Marking Scheme

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Senior School Certificate Examination, 2023

MATHEMATICS PAPER CODE 65/1/1

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**“Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its’ leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc. may invite action under various rules of the Board and IPC.”**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one’s own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS ‘X” be marked. Evaluators will not put right (✓) while evaluating which gives the impression that answer is correct, and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totalled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	<u>**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other**</u>

	answer scored out with a note "Extra Question".
10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks __________ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) must be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.

14	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totalling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totalling on the title page. ● Wrong totalling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any unassessed portion, non-carrying over of marks to the title page, or totalling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totalled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

EXPECTED ANSWER/VALUE POINTS

SECTION A

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A (Question nos. 1 to 18 are Multiple Choice Questions carrying 1 mark each)	
1	1. If for a square matrix A, $A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of x + y is : (a) -2 (b) 2 (c) 3 (d) -3	
Ans	(b) 2	**1**
2.	2. If $\lvert A\rvert = 2$, where A is a 2×2 matrix, then $\lvert 4A^{-1}\rvert$ equals : (a) 4 (b) 2 (c) 8 (d) $\frac{1}{32}$	
Ans	(c) 8	**1**
3.	3. Let A be a 3×3 matrix such that $\lvert \text{adj } A\rvert = 64$. Then $\lvert A\rvert$ is equal to : (a) 8 only (b) -8 only (c) 64 (d) 8 or -8	
Ans	(d) 8 or – 8	**1**
4.	4. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and 2A + B is a null matrix, then B is equal to : (a) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$	
Ans	(b) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$	**1**
5.	5. If $\frac{d}{dx}(f(x)) = \log x$, then f(x) equals : (a) $-\frac{1}{x} + C$ (b) $x(\log x - 1) + C$ (c) $x(\log x + x) + C$ (d) $\frac{1}{x} + C$	
Ans	(b) $x(\log x - 1) + C$	**1**

6.	6. $\int_0^{\frac{\pi}{6}} \sec^2(x - \frac{\pi}{6})\, dx$ is equal to : (a) $\frac{1}{\sqrt{3}}$ (b) $-\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $-\sqrt{3}$	
Ans	(a) $\frac{1}{\sqrt{3}}$	**1**
7.	7. The sum of the order and the degree of the differential equat $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ is : (a) 5 (b) 2 (c) 3 (d) 4	
Ans	(c) 3	**1**
8.	8. The value of p for which the vectors $2\hat{i} + p\hat{j} + \hat{k}$ and $-4\hat{i} - 6\hat{j} + 26\hat{k}$ are perpendicular to each other, is : (a) 3 (b) -3 (c) $-\frac{17}{3}$ (d) $\frac{17}{3}$	
Ans	(a) 3	**1**
9.	9. The value of $(\hat{i} \times \hat{j}) . \hat{j} + (\hat{j} \times \hat{i}) . \hat{k}$ is : (a) 2 (b) 0 (c) 1 (d) -1	
Ans	(d) *-1*	**1**
10.	10. If $\vec{a} + \vec{b} = \hat{i}$ and $\vec{a} = 2\hat{i} - 2\hat{j} + 2\hat{k}$, then $\lvert \vec{b} \rvert$ equals : (a) $\sqrt{14}$ (b) 3 (c) $\sqrt{12}$ (d) $\sqrt{17}$	
Ans	(b) 3	**1**
11.	11. Direction cosines of the line $\frac{x-1}{2} = \frac{1-y}{3} = \frac{2z-1}{12}$ are : (a) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$ (b) $\frac{2}{\sqrt{157}}, -\frac{3}{\sqrt{157}}, \frac{12}{\sqrt{157}}$ (c) $\frac{2}{7}, -\frac{3}{7}, -\frac{6}{7}$ (d) $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$	
Ans	(d) $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$	**1**

