

QUESTION PAPER CODE 65/1
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|A| = 8.$ 1
2. $k = 12.$ 1
3. $-\log |\sin 2x| + c$ OR $\log |\sec x| - \log |\sin x| + c.$ 1
4. Writing the equations as $\left. \begin{array}{l} 2x - y + 2z = 5 \\ 2x - y + 2z = 8 \end{array} \right\}$ $\frac{1}{2}$
 $\Rightarrow$ Distance = 1 unit $\frac{1}{2}$

SECTION B

5. Any skew symmetric matrix of order 3 is $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ 1
 $\Rightarrow |A| = -a(bc) + a(bc) = 0$ 1

OR

- Since A is a skew-symmetric matrix $\therefore A^T = -A$ $\frac{1}{2}$
 $\therefore |A^T| = |-A| = (-1)^3 |A|$ $\frac{1}{2}$
 $\Rightarrow |A| = -|A|$ $\frac{1}{2}$
 $\Rightarrow 2|A| = 0$ or $|A| = 0.$ $\frac{1}{2}$
6. $f(x) = x^3 - 3x$ $\frac{1}{2}$
 $\therefore f'(c) = 3c^2 - 3 = 0$ $\frac{1}{2}$
 $\therefore c^2 = 1 \Rightarrow c = \pm 1.$ $\frac{1}{2}$
 Rejecting $c = 1$ as it does not belong to $(-\sqrt{3}, 0),$ $\frac{1}{2}$
 we get $c = -1.$ $\frac{1}{2}$

7. Let V be the volume of cube, then $\frac{dV}{dt} = 9 \text{ cm}^3/\text{s}$.

Surface area (S) of cube = $6x^2$, where x is the side.

$$\text{then } V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{1}{3x^2} \cdot \frac{dV}{dt} \quad 1$$

$$S = 6x^2 \Rightarrow \frac{dS}{dt} = 12x \frac{dx}{dt} = 12x \cdot \frac{1}{3x^2} \frac{dV}{dt} \quad \frac{1}{2}$$

$$= 4 \cdot \frac{1}{10} \cdot 9 = 3.6 \text{ cm}^2/\text{s} \quad \frac{1}{2}$$

8. $f(x) = x^3 - 3x^2 + 6x - 100$

$$f'(x) = 3x^2 - 6x + 6 \quad \frac{1}{2}$$

$$= 3[x^2 - 2x + 2] = 3[(x - 1)^2 + 1] \quad 1$$

$$\text{since } f'(x) > 0 \quad \forall x \in \mathbb{R} \quad \therefore f(x) \text{ is increasing on } \mathbb{R} \quad \frac{1}{2}$$

9. Equation of line PQ is $\frac{x-2}{3} = \frac{y-2}{-1} = \frac{z-1}{-3} \quad \frac{1}{2}$

$$\text{Any point on the line is } (3\lambda + 2, -\lambda + 2, -3\lambda + 1) \quad \frac{1}{2}$$

$$3\lambda + 2 = 4 \Rightarrow \lambda = \frac{2}{3} \quad \therefore z \text{ coord.} = -3\left(\frac{2}{3}\right) + 1 = -1. \quad \frac{1}{2} + \frac{1}{2}$$

OR

$$\begin{array}{ccc} \text{P} & \text{R} & \text{Q} \\ (2, 2, 1) & (4, y, z) & (5, 1, -2) \end{array}$$

Let R(4, y, z) lying on PQ divides PQ in the ratio k : 1

$$\Rightarrow 4 = \frac{5k+2}{k+1} \Rightarrow k = 2. \quad 1$$

$$\therefore z = \frac{2(-2)+1(1)}{2+1} = \frac{-3}{3} = -1. \quad 1$$

10. Event A: Number obtained is even

B: Number obtained is red.

$$P(A) = \frac{3}{6} = \frac{1}{2}, P(B) = \frac{3}{6} = \frac{1}{2} \quad \frac{1}{2} + \frac{1}{2}$$

$$P(A \cap B) = P(\text{getting an even red number}) = \frac{1}{6} \quad \frac{1}{2}$$

$$\text{Since } P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \neq P(A \cap B) \text{ which is } \frac{1}{6} \quad \frac{1}{2}$$

$\therefore$ A and B are not independent events.

