{2}\right) + \frac{3}{5}\tan^{-1}\left(\frac{x}{3}\right) + C$	$\frac{1}{2}$ $1\frac{1}{2}$ 1
	OR	
28(b).	Evaluate : $\int_1^3 (\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert)\, dx$	
Sol.	$\int_1^3 (\lvert x-1\rvert + \lvert x-2\rvert + \lvert x-3\rvert) dx$ $= \int_1^3 (x-1)dx + \int_1^2 -(x-2)dx + \int_2^3 (x-2)dx - \int_1^3 (x-3)dx$ $= \int_1^3 2\ dx + \int_1^2 (2-x)\ dx + \int_2^3 (x-2)dx$ $= [2x]_1^3 + \left[\frac{(2-x)^2}{-2}\right]_1^2 + \left[\frac{(x-2)^2}{2}\right]_2^3$ $= 4 + \frac{1}{2} + \frac{1}{2} = 5$	$1\frac{1}{2}$ $1\frac{1}{2}$
29.	Find the particular solution of the differential equation given by $x^2\frac{dy}{dx} - xy = x^2\cos^2\left(\frac{y}{2x}\right)$, given that when $x = 1, y = \frac{\pi}{2}$.	
Sol.	$\frac{dy}{dx} = \frac{y}{x} + \cos^2\left(\frac{y}{2x}\right)$ Put $y = vx$ so that $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\Rightarrow v + x\frac{dv}{dx} = v + \cos^2\left(\frac{v}{2}\right)$ $\Rightarrow \int \sec^2\left(\frac{v}{2}\right) dv = \int \frac{1}{x} dx$ $\Rightarrow 2\tan\left(\frac{v}{2}\right) = \log\lvert x\rvert + C$ $\Rightarrow 2\tan\left(\frac{y}{2x}\right) = \log\lvert x\rvert + C$ $2\tan\frac{\pi}{4} = \log 1 + C \Rightarrow C = 2 \Rightarrow 2\tan\left(\frac{y}{2x}\right) = \log\lvert x\rvert + 2$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$

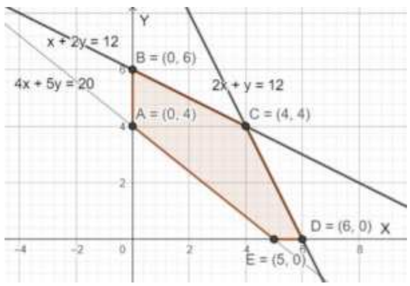
30. Solve the following linear programming problem graphically :

Maximise $z = 500x + 300y$,

subject to constraints

$$x + 2y \le 12$$
$$2x + y \le 12$$
$$4x + 5y \ge 20$$
$$x \ge 0, y \ge 0$$

Sol. Max $z = 500x + 300y$



Correct Graph - $1\frac{1}{2}$

Corner Point	Z
A (0,4)	1200
B (0,6)	1800
C (4,4)	3200
D (6,0)	3000
E (5,0)	2500

Correct Table - 1

Max $z = 3200$ at $x = 4, y = 4$ — $\frac{1}{2}$

31. E and F are two independent events such that $P(\bar{E}) = 0{\cdot}6$ and $P(E \cup F) = 0{\cdot}6$. Find $P(F)$ and $P(\bar{E} \cup \bar{F})$.

Sol.

$P(\bar{E}) = 0.6 \Rightarrow P(E) = 0.4$ — $\frac{1}{2}$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$ — $\frac{1}{2}$

$\Rightarrow 0.6 = 0.4 + P(F) - 0.4\,P(F) \Rightarrow P(F) = \frac{1}{3}$ — 1

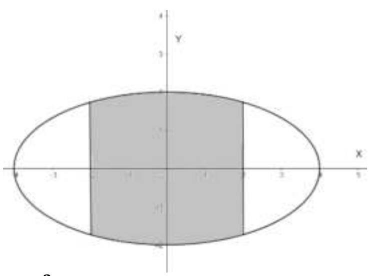
$P(\bar{E} \cup \bar{F}) = 1 - P(E \cap F)$ — $\frac{1}{2}$

$= 1 - 0.4 \times \frac{1}{3} = \frac{13}{15}$ — $\frac{1}{2}$

SECTION D

In this section there are 4 long answer type questions of 5 marks each.