12.	**12.** If $P\left(\frac{A}{B}\right) = 0{\cdot}3$, $P(A) = 0{\cdot}4$ and $P(B) = 0{\cdot}8$, then $P\left(\frac{B}{A}\right)$ is equal to : (a) 0·6 (b) 0·3 (c) 0·06 (d) 0·4	
Ans	(a) 0.6	**1**
13.	**13.** The value of k for which $f(x) = \begin{cases} 3x+5, & x \geq 2 \\ kx^2, & x < 2 \end{cases}$ is a continuous function, is : (a) $-\frac{11}{4}$ (b) $\frac{4}{11}$ (c) 11 (d) $\frac{11}{4}$	
Ans	(d) $\frac{11}{4}$	**1**
14.	**14.** If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(3I + 4A)(3I - 4A) = x^2 I$, then the value(s) x is/are : (a) $\pm\sqrt{7}$ (b) 0 (c) ± 5 (d) 25	
Ans	(c) ± 5	**1**
15.	**15.** The general solution of the differential equation $x\,dy - (1 + x^2)\,dx = dx$ is : (a) $y = 2x + \frac{x^3}{3} + C$ (b) $y = 2\log x + \frac{x^3}{3} + C$ (c) $y = \frac{x^2}{2} + C$ (d) $y = 2\log x + \frac{x^2}{2} + C$	
Ans	(d) $y = 2\log x + \frac{x^2}{2} + C$	**1**
16.	**16.** If $f(x) = a(x - \cos x)$ is strictly decreasing in $\mathbb{R}$, then 'a' belongs to (a) {0} (b) $(0, \infty)$ (c) $(-\infty, 0)$ (d) $(-\infty, \infty)$	
Ans	(c) $(-\infty, 0)$	**1**

17.	**17.** The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then : (a) z is maximum at (2, 72), minimum at (15, 20) (b) z is maximum at (15, 20), minimum at (40, 15) (c) z is maximum at (40, 15), minimum at (15, 20) (d) z is maximum at (40, 15), minimum at (2, 72)	
Ans	(c) z is maximum at (40, 15) and minimum at (15, 20)	**1**
18.	**18.** The number of corner points of the feasible region determined by the constraints $x - y \geq 0$, $2y \leq x + 2$, $x \geq 0$, $y \geq 0$ is : (a) 2 (b) 3 (c) 4 (d) 5	
Ans	(a) *2*	**1**
	(Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each)	
19.	**19.** *Assertion (A) :* The range of the function $f(x) = 2\sin^{-1} x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$. *Reason (R) :* The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$.	
Ans	(c) Assertion is True, Reason is False	**1**
20.	**20.** *Assertion (A) :* Equation of a line passing through the points (1, 2, 3) and (3, –1, 3) is $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-3}{0}$. *Reason (R) :* Equation of a line passing through points (x_1, y_1, z_1), (x_2, y_2, z_2) is given by $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$.	
Ans	(d) Assertion is False, Reason is True	**1**
	SECTION-B **(Question nos. 21 to 25 are very short Answer type questions carrying 2 marks each)**	
21.	**21.** (a) A function $f : A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B. **OR** (b) Evaluate : $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) + \cos^{-1}\left(\cos\frac{3\pi}{4}\right) + \tan^{-1}(1)$	
Ans	(a) $f(1) = 2, f(2) = 4, f(3) = 6, f(4) = 8$ $\therefore B = \{2, 4, 6, 8\}$ **OR** (b) Required value = $\frac{\pi}{4} + \frac{3\pi}{4} + \frac{\pi}{4}$	**$1\frac{1}{2}$** **½** **$1\frac{1}{2}$**

