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Senior Secondary School Certificate Examination

September 2021

Marking Scheme — Mathematics XII 65/1/1

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2. "Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/ Website etc may invite action under IPC."
3. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them.
4. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
5. Evaluators will mark (✓) wherever answer is correct. For wrong answer 'X" be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
6. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
7. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
8. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
9. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
10. A full scale of marks __________(example 0-80 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.

11. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
12. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrongquestion wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
13. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
14. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
15. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
16. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
17. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65/1/1
EXPECTED ANSWER/VALUE POINTS
PART A
SECTION-I

1. IF A is a square matrix of order 3 such that $A(\text{adj } A) = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then find |A|.

Ans. $A \cdot (\text{adj } A) = -2I$ $\frac{1}{2}$

$\therefore |A| = -2$ $\frac{1}{2}$

2. (a) Find the order of the matrix A such that

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

Ans. Order of matrix A is 2×2 1

OR

(b) If $B = \begin{bmatrix} 1 & -5 \\ 0 & -3 \end{bmatrix}$ and $A + 2B = \begin{bmatrix} 0 & 4 \\ -7 & 5 \end{bmatrix}$, find the matrix A.

Ans. $A = \begin{bmatrix} 0 & 4 \\ -7 & 5 \end{bmatrix} - \begin{bmatrix} 2 & -10 \\ 0 & -6 \end{bmatrix}$ $\frac{1}{2}$

$A = \begin{bmatrix} -2 & 14 \\ -7 & 11 \end{bmatrix}$ $\frac{1}{2}$

3. Write the smallest reflexive relation on set A = {a, b, c}.

Ans. The smallest reflexive relation is {(a, a), (b, b), (c, c)} 1

4. (a) Find:

$$\int e^x \left(\log \sqrt{x} + \frac{1}{2x} \right) dx$$

Ans. Here $f(x) = \log \sqrt{x}, f'(x) = \frac{1}{2x}$

$$\therefore \int e^x (\log\sqrt{x} + \frac{1}{2x})dx = e^x.\log\sqrt{x} + c$$ 1

OR

(b) Find:

$$\int e^{2\log x}\, dx$$

Ans. $\int e^{2\log x}\, dx = \int x^2\, dx + c$ $\frac{1}{2}$

$$= \frac{x^3}{3} + c$$ $\frac{1}{2}$

5. (a) Find the angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$.

Ans. Let θ be the angle,

$$\therefore \cos\theta = \frac{(\hat{i}-\hat{j})\cdot(\hat{j}-\hat{k})}{|\hat{i}-\hat{j}||\hat{j}-\hat{k}|} = -\frac{1}{2}$$ $\frac{1}{2}$

$$\Rightarrow \theta = \frac{2\pi}{3}$$ $\frac{1}{2}$

OR

(b) Write the projection of the vector $\vec{r} = 3\hat{i} - 4\hat{j} + 12\hat{k}$ on (i) x-axis, and (ii) y-axis.

Ans. (i) Projection of the vector $\vec{r}$ on x-axis = 3 $\frac{1}{2}$

(ii) Projection of the vector $\vec{r}$ on y-axis = 4 $\frac{1}{2}$

6. If $\vec{a} = \alpha\hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} - \beta\hat{k}$, find the value of α and β so that $\vec{a}$ and $\vec{b}$ may be collinear.

Ans. $\vec{a}$ and $\vec{b}$ are collinear.

$$\therefore \frac{\alpha}{2} = \frac{3}{-1} = \frac{-6}{-\beta} \Rightarrow \alpha = -6, \beta = -2$$ $\frac{1}{2} + \frac{1}{2}$

7. If f = {(1, 2), (2, 4), (3, 1), (4, k)} is a one-one function from set A to A, where A = {1, 2, 3, 4}, then find the value of k.

Ans. k = 3 1

8. (a) Check whether the relation R defind on the set {1, 2, 3, 4} as R = {(a, b) : b = a + 1} is transitive. Justify your answer.