32(a).	If $A=\begin{bmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}$, find A^{-1} and use it to solve the following system of equations : $x-2y=10,\ 2x-y-z=8,\ -2y+z=7$	
Sol.	$\lvert A\rvert = 1 \neq 0$ hence A^{-1} exists. $Adj\ A=\begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}$ $A^{-1}=\begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}$ $AX=B \Rightarrow \begin{bmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}$ $X=A^{-1}B \Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}\begin{bmatrix}10\\ 8\\ 7\end{bmatrix}=\begin{bmatrix}0\\ -5\\ -3\end{bmatrix}$ $\Rightarrow x=0, y=-5, z=-3$	1 2 $\frac{1}{2}$ $1\frac{1}{2}$
	OR	
32(b).	If $A=\begin{bmatrix}-1 & a & 2\\ 1 & 2 & x\\ 3 & 1 & 1\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1 & -1 & 1\\ -8 & 7 & -5\\ b & y & 3\end{bmatrix}$, find the value of $(a+x)-(b+y)$.	
Sol.	$AA^{-1}=I$ $\begin{bmatrix}-1 & a & 2\\ 1 & 2 & x\\ 3 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & -1 & 1\\ -8 & 7 & -5\\ b & y & 3\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ $\begin{bmatrix}-1-8a+2b & 1+7a+2y & 5-5a\\ -15+bx & 13+xy & 3x-9\\ -5+b & 4+y & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ $-5+b=0 \Rightarrow b=5$, $5-5a=0 \Rightarrow a=1$ $4+y=0 \Rightarrow y=-4$, $3x-9=0 \Rightarrow x=3$ $\therefore (a+x)-(b+y)=(1+3)-(5-4)=3$	1 $1\frac{1}{2}$ 1 1 $\frac{1}{2}$
33(a).	Evaluate : $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}\,dx$	
Sol.	Let $I=\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}\,dx$	

	Put sin x − cos x = t, so that (cos x + sin x) dx = dt $\sin^2 x + \cos^2 x - \sin 2x = t^2 \Rightarrow \sin 2x = 1 - t^2$ $I = \int_{-1}^{0} \frac{dt}{25-16t^2}$ $= \frac{1}{16}\int_{-1}^{0} \frac{dt}{(\frac{5}{4})^2 - t^2}$ $= \frac{1}{40}\left[\log\left\|\frac{5+4t}{5-4t}\right\|\right]_{-1}^{0}$ $= \frac{1}{40}\left[\log 1 - \log\left(\frac{1}{9}\right)\right] = \frac{1}{40}\log 9 \ or \ \frac{1}{20}\log 3$	1 $\frac{1}{2}$ 1 $1\frac{1}{2}$ 1
	OR	
33(b).	Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\, dx$	
Sol.	Let $I = \int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\, dx$ Put sin x = t so that cos x dx = dt $I = 2\int_0^1 t \tan^{-1} t \, dt$ $= 2\left[\tan^{-1} t \left(\frac{t^2}{2}\right) - \frac{1}{2}\int \frac{t^2}{1+t^2} dt\right]_0^1$ $= 2\left[\left(\frac{t^2}{2}\right)\tan^{-1} t - \frac{1}{2}t + \frac{1}{2}\tan^{-1} t\right]_0^1$ $= 2\left(\frac{\pi}{4} - \frac{1}{2}\right) = \frac{\pi}{2} - 1$	1 1 $1\frac{1}{2}$ 1 $\frac{1}{2}$
34.	Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines x = – 2 and x = 2.	
Sol.	 Area $= 4\int_0^2 y\, dx$ $= 4\left[\frac{1}{2}\int_0^2 \sqrt{4^2 - x^2}\, dx\right]$ $= 2\left[\frac{x}{2}\sqrt{4^2 - x^2} + 8\sin^{-1}(\frac{x}{4})\right]_0^2$ $= 2\left[\sqrt{12} + \frac{8\pi}{6}\right] = 4\sqrt{3} + \frac{8\pi}{3}$	Correct graph-1 1 2 1

35. The image of point P(x, y, z) with respect to line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is P′ (1, 0, 7). Find the coordinates of point P.