	$= \frac{5\pi}{4}$	$\frac{1}{2}$
22.	**22.** Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i} + \hat{j} + \hat{k}$.	
Ans	Unit vector along $\hat{i} + \hat{j} + \hat{k}$ is $\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$ Required vectors are $3\hat{i} + 3\hat{j} + 3\hat{k}$ and $-3\hat{i} - 3\hat{j} - 3\hat{k}$	1 $\frac{1}{2}+\frac{1}{2}$
23.	**23.** (a) Position vectors of the points A, B and C as shown in the figure below are $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.  If $\overrightarrow{AC} = \frac{5}{4}\overrightarrow{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$. **OR** (b) Check whether the lines given by equations $x = 2\lambda + 2$, $y = 7\lambda + 1$, $z = -3\lambda - 3$ and $x = -\mu - 2$, $y = 2\mu + 8$, $z = 4\mu + 5$ are perpendicular to each other or not.	
Ans	(a) According to question, $\vec{c} - \vec{a} = \frac{5}{4}(\vec{b} - \vec{a})$ $\therefore \vec{c} = \frac{5\vec{b}}{4} - \frac{\vec{a}}{4}$ OR (b) D.r.s. of lines are $< 2, 7, -3 >$ and $< -1, 2, 4 >$ Now $2.-1 + 7\cdot 2 + -3\cdot 4 = 0$ $\therefore$ given lines are perpendicular	$1\frac{1}{2}$ $\frac{1}{2}$ 1 1
24.	**24.** If $y = (x + \sqrt{x^2 - 1})^2$, then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.	
Ans	$\frac{dy}{dx} = 2\left(x + \sqrt{x^2-1}\right)\left(1 + \frac{x}{\sqrt{x^2-1}}\right) = \frac{2\left(x+\sqrt{x^2-1}\right)^2}{\sqrt{x^2-1}}$ $\sqrt{x^2-1}\,\frac{dy}{dx} = 2y$ $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$	$1\frac{1}{2}$ $\frac{1}{2}$

25.	**25.** Show that the function $f(x) = \frac{16 \sin x}{4 + \cos x} - x$, is strictly decreasing in $\left(\frac{\pi}{2}, \pi\right)$.	
Ans	$f'(x) = \frac{16[4 + \cos x]\cos x + 16 \sin^2 x}{(4 + \cos x)^2} - 1$ $= \frac{\cos x\,(56 - \cos x)}{(4 + \cos x)^2}$ in $(\frac{\pi}{2}, \pi)$, $\cos x < 0 \Rightarrow f'(x) < 0$ $\therefore$ f(x) in strictly decreasing in $(\frac{\pi}{2}, \pi)$	1 ½ ½
SECTION-C **(Question nos. 26 to 31 are short Answer type questions carrying 3 marks each)**		
26.	**26.** Evaluate : $\int_0^{\frac{\pi}{2}} [\log(\sin x) - \log(2\cos x)]\, dx.$	
Ans	Let $I = \int_0^{\pi/2} [\log \sin x - \log(2\cos x)]dx = \int_0^{\pi/2} \log\left(\frac{\tan x}{2}\right)dx$ Using property $\int_0^a f(x)\,dx = \int_0^a f(a-x)dx$ We get, $I = \int_0^{\pi/2} \log\left(\frac{\cot x}{2}\right)dx$ $\therefore 2I = \int_0^{\pi/2} \log\left(\frac{\tan x}{2} \times \frac{\cot x}{2}\right)dx = \int_0^{\pi/2} \log\left(\frac{1}{4}\right)dx$ $2I = \log\left(\frac{1}{4}\right)x\Big\|_0^{\pi/2} = \frac{\pi}{2}\log\frac{1}{4}$ $I = \frac{\pi}{4}\log\frac{1}{4}$ OR $-\frac{\pi}{2}\log 2$	½ 1 ½ ½ ½
27.	**27.** Find : $\int \frac{1}{\sqrt{x}(\sqrt{x}+1)(\sqrt{x}+2)}dx$	
Ans	Let $I = \int \frac{dx}{\sqrt{x}\,(\sqrt{x}+1)(\sqrt{x}+2)}$	