11. Let A works for x day and B for y days.

$$\therefore \text{ L.P.P. is Minimize } C = 300x + 400y \quad \frac{1}{2}$$

$$\text{Subject to: } \begin{cases} 6x + 10y \geq 60 \\ 4x + 4y \geq 32 \\ x \geq 0, y \geq 0 \end{cases} \quad 1 \frac{1}{2}$$

$$12. \int \frac{dx}{5 - 8x - x^2} = \int \frac{dx}{(\sqrt{21})^2 - (x + 4)^2} \quad 1$$

$$= \frac{1}{2\sqrt{21}} \log \left| \frac{\sqrt{21} + (x + 4)}{\sqrt{21} - (x + 4)} \right| + c \quad 1$$

SECTION C

$$13. \tan^{-1} \frac{x-3}{x-4} + \tan^{-1} \frac{x+3}{x+4} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{x-3}{x-4} + \frac{x+3}{x+4}}{1 - \frac{x-3}{x-4} \cdot \frac{x+3}{x+4}} \right) = \frac{\pi}{4} \quad 1 \frac{1}{2}$$

$$\Rightarrow \frac{2x^2 - 24}{-7} = 1 \Rightarrow x^2 = \frac{17}{2} \quad 1 \frac{1}{2}$$

$$\Rightarrow x = \pm \sqrt{\frac{17}{2}} \quad 1$$

$$14. \Delta = \begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$$R_1 \rightarrow R_1 - R_2 \text{ and } R_2 \rightarrow R_2 - R_3$$

$$\Delta = \begin{vmatrix} a^2 - 1 & a - 1 & 0 \\ 2(a - 1) & a - 1 & 0 \\ 3 & 3 & 1 \end{vmatrix} \quad 1+1$$

$$= (a - 1)^2 \begin{vmatrix} a + 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix} \quad 1$$

Expanding

$$(a - 1)^2 \cdot (a - 1) = (a - 1)^3. \quad 1$$

OR

$$\text{Let } \begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix} \quad 1$$

$$\Rightarrow \begin{pmatrix} 2a - c & 2b - d \\ a & b \\ -3a + 4c & -3b + 4d \end{pmatrix} = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix} \quad 1$$

$$\Rightarrow 2a - c = -1, \quad 2b - d = -8$$

$$a = 1, \quad b = -2 \quad 1$$

$$-3a + 4c = 9, \quad -3b + 4d = 22$$

Solving to get $a = 1, b = -2, c = 3, d = 4$

$$\therefore A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} \quad 1$$

$$15. x^y + y^x = a^b$$

Let $u + v = a^b$, where $x^y = u$ and $y^x = v$.

$$\therefore \frac{du}{dx} + \frac{dv}{dx} = 0 \quad \dots(i) \quad \frac{1}{2}$$

$$y \log x = \log u \Rightarrow \frac{du}{dx} = x^y \left[\frac{y}{x} + \log x \cdot \frac{dy}{dx} \right] \quad 1$$

$$x \log y = \log v \Rightarrow \frac{dv}{dx} = y^x \left[\frac{x}{y} \frac{dy}{dx} + \log y \right] \quad 1$$

$$\text{Putting in (i) } x^y \left[\frac{y}{x} + \log x \frac{dy}{dx} \right] + y^x \left[\frac{x}{y} \frac{dy}{dx} + \log y \right] = 0 \quad \frac{1}{2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y^x \log y + y \cdot x^{y-1}}{x^y \cdot \log x + x \cdot y^{x-1}} \quad 1$$

OR

$$e^y \cdot (x+1) = 1 \Rightarrow e^y \cdot 1 + (x+1) \cdot e^y \cdot \frac{dy}{dx} = 0 \quad 1 \frac{1}{2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{1}{(x+1)} \quad 1$$

$$\frac{d^2y}{dx^2} = +\frac{1}{(x+1)^2} = \left(\frac{dy}{dx} \right)^2 \quad 1 \frac{1}{2}$$

$$16. \quad I = \int \frac{\cos \theta}{(4 + \sin^2 \theta)(5 - 4 \cos^2 \theta)} d\theta = \int \frac{\cos \theta}{(4 + \sin^2 \theta)(1 + 4 \sin^2 \theta)} d\theta \quad \frac{1}{2}$$