Ans. (1, 2), (2, 3) $\in$ R but (1, 3) $\notin$ R $\therefore$ R is not transitive. 1

(Note: Any Similar other pair can be taken to show that R is not a transitive relation.)

OR

(b) If the relation R on the set A = {x : 0 ≤ x ≤ 12} given by R = {(a, b) : a = b} is an equivalenc relation, then find the set of all elements related to 1.

Ans. Set of all elements related to 1 is {1}. 1

9. If A = [1 0 4] and B = $\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$, find AB.

Ans. AB = [26] 1

10. (a) Write the order and degree of the differential equation:

$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$$

Ans. Order = 2 $\frac{1}{2}$

Degree is not defined. $\frac{1}{2}$

OR

(b) Find the general solution of the differential equation $\frac{dy}{dx} = a$, where a is an ab\rbitrary constant.

Ans. $\frac{dy}{dx} = a, \quad \int dy = \int a.dx$ $\frac{1}{2}$

$\Rightarrow y = ax + c$ $\frac{1}{2}$

11. Show that the function $f(x) = \frac{3}{x} + 7$ is strictly decreasing for x $\in$ R – {0}.

Ans. $f'(x) = -\frac{3}{x^2} < 0$ for all x $\in$ R – {0} 1

$\therefore$ f(x) is strictly decreasing.

12. Find the magnitude of vector $\vec{a}$ given by $\vec{b} = (\hat{i}+3\hat{j}-2\hat{k})\times(-\hat{i}+3\hat{k})$.

Ans. $\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ -1 & 0 & 3 \end{vmatrix} = 9\hat{i}-\hat{j}+3\hat{k}$ $\frac{1}{2}$

$\therefore\ |\vec{a}| = \sqrt{91}$ $\frac{1}{2}$

13. Write the equation of the plane that cuts the coordinate axes at (2, 0, 0), (0, 4, 0) and (0, 0, 7).

Ans. Equation of the plane is $\left.\begin{array}{c} \frac{x}{2}+\frac{y}{4}+\frac{z}{7}=1 \\ \text{OR} \\ 14x+7y+4z=28 \end{array}\right\}$ 1

14. Find the distance between the two parallel planes 3x + 5y + 7z = 3 and 9x + 15y + 21z = 12.

Ans. Distance between the two parallel planes = $\left|\frac{4-3}{\sqrt{9+25+49}}\right| = \frac{1}{\sqrt{83}}$ 1

15. If A and B are two independent events and P(A) = $\frac{1}{3}$ and P(B) = $\frac{1}{2}$, find $P(\bar{A}\mid\bar{B})$.

Ans. A and B are independent $\Rightarrow$ $\bar{A}$ and $\bar{B}$ are independent

$\therefore\ P(\bar{A}/\bar{B}) = P(\bar{A}) = 1-P(A) = 1-\frac{1}{3} = \frac{2}{3}$ 1

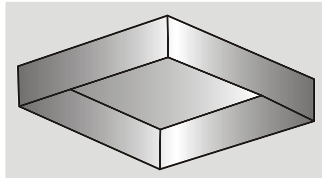
16. A coin is tossed once. If head comes up, a die is thrown, but if tail comes up, the coin is tossed again. Find the probability of obtaining head and number 6.

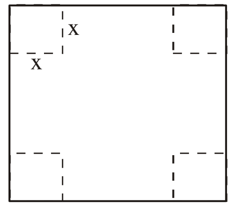
Ans. Probability of obtaining head and number $6 = \frac{1}{2}\times\frac{1}{6} = \frac{1}{12}$ 1

SECTION-II

Both the case study based question (17 & 18) are compulsory. Attempt any 4 subparts out of 5 from each of question number 17 and 18. Each subpart carries 1 mark.

17. A factory makes an open cardboard box for a jewellery shop from a square sheet of side 18 cm by cutting off squares from each corner and folding up the flaps.