	Let $\sqrt{x} = t,\ \frac{1}{2\sqrt{x}}\,dx = dt$ $\therefore\ I = 2\int \frac{dt}{(t+1)(t+2)}$ $= 2\int\left(\frac{1}{t+1} - \frac{1}{t+2}\right)dt$ $= 2[\log\lvert t+1\rvert - \log\lvert t+2\rvert] + C$ $= 2[\log(\sqrt{x}+1) - \log(\sqrt{x}+2)] + C$ or $2\log\left(\frac{\sqrt{x}+1}{\sqrt{x}+2}\right) + C$	½ 1 1 ½
28.	**28.** (a) Find the particular solution of the differential equation $\frac{dy}{dx} + \sec^2 x \cdot y = \tan x \cdot \sec^2 x$, given that $y(0) = 0$. **OR** (b) Solve the differential equation given by $x\,dy - y\,dx - \sqrt{x^2+y^2}\,dx = 0$.	
Ans	**Let** (a) Integrating factor = $e^{\int \sec^2 x\,dx} = e^{\tan x}$ Solution is $ye^{\tan x} = \int \tan x \sec^2 x\, e^{\tan x}\,dx + C$ Let $\tan x = t$ $\sec^2 x\,dx = dt$ $\therefore \int e^{\tan x}\tan x \sec^2 x\,dx = \int e^t\, t\,dt = e^t(t-1)$ $\therefore\ ye^{\tan x} = e^{\tan x}(\tan x - 1) + C$ $y(0) = 0$ gives $C = 1$ Particular solution is $ye^{\tan x} = e^{\tan x}(\tan x - 1) + 1$ or $y = \tan x - 1 + e^{-\tan x}$ OR (b)Given differential equation can be written as $\frac{dy}{dx} = \frac{y}{x} + \sqrt{1 + \left(\frac{y}{x}\right)^2}$ --------- (i) Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ substituting in (i) We get $v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$ $\Rightarrow \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$ Integrating both sides, we get $\log\lvert\sqrt{1+v^2} + v\rvert = \log\lvert x\rvert + \log C$ $y + \sqrt{x^2+y^2} = C x^2$	½ ½ ½ ½ ½ ½ ½ ½ ½ 1 ½

29.

29. Solve graphically the following linear programming problem :

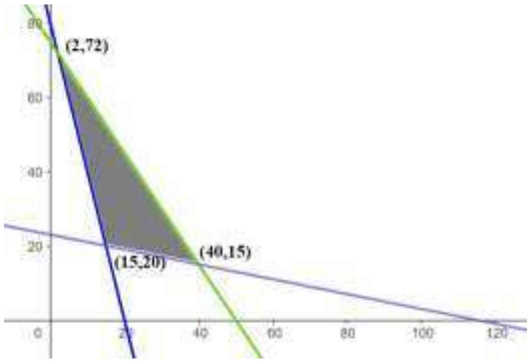
Maximise $z = 6x + 3y$,

subject to the constraints

$$4x + y \geq 80,$$
$$3x + 2y \leq 150,$$
$$x + 5y \geq 115,$$
$$x \geq 0, y \geq 0.$$

Ans

Correct Graph — **2**



Corner points	Value of Z
(2, 72)	(12 + 216 = 228)
(15, 20)	(90 + 60 = 150)
(40, 15)	(240 + 45 = 285) Maximum

1

30.

30. (a) The probability distribution of a random variable X is given below :

X	1	2	3
P(X)	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{6}$

(i) Find the value of k.

(ii) Find $P(1 \leq X < 3)$.

(iii) Find E(X), the mean of X.

OR

(b) A and B are independent events such that $P(A \cap \overline{B}) = \frac{1}{4}$ and $P(\overline{A} \cap B) = \frac{1}{6}$. Find P(A) and P(B).

Ans

(a) (i) $\frac{k}{2} + \frac{k}{3} + \frac{k}{6} = 1$ — **1**

Gives k = 1 — **½**

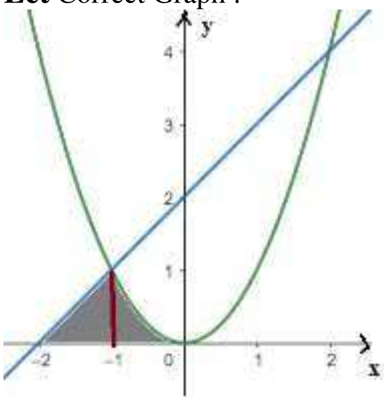
(ii) $P(1 \leq X < 3) = \frac{5k}{6} = \frac{5}{6}$ — **½**

(iii) $E(X) = \sum p_i . x_i = \frac{k}{2} + \frac{2k}{3} + \frac{k}{2} = \frac{5k}{3}$ — **½**