$$= \int \frac{dt}{(4 + t^2)(1 + 4t^2)}, \text{ where } \sin \theta = t \quad 1$$

$$= \int \frac{-\frac{1}{15}}{4 + t^2} dt + \int \frac{\frac{4}{15}}{1 + 4t^2} dt \quad 1$$

$$= -\frac{1}{30} \tan^{-1} \left(\frac{t}{2} \right) + \frac{4}{30} \tan^{-1}(2t) + c \quad 1$$

$$= -\frac{1}{30} \tan^{-1} \left(\frac{\sin \theta}{2} \right) + \frac{2}{15} \tan^{-1}(2 \sin \theta) + c \quad \frac{1}{2}$$

$$17. \quad I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx = \int_0^{\pi} \frac{(\pi - x) \tan x}{\sec x + \tan x} dx \quad 1$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} dx = \pi \int_0^{\pi} \tan x (\sec x - \tan x) dx$$

$$I = \frac{\pi}{2} \int_0^{\pi} (\sec x \tan x - \sec^2 x + 1) dx \quad 1$$

$$= \frac{\pi}{2} [\sec x - \tan x + x]_0^{\pi} \quad 1$$

$$= \frac{\pi(\pi - 2)}{2} \quad 1$$

OR

$$I = \int_1^4 \{|x - 1| + |x - 2| + |x - 4|\} dx$$

$$= \int_1^4 (x - 1) dx - \int_1^2 (x - 2) dx + \int_2^4 (x - 2) dx - \int_1^4 (x - 4) dx \quad 2$$

$$= \left[\frac{(x - 1)^2}{2} \right]_1^4 - \left[\frac{(x - 2)^2}{2} \right]_1^2 + \left[\frac{(x - 2)^2}{2} \right]_2^4 - \left[\frac{(x - 4)^2}{2} \right]_1^4 \quad 1$$

$$= \frac{9}{2} + \frac{1}{2} + 2 + \frac{9}{2} = 11\frac{1}{2} \text{ or } \frac{23}{2} \quad 1$$

18. Given differential equation can be written as

$$(1 + x^2) \frac{dy}{dx} + y = \tan^{-1} x \Rightarrow \frac{dy}{dx} + \frac{1}{1 + x^2} y = \frac{\tan^{-1} x}{1 + x^2} \quad 1$$

$$\text{Integrating factor} = e^{\tan^{-1} x}. \quad 1$$

$$\therefore \text{Solution is } y \cdot e^{\tan^{-1} x} = \int \tan^{-1} x \cdot e^{\tan^{-1} x} \frac{1}{1 + x^2} dx \quad 1$$

$$\Rightarrow y \cdot e^{\tan^{-1} x} = e^{\tan^{-1} x} \cdot (\tan^{-1} x - 1) + c \quad 1$$

$$\text{or } y = (\tan^{-1} x - 1) + c \cdot e^{-\tan^{-1} x}$$

19. $\overrightarrow{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}, \overrightarrow{BC} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$ 1

Since $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}$, are not parallel vectors, and $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0} \therefore A, B, C$ form a triangle 1

Also $\overrightarrow{BC} \cdot \overrightarrow{CA} = 0 \therefore A, B, C$ form a right triangle 1

Area of $\Delta = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{BC}| = \frac{1}{2} \sqrt{210}$ 1

20. Given points, A, B, C, D are coplanar, if the

vectors $\overrightarrow{AB}, \overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar, i.e.

$\overrightarrow{AB} = -2\hat{i} - 4\hat{j} - 6\hat{k}, \overrightarrow{AC} = -\hat{i} - 3\hat{j} - 8\hat{k}, \overrightarrow{AD} = \hat{i} + (\lambda - 9)\hat{k}$ $1\frac{1}{2}$

are coplanar

i.e., $\begin{vmatrix} -2 & -4 & -6 \\ -1 & -3 & -8 \\ 1 & 0 & \lambda - 9 \end{vmatrix} = 0$ 1

$-2[-3\lambda + 27] + 4[-\lambda + 17] - 6(3) = 0$ 1

$\Rightarrow \lambda = 2.$ $\frac{1}{2}$

21. Writing

+	1	3	5	7
1	×	4	6	8
3	4	×	8	10
5	6	8	×	12
7	8	10	12	×

$\therefore X :$ 4 6 8 10 12 1

$P(X) :$ $\frac{2}{12}$ $\frac{2}{12}$ $\frac{4}{12}$ $\frac{2}{12}$ $\frac{2}{12}$