Based on the above information, answer any four of the following five question, if x is the length of each square cut from corners.

(i) The volume of the open box is:

(A) $4x\,(x^2 - 18x + 81)$

(B) $2x(2x^2 + 36x + 162)$

(C) $2x(2x^2 + 36x - 162)$

(D) $4x(x^2 + 18x + 81)$

Ans. (A) $4x\,(x^2 - 18x + 81)$ 1

(ii) The condition for the volume (V) to be maximum is:

(A) $\frac{dV}{dx} = 0 \text{ and } \frac{d^2V}{dx^2} < 0$

(B) $\frac{dV}{dx} = 0 \text{ and } \frac{d^2V}{dx^2} > 0$

(C) $\frac{dV}{dx} > 0 \text{ and } \frac{d^2V}{dx^2} = 0$

(D) $\frac{dV}{dx} < 0 \text{ and } \frac{d^2V}{dx^2} = 0$

Ans. (A) $\frac{dV}{dx} = 0 \text{ and } \frac{d^2V}{dx^2} < 0$ 1

(iii) What should be the side of square to be cut off so that the volume is maximum?

(A) 6 cm

(B) 9 cm

(C) 3 cm

(D) 4 cm

Ans. (C) 3 cm 1

(iv) Maximum volume of the open box is:

(A) 423 cm^3

(B) 432 cm^3

(C) 400 cm^3

(D) 216 cm^3

Ans. (B) 432 cm^3 1

(v) The total area of the removed squares is:

(A) 324 cm^2

(B) 144 cm^2

(C) 36 cm^2

(D) 64 cm^2

Ans. (C) 36 cm^2 1

18. In answering a multiple choice test for class XII, a student either knows or guesses or copies the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied is $\frac{1}{8}$.

Let E_1, E_2, E_3 be the events that the student guesses, copies or knows the answer respectively and A is the event that the student answrs correctly.

Based on the above information, answer any four of the following five questions:

(i) What is the probability that the student knows the answer?

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{1}{4}$

Ans. (B) $\frac{1}{2}$ 1

(ii) What is the probability that he answers correctly given that he knew the answer?

(A) 1

(B) 0

(C) $\frac{1}{4}$

(D) $\frac{1}{8}$

Ans. (A) 1 1

(iii) What is the probability that he answers correctly given that he had made a guess?

(A) $\frac{1}{4}$

(B) 0

(C) 1

(D) $\frac{1}{8}$

Ans. (A) $\frac{1}{4}$ 1

(iv) What is the probability that he knew the answer to the question, given that he answered it correctly?

(A) $\frac{24}{29}$

(B) $\frac{4}{29}$

(C) $\frac{1}{29}$

(D) $\frac{3}{29}$

Ans. (A) $\frac{24}{29}$ 1

(v) $\sum_{k=1}^{3} P(E_k \mid A)$ is:

(A) 0

(B) $\frac{1}{3}$

(C) 1

(D) $\frac{11}{8}$

Ans. (C) 1 1

PART B

SECTION-III

Question numbers 19 to 28 carry 2 marks each.

19. A random variable X has the probability distribution:

X	0	1	2	3	4
P(X):	0	K	4K	3K	2K

Find the value of K and $P(X \le 2)$.

Ans. $0 + K + 4K + 3K + 2K = 1 \Rightarrow K = \frac{1}{10}$ 1

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2) = 5K = \frac{1}{2}$ 1

20. Simplify $\sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$.

Ans. Let $x = \cos\theta \quad \therefore\ \theta = \cos^{-1} x$ $\frac{1}{2}$

$\sec^{-1}\left(\frac{1}{2x^2-1}\right) = \sec^{-1}\left(\frac{1}{2\cos^2\theta-1}\right) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = \sec^{-1}(\sec 2\theta)$ $\frac{1}{2}$