	E(X) = $\frac{5}{3}$	½
	OR	
	(b) $P(A)\,P(\bar{B}) = \frac{1}{4}$ $\quad P(\bar{A})\,P(B) = \frac{1}{6}$	½
	Let P(A) = x $\quad$ P(B) = y	
	$x(1-y) = \frac{1}{4},\ \ (1-x)y = \frac{1}{6} \Rightarrow x - y = \frac{1}{12}$	
	eliminating y, we get $12x^2 - 13x + 3 = 0$	1
	gives $x = \frac{1}{3}, \frac{3}{4}$	1
	$P(A) = \frac{1}{3} \Rightarrow P(B) = \frac{1}{4}$ $P(A) = \frac{3}{4} \Rightarrow P(B) = \frac{2}{3}$	½
31.	**31.** (a) Evaluate : $$\int_0^{\frac{\pi}{2}} e^x \sin x\, dx$$ **OR** (b) Find : $$\int \frac{1}{\cos(x-a)\ \cos(x-b)}\, dx$$	
Ans	(a)Let $I = \int e^x \sin x\, dx$	
	$= e^x \sin x - \int \cos x\, e^x\, dx$	**1**
	$= e^x \sin x - \cos x\, e^x - I$	½
	$\therefore\ I = \frac{1}{2} e^x (\sin x - \cos x)$	½
	$\therefore \int_0^{\pi/2} e^x \sin x\, dx = \frac{1}{2} e^{\pi/2} + \frac{1}{2}$ or $\frac{1}{2}(e^{\pi/2} + 1)$	**1**
	OR	
	(b)Let $I = \int \frac{1}{\cos(x-a)\ \cos(x-b)} dx$	
	$= \frac{1}{\sin(a-b)} \int \frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)} dx$	1
	$= \frac{1}{\sin(a-b)} \left[\int \frac{\sin(x-b)\cos(x-a)}{\cos(x-a)\cos(x-b)} - \frac{\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)}\right] dx$	½
	$= \frac{1}{\sin(a-b)} \left[\int [\tan(x-b) - \tan(x-a)]\, dx\right]$	½

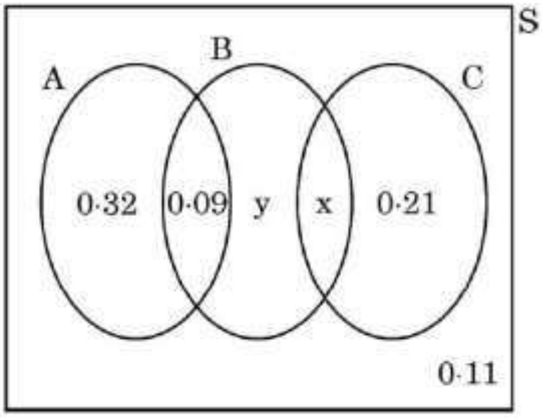
	$= \frac{1}{\sin(a-b)}$ [log $\lvert \sec(x-b) \rvert$ – log $\lvert \sec(x-a) \rvert$] + C	1
SECTION-D **(Question nos. 32 to 35 are Long Answer type questions carrying 5 marks each)**		
32.	**32.** A relation R is defined on a set of real numbers $\mathbb{R}$ as R = {(x, y) : x . y is an irrational number}. Check whether R is reflexive, symmetric and transitive or not.	
Ans	For reflexive (1, 1) $\notin$R as 1^2 is rational (or any other counter example) R is not reflexive For symmetric Let (x, y) $\in$R $\therefore$ x.y is an irrational number $\therefore$ (y.x) is an irrational number $\therefore$ (y, x) $\in$R $\therefore$ R is symmetric For Transitive (1, $\sqrt{2}$) $\in$R, ($\sqrt{2}$, 2) $\in$R (or any other counter example) but (1, 2) $\notin$R $\therefore$ R is not transitive	**1 ½** **1 ½** **2**
33.	**33.** (a) If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$. **OR** (b) Solve the following system of equations by matrix method : x + 2y + 3z = 6 2x – y + z = 2 3x + 2y – 2z = 3	
Ans	(a)$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ $(AB)^{-1} = B^{-1}A^{-1}$ $\lvert A \rvert = 1(3) - 2(-1) - 2(2) = 3 + 2 - 4 = 1 \neq 0$ **adj(A)** = $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ $\mathbf{A^{-1}} = \frac{1}{1}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ $\therefore B^{-1}A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$	½ **1** 2 ½