Sol.

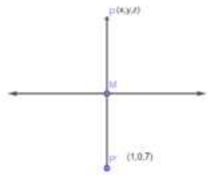
Let foot of the perpendicular on the given line from point P be M $(\lambda, 2\lambda+1, 3\lambda+2)$ — 1

D. ratios of PP' are $\lambda - 1,\ 2\lambda+1,\ 3\lambda-5$ — 1

$1(\lambda - 1) + 2\,(2\lambda + 1) + 3\,(3\lambda - 5) = 0$

$\Rightarrow\ \lambda = 1$ — 1

Coordinates of M(1,3,5) — 1



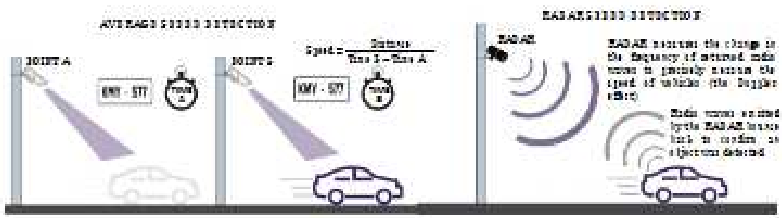
$\frac{x+1}{2} = 1,\ \frac{y+0}{2} = 3,\ \frac{z+7}{2} = 5$

$\Rightarrow x = 1,\ y = 6,\ z = 3\ \Rightarrow P(1, 6, 3)$ — 1

SECTION E

In this section there are 3 case-study based questions of 4 marks each.

36. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.



A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.

On the basis of the above information, answer the following questions :

(i) Express θ in terms of height of the camera installed on the pole and x. — *1*

(ii) Find $\frac{d\theta}{dx}$. — *1*

(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. — *2*

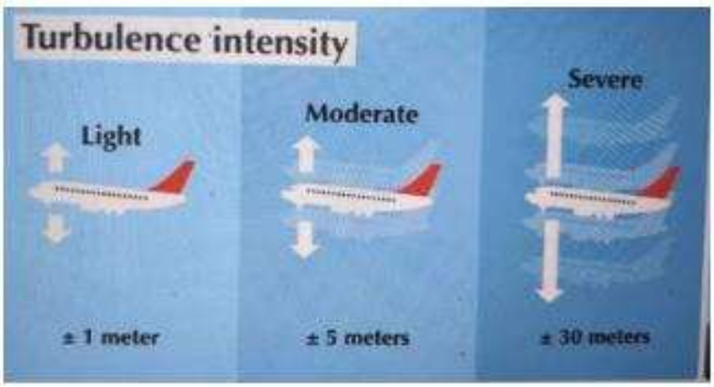
OR

(iii) (b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car. — *2*

Sol.

(i) $\tan\theta = \frac{5}{x} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{x}\right)$ — 1