$= 2\theta$ $\frac{1}{2}$

$= 2\cos^{-1} x$ $\frac{1}{2}$

21. If the matrix $A = \begin{bmatrix} 0 & 6-5x \\ x^2 & x+3 \end{bmatrix}$ is symmetric, find the values of x.

Ans. Matrix A is symmetric $\Rightarrow x^2 = 6 - 5x$ 1

$\therefore\ x^2 + 5x - 6 = 0 \Rightarrow (x + 6)(x - 1) = 0 \qquad \therefore\ x = -6, 1$ 1

22. (a) Find the relationship between a and b so that the function f defined by

$$f(x) = \begin{cases} ax + 1 & \text{if} \quad x \le 3 \\ bx + 3 & \text{if} \quad x > 3 \end{cases}$$

is continuous at x = 3.

Ans. f is continuous at x = 3 $\Rightarrow \lim_{x\to 3^-}(ax+1) = \lim_{x\to 3^+}(bx+3)$ 1

$\Rightarrow 3a + 1 = 3b + 3$ $\frac{1}{2}$

$\Rightarrow 3a - 3b = 2\left(\text{or } a - b = \frac{2}{3}\right)$ $\frac{1}{2}$

OR

(b) Check the differentiability of f(x) = |x – 3| at x = 3.

Ans. L.H.D. (x = 3) = $\lim_{x\to 3^-}\frac{|x-3|-0}{x-3} = \lim_{x\to 3^-}\frac{-(x-3)}{x-3} = -1$ 1

R.H.D (x = 3) = $\lim_{x\to 3^+}\frac{|x-3|-0}{x-3} = \lim_{x\to 3^+}\frac{x-3}{x-3} = 1$ $\frac{1}{2}$

L.H.D. ≠ R.H.D, $\therefore$ f(x) is not differentiable at x = 3 $\frac{1}{2}$

23. Find:

$$\int \frac{x^2+2}{x^2+1}\,dx$$

Ans. $\int \frac{x^2+2}{x^2+1}\,dx = \int\left(1+\frac{1}{x^2+1}\right)dx = x + \tan^{-1}x + c$ 1+1

24. (a) Evaluate:

$$\int_{-1}^{1}\frac{|x|}{x}\,dx$$

Ans. Let $f(x)=\frac{|x|}{x}$, $\therefore f(-x)=\frac{|-x|}{-x}=-\frac{|x|}{x}=-f(x)$ 1

$\therefore \int_{-1}^{1}\frac{|x|}{x}=0$ 1

OR

(b) Evaluate:

$$\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$$

Ans. Let $I=\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ ...(i)

$\therefore\ I=\int_0^{\pi/2}\log\left(\frac{4+3\sin(\pi/2-x)}{4+3\cos(\pi/2-x)}\right)dx$

$=\int_0^{\pi/2}\log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx$ ...(ii) $\frac{1}{2}$

Adding (i) and (ii), we get

$2I=\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\times\frac{4+3\cos x}{4+3\sin x}\right)dx=\int_0^{\pi/2}\log 1dx=0$ $\frac{1}{2}+\frac{1}{2}$

$\therefore\ I=0$ $\frac{1}{2}$

25. Find the integrating factor of $x\frac{dy}{dx}+(1+x\cot x)y=x$.

Ans. The differential equation can be written as: $\frac{dy}{dx}+\left(\frac{1}{x+\cot x}\right)y=1$ $\frac{1}{2}$

Integrating factor $=e^{\int\left(\frac{1}{x}+\cot x\right)dx}=e^{(\log x+\log\sin x)}=e^{\log(x\sin x)}=x.\sin x$ $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

26. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors, find the value of $|\vec{a}+2\vec{b}+3\vec{c}|$.

Ans. $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$ and $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$ $\frac{1}{2}$

$|\vec{a}+2\vec{b}+3\vec{c}|^2=\vec{a}^2+4\vec{b}^2+9\vec{c}^2=1+4+9=14$ 1

$\therefore$ $|\vec{a}+2\vec{b}+3\vec{c}|=\sqrt{14}$ $\frac{1}{2}$

27. If the side AB and BC of a parallelogram ABCD are represented as vectors $\overrightarrow{AB}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\overrightarrow{BC}=\hat{i}+2\hat{j}+3\hat{k}$, then find the unit vector along diagonal AC.