$= \frac{1}{6}$ $\frac{1}{6}$ $\frac{2}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ 1

$xP(X) :$ $\frac{4}{6}$ $\frac{6}{6}$ $\frac{16}{6}$ $\frac{10}{6}$ $\frac{12}{6}$

$x^2P(X) :$ $\frac{16}{6}$ $\frac{36}{6}$ $\frac{128}{6}$ $\frac{100}{6}$ $\frac{144}{6}$

$$\Sigma xP(x) = \frac{48}{6} = 8 \therefore \text{Mean} = 8$$

1

$$\text{Variance} = \Sigma x^2 P(x) - [\Sigma xP(x)]^2 = \frac{424}{6} - 64 = \frac{20}{3}$$

1

22. Let E_1 : Selecting a student with 100% attendance
 E_2 : Selecting a student who is not regular

1

A: selected student attains A grade.

$$P(E_1) = \frac{30}{100} \text{ and } P(E_2) = \frac{70}{100}$$

 $\frac{1}{2}$

$$P(A/E_1) = \frac{70}{100} \text{ and } P(A/E_2) = \frac{10}{100}$$

 $\frac{1}{2}$

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

$$= \frac{\frac{30}{100} \times \frac{70}{100}}{\frac{30}{100} \times \frac{70}{100} + \frac{70}{100} \times \frac{10}{100}}$$

$$= \frac{3}{4}$$

1

Regularity is required everywhere or any relevant value

1

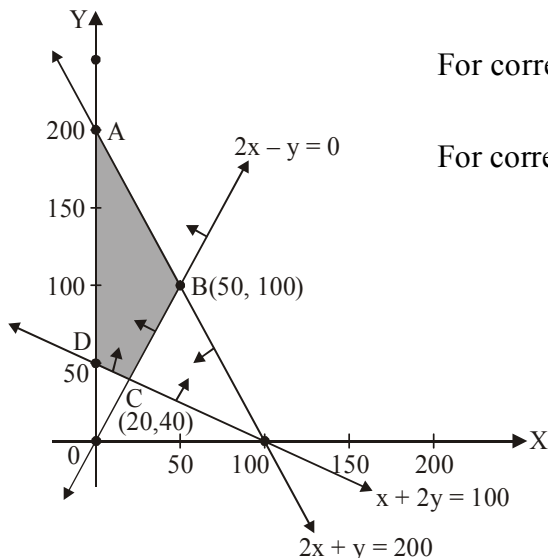
23.

$$Z = x + 2y \text{ s.t } x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200, x, y \geq 0$$

For correct graph of three lines

 $\frac{1}{2}$

For correct shading

 $\frac{1}{2}$ 

$$Z(A) = 0 + 400 = 400$$

$$Z(B) = 50 + 200 = 250$$

$$Z(C) = 20 + 80 = 100$$

$$Z(D) = 0 + 100 = 100$$

$$\therefore \text{Max (= 400) at } x = 0, y = 200$$

1

SECTION D

24. Getting $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$... (i) $1 \frac{1}{2}$

Given equations can be written as $\begin{pmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$ 1

$\Rightarrow AX = B$

From (i) $A^{-1} = \frac{1}{8} \begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix}$ 1

$\therefore X = A^{-1}B = \frac{1}{8} \begin{pmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 9 \\ 1 \end{pmatrix}$ 1

$= \frac{1}{8} \begin{pmatrix} 24 \\ -16 \\ -8 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ 1

$\Rightarrow x = 3, y = -2, z = -1$ $\frac{1}{2}$

25. Let $x_1, x_2 \in \mathbb{R} - \left\{ -\frac{4}{3} \right\}$ and $f(x_1) = f(x_2)$ $\frac{1}{2}$

$\Rightarrow \frac{4x_1 + 3}{3x_1 + 4} = \frac{4x_2 + 3}{3x_2 + 4} \Rightarrow (4x_1 + 3)(3x_2 + 4) = (3x_1 + 4)(4x_2 + 3)$