(ii) $\frac{d\theta}{dx} = \frac{-5}{5^2 + x^2}$ — 1

	(iii) (a) $\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt} = \frac{-5}{5^2+x^2} \times 20\Big]_{x=50}$ $= \frac{-100}{2525}$ or $\frac{-4}{101}$ rad/s OR (b) $\frac{d\theta}{dt} = \frac{d\theta}{dx} \times \frac{dx}{dt} \Rightarrow \frac{3}{101} = \frac{-5}{5^2+x^2}\Big]_{x=50} \times \frac{dx}{dt}$ $\Rightarrow \frac{3}{101} = \frac{-5}{2525} \times \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -15\ m/s$ Hence the speed is 15 m/s	$1\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$
37.	According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights. Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.  On the basis of the above information, answer the following questions : (i) Find the probability that an airplane reached its destination late. 2 (ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence. 2	
Sol.	(i) Let A denote the event of airplane reaching its destination late E_1 = severe turbulence E_2 = moderate turbulence E_3 = light turbulence P(A) = $P(E_1)\,P(A\|E_1) + P(E_2)P(A\|E_2) + P(E_3)P(A\|E_3)$ $= \frac{1}{3} \times \frac{55}{100} + \frac{1}{3} \times \frac{37}{100} + \frac{1}{3} \times \frac{17}{100}$ $= \frac{1}{3}\left(\frac{109}{100}\right) = \frac{109}{300}$	$\frac{1}{2}$ 1 $\frac{1}{2}$

(ii) $P(E_2|A) = \frac{P(E_2)P(A|E_2)}{P(A)}$

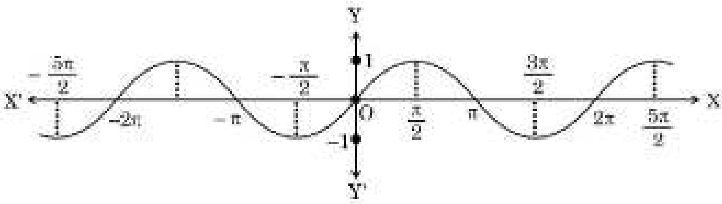
$= \frac{\frac{1}{3}\times\frac{37}{100}}{\frac{109}{300}}$ $1\frac{1}{2}$

$= \frac{37}{109}$ $\frac{1}{2}$

38.

If a function f : X → Y defined as f(x) = y is one-one and onto, then we can define a unique function g : Y → X such that g(y) = x, where x ∈ X and y = f(x), y ∈ Y. Function g is called the inverse of function f.

The domain of sine function is R and function sine : R → R is neither one-one nor onto. The following graph shows the sine function.



y = sin x

Let sine function be defined from set A to [– 1, 1] such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from [– 1, 1] to A.

On the basis of the above information, answer the following questions :

(i) If A is the interval other than principal value branch, give an example of one such interval. *1*

(ii) If $\sin^{-1}(x)$ is defined from [– 1, 1] to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$. *1*

(iii) (a) Draw the graph of $\sin^{-1} x$ from [– 1, 1] to its principal value branch. *2*

OR

(iii) (b) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$. *2*

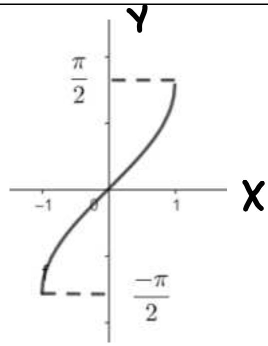
Sol.

(i) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ **or any other interval corresponding to the domain [-1,1]** **1**

(ii) $\sin^{-1}\left(\frac{-1}{2}\right) - \sin^{-1}(1)$

$= \frac{-\pi}{6} - \frac{\pi}{2}$

$= \frac{-4\pi}{6}$ or $\frac{-2\pi}{3}$ **1**

	(iii) (a) 	Correct graph - 2
	OR	
	(b) $f(x) = 2\, sin^{-1}(1 - x)$	
	$-1 \leq 1 - x \leq 1$	$\frac{1}{2}$
	$\Rightarrow -2 \leq -x \leq 0$	
	$\Rightarrow 0 \leq x \leq 2$	
	Domain $= [0, 2]$	$\frac{1}{2}$
	$\frac{-\pi}{2} \leq \sin^{-1}(1 - x) \leq \frac{\pi}{2}$	$\frac{1}{2}$
	$-\pi \leq 2\, \sin^{-1}(1 - x) \leq \pi$	
	So range $= [-\pi, \pi]$	$\frac{1}{2}$