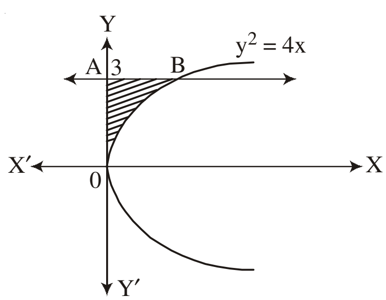
Ans. $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=(2\hat{i}+4\hat{j}-5\hat{k})+(\hat{i}+2\hat{j}+3\hat{k})$

$\therefore$ $\overrightarrow{AC}=3\hat{i}+46-2\hat{k}$ 1

Unit vector along AC $=\dfrac{3\hat{i}+6\hat{j}-2\hat{k}}{|3\hat{i}+6\hat{j}-2\hat{k}|}=\dfrac{3}{7}\hat{i}+\dfrac{6}{7}\hat{j}-\dfrac{2}{7}\hat{k}$ 1

28. (a) Using integration, find the area bounded by the curve $y^2 = 4x$, y-axis and y = 3.

Ans.



Correct Fig. $\frac{1}{2}$

Required area = ar(OAB)

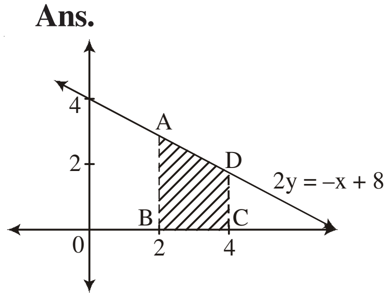
$=\dfrac{1}{4}\displaystyle\int_0^3 y^2\,dy$ $\frac{1}{2}$

$=\dfrac{1}{12}\cdot y^3\Big]_0^3=\dfrac{9}{4}$ sq. units $\frac{1}{2}+\frac{1}{2}$

OR

(b) Using integration, find the area of the region bounded by the line 2y = –x + 8, x-axis, x = 2 and x = 4.

Ans.



Correct Graph $\frac{1}{2}$

Required area = ar(ABCD)

$=\displaystyle\int_2^4 \dfrac{8-x}{2}\,dx$ $\frac{1}{2}$

or 1 if graph is not drawn.

$$= -\frac{1}{4}(8-x)^2\Big]_2^4$$ $\frac{1}{2}$

$$= -\frac{1}{4}(4^2 - 6^2) = 5 \text{ sq. units}$$ $\frac{1}{2}$

SECTION-IV

Question number 29 to 35 carry 3 marks each.

29. Show that the function $f : R - \{-1\} \to R - \{1\}$ given by $f(x) = \frac{x}{x+1}$ is bijective.

Ans. One-One:

$$\left.\begin{array}{l} \text{Let } f(x_1) = f(x_2),\ x_1, x_2 \in R - \{-1\} \\ \Rightarrow \frac{x_1}{x_1+1} = \frac{x_2}{x_2+1} \\ \Rightarrow x_1x_2 + x_1 = x_1x_2 + x_2 \\ \Rightarrow x_1 = x_2 \end{array}\right\}$$ 1

$\therefore$ f is one-one.

Onto: Let any $y \in R - \{1\}$ such that $y = f(x)$

$$\Rightarrow y = \frac{x}{x+1} \Rightarrow x = \frac{y}{1-y}$$ 1

$\therefore$ For each $y \in R - \{1\}$ there exists $x \in R - \{-1\}$. Such that $y = f(x)$ $\frac{1}{2}$

$\therefore$ 'f' is an onto function.