	$= \begin{bmatrix} 10 & 7 & 21 \\ -49 & -34 & -103 \\ 17 & 12 & 36 \end{bmatrix}$	1
	OR	
	(b)Given system is $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 2 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$ $A \cdot X = B \Rightarrow X = A^{-1}B$ $\|A\| = 35 \neq 0$ $A_{11} = 0 \quad A_{12} = 7 \quad A_{13} = 7$ $A_{21} = 10 \quad A_{22} = -11 \quad A_{23} = 4$ $A_{31} = 5 \quad A_{32} = 5 \quad A_{33} = -5$ $\therefore A^{-1} = \frac{1}{35}\begin{bmatrix} 0 & 10 & 5 \\ 7 & -11 & 5 \\ 7 & 4 & -5 \end{bmatrix}$ $\Rightarrow X = \frac{1}{35}\begin{bmatrix} 0 & 10 & 5 \\ 7 & -11 & 5 \\ 7 & 4 & -5 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 35 \\ 35 \\ 35 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\therefore x = 1 \quad y = 1 \quad z = 1$	 ½ 1 1½ 1 1
34.	**34.** (a) Find the vector and the Cartesian equations of a line passing through the point (1, 2, – 4) and parallel to the line joining the points A(3, 3, – 5) and B(1, 0, – 11). Hence, find the distance between the two lines. **OR** (b) Find the equations of the line passing through the points A(1, 2, 3) and B(3, 5, 9). Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B.	
Ans	(a) Vector equation of required line through (1, 2, – 4) is $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ and cartesian equation: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$ Equation of line through A(3, 3, – 5) and B(1, 0, – 11) is $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$ Distance between parallel lines is given by $d = \frac{\|(\vec{a_2} - \vec{a_1}) \times \vec{b}\|}{\|\vec{b}\|}$ Here $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$ $(\vec{a}_2 - \vec{a}_1) = 2\hat{i} + \hat{j} - \hat{k}$ $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = 9\hat{i} - 14\hat{j} + 4\hat{k}$	 **1** **1** **½** **½** **1**

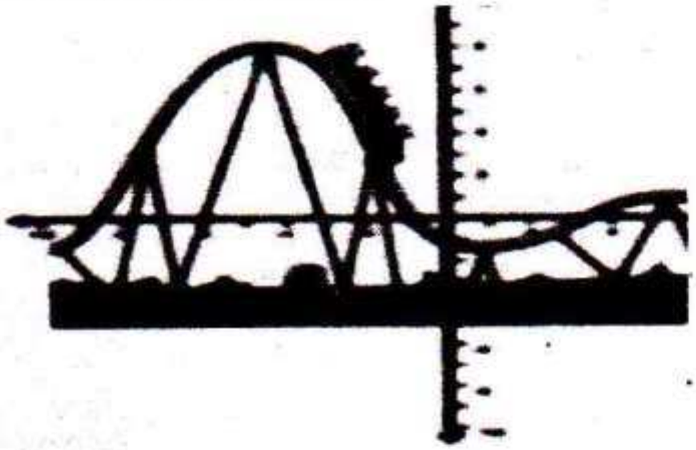
	$\therefore$ d = $\frac{\sqrt{293}}{7}$ **OR** (b)Equation of line AB is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{6}$ Let coordinates of required point on AB be $(2\lambda + 1, 3\lambda + 2, 6\lambda + 3)$ for some λ According to Question $(2\lambda - 2)^2 + (3\lambda - 3)^2 + (6\lambda - 6)^2 = 14^2$ gives $\lambda^2 - 2\lambda - 3 = 0$ Solving we get λ= 3 and -1 $\therefore$ required points are (7, 11, 21) and (– 1, – 1, – 3)	**1** **1** **1** **1** **1** **1**
35.	**35.** Find the area of the region bounded by the curves $x^2 = y$, $y = x + 2$ and x-axis, using integration.	
Ans	**Let** Correct Graph :  x coordinates of point of intersection are – 1, 2 Required area = $\int_{-2}^{-1} (x + 2)\,dx + \int_{-1}^{0} x^2\,dx$ $= \left.\frac{(x+2)^2}{2}\right\vert_{-2}^{-1} + \left.\frac{x^3}{3}\right\vert_{-1}^{0}$ $= \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$	**1 ½** ½ **1½** **1** ½