$\Rightarrow 12x_1x_2 + 16x_1 + 9x_2 + 12 = 12x_1x_2 + 16x_2 + 9x_1 + 12$

$\Rightarrow 16(x_1 - x_2) - 9(x_1 - x_2) = 0 \Rightarrow x_1 - x_2 = 0 \Rightarrow x_1 = x_2$

Hence f is a 1-1 function 2

Let $y = \frac{4x + 3}{3x + 4}$, for $y \in \mathbb{R} - \left\{ \frac{4}{3} \right\}$

$3xy + 4y = 4x + 3 \Rightarrow 4x - 3xy = 4y - 3$

$\Rightarrow x = \frac{4y - 3}{4 - 3y} \therefore \forall y \in \mathbb{R} - \left\{ \frac{4}{3} \right\}, x \in \mathbb{R} - \left\{ -\frac{4}{3} \right\}$

Hence f is ONTO and so bijective

2

$$\text{and } f^{-1}(y) = \frac{4y-3}{4-3y}; y \in \mathbb{R} - \left\{\frac{4}{3}\right\}$$

1

$$f^{-1}(0) = -\frac{3}{4}$$

 $\frac{1}{2}$

$$\text{and } f^{-1}(x) = 2 \Rightarrow \frac{4x-3}{4-3x} = 2$$

$$\Rightarrow 4x - 3 = 8 - 6x$$

$$\Rightarrow 10x = 11 \Rightarrow x = \frac{11}{10}$$

 $\frac{1}{2}$

OR

$$(a, b) * (c, d) = (ac, b + ad); (a, b), (c, d) \in A$$

$$(c, d) * (a, b) = (ca, d + bc)$$

Since $b + ad \neq d + bc \Rightarrow *$ is NOT comutative

 $1\frac{1}{2}$

for associativity, we have,

$$[(a, b) * (c, d)] * (e, f) = (ac, b + ad) * (e, f) = (ace, b + ad + acf)$$

$$(a, b) * [(c, d) * (e, f)] = (a, b) * (ce, d + cf) = (ace, b + ad + acf)$$

 $1\frac{1}{2}$

$\Rightarrow *$ is associative

(i) Let (e, f) be the identity element in A

$$\text{Then } (a, b) * (e, f) = (a, b) = (e, f) * (a, b)$$

$$\Rightarrow (ae, b + af) = (a, b) = (ae, f + be)$$

$$\Rightarrow e = 1, f = 0 \Rightarrow (1, 0) \text{ is the identity element}$$

 $1\frac{1}{2}$

(ii) Let (c, d) be the inverse element for (a, b)

$$\Rightarrow (a, b) * (c, d) = (1, 0) = (c, d) * (a, b)$$

$$\Rightarrow (ac, b + ad) = (1, 0) = (ac, d + bc)$$

$$\Rightarrow ac = 1 \Rightarrow c = \frac{1}{a} \text{ and } b + ad = 0 \Rightarrow d = -\frac{b}{a} \text{ and } d + bc = 0 \Rightarrow d = -bc = -b\left(\frac{1}{a}\right)$$

$$\Rightarrow \left(\frac{1}{a}, -\frac{b}{a}\right), a \neq 0 \text{ is the inverse of } (a, b) \in A$$

 $1\frac{1}{2}$

26. Let the sides of cuboid be x, x, y

$$\Rightarrow x^2y = k \text{ and } S = 2(x^2 + xy + xy) = 2(x^2 + 2xy)$$

$$\frac{1}{2} + 1$$

$$\therefore S = 2 \left[x^2 + 2x \frac{k}{x^2} \right] = 2 \left[x^2 + \frac{2k}{x} \right]$$

$$1$$

$$\frac{ds}{dx} = 2 \left[2x - \frac{2k}{x^2} \right]$$

$$1$$

$$\therefore \frac{ds}{dx} = 0 \Rightarrow x^3 = k = x^2y \Rightarrow x = y$$

$$1$$

$$\frac{d^2s}{dx^2} = 2 \left[2 + \frac{4k}{x^3} \right] > 0 \therefore x = y \text{ will give minimum surface area}$$

$$1$$

and $x = y$, means sides are equal

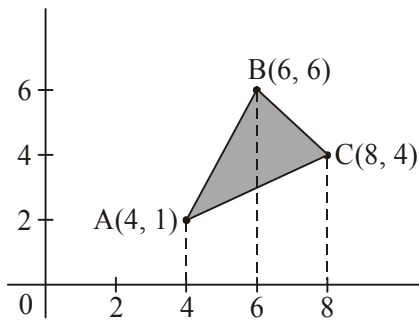
$\therefore$ Cube will have minimum surface area

$$\frac{1}{2}$$

27.