$\therefore$ f is a bijective function. $\frac{1}{2}$

30. (a) If $x = a \cos\theta + b \sin\theta$, $y = a \sin\theta - b \cos\theta$, then show that $\frac{dy}{dx} = -\frac{x}{y}$ and hene show that

$$y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$$

Ans. $x = a \cos\theta + b \sin\theta$, $y = a \sin\theta - b \cos\theta$

$$\frac{dx}{d\theta} = -a\sin\theta + b\sin\theta, \frac{dy}{d\theta} = a\cos\theta + \sin\theta$$ $\frac{1}{2} + \frac{1}{2}$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\cos\theta + b\sin\theta}{-a\sin\theta + b\cos\theta} = -\frac{x}{y}$$ $\frac{1}{2}$

Differentiate $\frac{dy}{dx} = -\frac{x}{y}$ with respect to 'x'

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1.y - x\frac{dy}{dx}}{y^2}$$ 1

$$\Rightarrow y^2\frac{d^2y}{dx^2} = -y + x\frac{dy}{dx}$$

$$\Rightarrow y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$$ $\frac{1}{2}$

OR

(b) If $e^{y-x} = y^x$, prove that $\frac{dy}{dx} = \frac{y(1+\log y)}{x\log y}$

Ans. Taking log on both side of $e^{y-x} = y^x$

$y - x = x \log y$ ...(i) $\frac{1}{2}$

Differentioate with respect to x

$$\frac{dy}{dx} - 1 = \log y + \frac{x}{y}\frac{dy}{dx}$$ $1\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx} = \frac{y(1+\log y)}{y-x} = \frac{y(1+\log y)}{x\log y}$$, using (i) 1

31. Differentiate $\sin^2 x$ w.r.t $e^{\cos x}$.

Ans. Let $u = \sin^2 x$, $v = e^{\cos x}$

$$\frac{du}{dv} = \frac{du/dv}{dv/dx} = \frac{2\sin x.\cos x}{e^{\cos x}.(-\sin x)} = -\frac{2\cos x}{e^{\cos x}}$$ $1+1\frac{1}{2}+\frac{1}{2}$

32. (a) Find the equation of the normal to the curve $y^2 = 4ax$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$.

Ans. $y^2 = 4ax$, differentiating with respect to 'x'

$$2y\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y} \Rightarrow \left.\frac{dy}{dx}\right]_{\left(\frac{a}{m^2},\frac{2a}{m}\right)} = m$$ 1+1

Equation of normal.

$$y - \frac{2a}{m} = -\frac{1}{m}\left(x - \frac{a}{m^2}\right)$$ 1

or $m^2x + m^3y - 2am^2 - a = 0$

OR

(b) Find the equation of the tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses x-axis.

Ans. The point where the curve crosses x-axis is (2, 0) $\frac{1}{2}$

Differentiating, $y(1 + x^2) = 2 - x$ with respect to 'x'

$$\frac{dy}{dx} = \frac{-2xy-1}{1+x^2}$$, slope of tangent at (2, 0) = $-\frac{1}{5}$ $1+\frac{1}{2}$

Equation of tangentt: $y - 0 = -\frac{1}{5}(x-2)$ 1

or $x + 5y - 2 = 0$

33. Find:

$$\int \frac{x^2}{(x-1)(x+1)^2} dx$$

Ans. $$\int \frac{x^2}{(x-1)(x+1)^2} dx = \frac{1}{4}\int \frac{1}{x-1} dx + \frac{3}{4}\int \frac{1}{x+1} dx - \frac{1}{2}\int \frac{1}{(x+1)^2} dx$$ $1\frac{1}{2}$

$$= \frac{1}{4}\log|x-1) + \frac{3}{4}\log|x+1| + \frac{1}{2(x+1)} + c$$ $1\frac{1}{2}$

34. If the solution of the differential equation $\frac{dy}{dx} = \frac{2xy - y^2}{2x^2}$ is $\frac{ax}{y} = b\log|x| + C,$ find the value of a and b.