SECTION-E

(Question nos. 36 to 38 are source based/case based/passage based/integrated units of assessment questions carrying 4 marks each)

36.	The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0·44.  On the basis of the above information, answer the following questions : (i) Find the value of x. (ii) Find the value of y. (iii) (a) Find $P\left(\frac{C}{B}\right)$. **OR** (iii) (b) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C.	
Ans	(i) $x + 0{\cdot}21 = 0{\cdot}44 \Rightarrow x = 0{\cdot}23$ (ii) $0{\cdot}41 + y + 0{\cdot}44 + 0.11 = 1 \Rightarrow y = 0{\cdot}04$ (iii) (a) $P\left(\frac{C}{B}\right) = \frac{P(C \cap B)}{P(B)}$ $P(B) = 0{\cdot}09 + 0{\cdot}04 + 0{\cdot}23 = 0{\cdot}36$ $P\left(\frac{C}{B}\right) = \frac{0{\cdot}23}{0{\cdot}36} = \frac{23}{36}$ **OR** (iii) (b) P(A or B but not C) $= 0{\cdot}32 + 0{\cdot}09 + 0.04$ $= 0{\cdot}45$	1 1 1 1 1 ½ ½

37.	**Case Study – 2** **37.** A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank.  A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of 2 cm^3/s. The semi-vertical angle of the conical tank is 45°. On the basis of given information, answer the following questions : (i) Find the volume of water in the tank in terms of its radius r. (ii) Find rate of change of radius at an instant when $r = 2\sqrt{2}$ cm. (iii) (a) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r = 2\sqrt{2}$ cm. **OR** (iii) (b) Find the rate of change of height 'h' at an instant when slant height is 4 cm.	
Ans	((i) $v = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$ [as θ = 45° gives r = h] (ii) $\frac{dv}{dt} = \pi r^2 \frac{dr}{dt}$ $\Rightarrow \left(\frac{dr}{dt}\right)_{r=2\sqrt{2}} = -\frac{1}{4\pi}$ cm/sec (iii)(a) $C = \pi r l = \pi r\sqrt{2}\, r = \sqrt{2}\,\pi r^2$ $\frac{dC}{dt} = \sqrt{2}\,\pi\, 2r\,\frac{dr}{dt}$ $\left(\frac{dC}{dt}\right)_{r=2\sqrt{2}} = -2\ cm^2/sec$ OR (iii)(b) $l^2 = h^2 + r^2$ $l = 4 \Rightarrow r = h = 2\sqrt{2}$ $h = r \Rightarrow \frac{dh}{dt} = \frac{dr}{dt} = -\frac{1}{4\pi}$ cm/sec	**1** **½** **½** **1** **½** **½** 1 1

38.	**Case Study – 3** **38.** The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y-axis at a point $(0, -1)$, answer the following :  (i) Find the value of 'a'. (ii) Find $f''(x)$ at $x = 1$.	
Ans	(i) $-1 = a(-27) \Rightarrow a = \frac{1}{27}$ (ii) $f(x) = \frac{1}{27}(x + 9)(x + 1)(x - 3)$ $= \frac{1}{27}(x^3 + 7x^2 - 21x - 27)$ $f'(x) = \frac{1}{27}(3x^2 + 14x - 21)$ $f''(x) = \frac{6x + 14}{27}$ $f''(1) = \frac{20}{27}$	**1 + 1** ½ ½ ½ ½