Ans. The given differential equation can be written as.

$$\frac{dy}{dx} = \frac{y}{x} - \frac{1}{2}\left(\frac{y}{x}\right)^2$$

Put $y = vx,\ \frac{dy}{dx} = v + x\frac{dv}{dx}$, we get $\frac{1}{2}$

$$v + x\frac{dv}{dx} = v - \frac{v^2}{2}$$

$\Rightarrow \quad \frac{1}{v^2}dv = \left(-\frac{1}{2}\right)\frac{1}{x}dx$, integrating both sides $\frac{1}{2}$

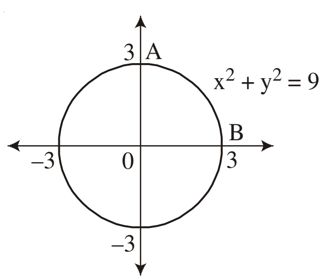
$\Rightarrow \quad -\frac{1}{v} = -\frac{1}{2}\log|x| + c$ $\frac{1}{2}$

$\Rightarrow \quad -\frac{x}{y} = -\frac{1}{2}\log|x| + c,\ \ a = -1,\ \ b = -\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

or $\quad \frac{x}{y} = \frac{1}{2}\log|x| + c, a = 1, b = \frac{1}{2}$

35. Using integration, find the area bounded by the circle $x^2 + y^2 = 9$.

Ans. Area of circle = $4.\text{ar}(AOB) = 4\int_0^3 y\,dx$ 1



$= 4\int_0^3 \sqrt{3^2 - x^2}\,dx$ $\frac{1}{2}$

$= 4\left\{\int \frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right\}\Big]_0^3$ 1

$= 4\left\{0 + \frac{9}{2}\times\frac{\pi}{2}\right\} - 0\Big]_0^3 = 9\pi$ sq. units $\frac{1}{2}$

SECTION-V

Question number 36 to 38 carry 5 marks each.

36. (a) If $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$, find A^{-1}.

Hence, soslve the following system of equations:

$3x + 4y + 2z = 8$

$2y - 3z = 3$

$x - 2y + 6z = -2$

Ans. $|A| = 2.$ $\frac{1}{2}$

co-factors of the elements of the matrix.

$$\left.\begin{array}{l} A_{11} = 6 \;\; A_{12} = -3 \;\; A_{13} = -2 \\ A_{21} = -28 \;\; A_{22} = 16 \;\; A_{23} = 10 \\ A_{31} = -16 \;\; A_{32} = 9 \;\; A_{33} = 6 \end{array}\right\}$$ 2

(1 mark for any 4 correct co-factors)

$$\therefore \; A^{-1} = \frac{1}{|A|}\cdot \operatorname{adj}(A) = \frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix}$$ 1

The given system of equations can be written as

$A \cdot X = B$ $\frac{1}{2}$

$$\text{where, } X = A^{-1}.B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

$\therefore \; x = -2, \; y = 3, \; z = 1$ 1

OR

(b) If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

Ans. $|B| = 1, \; \operatorname{adj}(B) = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ $\frac{1}{2}+2$

$$\therefore \quad B^{-1} = \frac{1}{|B|}\operatorname{adj}(B) = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$ 1

$$(AB)^{-1} = B^{-1} \cdot A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$ $\frac{1}{2}$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$ 1

37. (a) Find the shortest distance between the following lines:

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Ans. Vector equation of the lines:

$$\left.\begin{aligned} &\vec{a}_1 = -\hat{i} - \hat{j} - \hat{k}, \vec{b}_1 = (7\hat{i} - 6\hat{j} + \hat{k}) \\ \text{and} \quad &\vec{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k}, \vec{b}_2 = (\hat{i} - 2\hat{j} + \hat{k}) \end{aligned}\right\}$$ 1

$$\vec{a}_2 - \vec{a}_1 = 4\hat{i} + 6\hat{j} + 8\hat{k}, \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = -4\hat{i} - 6\hat{k} - 8\hat{k}$$ $1+1\frac{1}{2}$

$$\text{Shortest distance} = \left| \frac{(4\hat{i} + 6\hat{j} + 8\hat{k}) \cdot (-4\hat{i} - 6\hat{j} - 8\hat{k})}{|-4\hat{i} - 6\hat{j} - 8\hat{k}|} \right| = \sqrt{116}$$ $1\frac{1}{2}$

OR

(b) Find the distance of the point (–1, –5, –10) from the point of intersection of the line

$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5 \cdot$

Ans. General point on the line is: $\vec{r} = (2 + 3\lambda)\hat{i} + (-1 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}$ $1\frac{1}{2}$

For the point of intersection of the line with plane:

$$1(2 + 3\lambda) - 1(-1 + 4\lambda) + 1(2 + 2\lambda) = 5 \Rightarrow \lambda = 0$$ $1\frac{1}{2}$

$\therefore$ Point of intersection is : (2, –1, 2) 1

$$\text{Distance} = \sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = \sqrt{169} = 13$$ 1

38. (a) Solve the following linear programming problem graphically:

Maximise z = 3x + 9y

subject to constraints

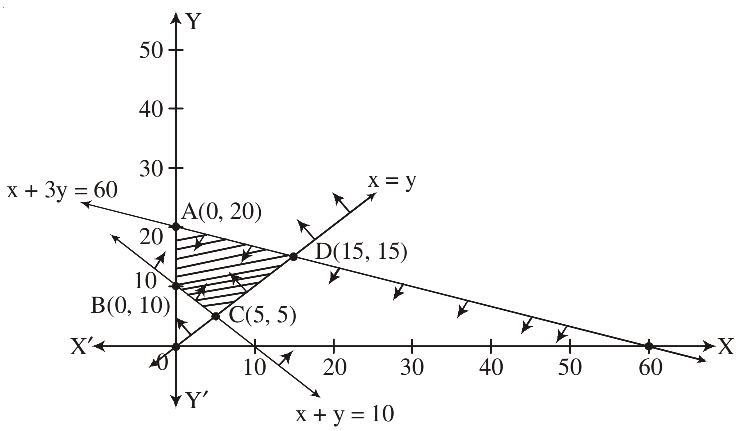
$x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

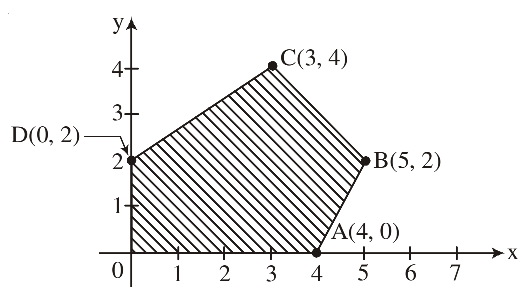
$x, y \geq 0$

Ans.



Correct graph	3
Value of z at corner points	
z(A) = 3(0) + 9(20) = 180	
z(B) = 0 + 90 = 90	
z(C) = 15 + 45 = 60	1
z(D) = 45 + 45 = 180	
Max(z) = 180 at any point on AD.	1

(b) The corner point of the feasible region determined by the system of linear inequations are as shown below:



Answer each of the following:

(i) Let z = 13x – 15y be the objective function. Find the maximum and minimum values of z and also the corresponding points at which the maximum and minimum values occur.

Ans. z(A) = 13(4) – 15(0) = 52

z(B) = 13(5) – 15(2) = 35

z(C) = 13(3) – 15(4) = –21 3

$z(D) = 13(0) - 15(2) = -30$

$z(0) = 0$ $\frac{1}{2}+\frac{1}{2}$

$\therefore$ Max(z) = 52 at A(4, 0), Min(z) = –30 at (0, 2)

(ii) Let z = kx + y be the objective function. Find k, if the value of z at A is same as the value of z at B.

Ans. $z(A) = z(B) \Rightarrow 4k + 0 = 5k + 2 \Rightarrow k = -2$ 1