Figure

$$1$$



$$\left. \begin{array}{l} \text{Equation of AB : } y = \frac{5}{2}x - 9 \\ \text{Equation of BC : } y = 12 - x \\ \text{Equation of AC : } y = \frac{3}{4}x - 2 \end{array} \right\}$$

$$1 \frac{1}{2}$$

$$\therefore \text{Area (A)} = \int_4^6 \left(\frac{5}{2}x - 9 \right) dx + \int_6^8 (12 - x) dx - \int_4^8 \left(\frac{3}{4}x - 2 \right) dx$$

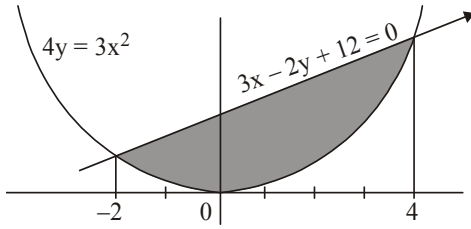
$$1$$

$$= \left[\frac{5}{4}x^2 - 9x \right]_4^6 + \left[12x - \frac{x^2}{2} \right]_6^8 - \left[\frac{3}{8}x^2 - 2x \right]_4^8$$

$$1 \frac{1}{2}$$

$$= 7 + 10 - 10 = 7 \text{ sq.units}$$

$$1$$



Figure

1

$$4y = 3x^2 \text{ and } 3x - 2y + 12 = 0 \Rightarrow 4\left(\frac{3x+12}{2}\right) = 3x^2$$

$$\Rightarrow 3x^2 - 6x - 24 = 0 \text{ or } x^2 - 2x - 8 = 0 \Rightarrow (x-4)(x+2) = 0$$

$$\Rightarrow \text{x-coordinates of points of intersection are } x = -2, x = 4$$

1

$$\therefore \text{Area (A)} = \int_{-2}^4 \left[\frac{1}{2}(3x+12) - \frac{3}{4}x^2 \right] dx \quad 1 \frac{1}{2}$$

$$= \left[\frac{1}{2} \frac{(3x+12)^2}{6} - \frac{3}{4} \frac{x^3}{3} \right]_{-2}^4 \quad 1 \frac{1}{2}$$

$$= 45 - 18 = 27 \text{ sq. units} \quad 1$$

$$28. \quad \frac{dy}{dx} = \frac{x+2y}{x-y} = \frac{1+\frac{2y}{x}}{1-\frac{y}{x}} \quad \frac{1}{2}$$

$$\frac{y}{x} = v \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx} \quad \therefore v + x \frac{dv}{dx} = \frac{1+2v}{1-v} \quad \frac{1}{2}$$

$$\Rightarrow x \frac{dv}{dx} = -\frac{1+2v-v+v^2}{v-1} \Rightarrow \int \frac{v-1}{v^2+v+1} dv = -\frac{dx}{x} \quad 1$$

$$\Rightarrow \int \frac{2v+1-3}{v^2+v+1} dv = \int -\frac{2}{x} dx \Rightarrow \int \frac{2v+1}{v^2+v+1} dv - 3 \int \frac{1}{\left(v+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dv = -\int \frac{2}{x} dx \quad 1+1$$

$$\Rightarrow \log |v^2+v+1| - 3 \cdot \frac{2}{\sqrt{3}} \tan^{-1} \left(\frac{2v+1}{\sqrt{3}} \right) = -\log |x|^2 + c \quad 1$$

$$\Rightarrow \log |y^2+xy+x^2| - 2\sqrt{3} \tan^{-1} \left(\frac{2y+x}{\sqrt{3}x} \right) = c \quad \frac{1}{2}$$

$$x = 1, y = 0 \Rightarrow c = -2\sqrt{3} \cdot \frac{\pi}{6} = -\frac{\sqrt{3}}{3} \pi \quad \frac{1}{2}$$

$$\therefore \log |y^2+xy+x^2| - 2\sqrt{3} \tan^{-1} \left(\frac{2y+x}{\sqrt{3}x} \right) + \frac{\sqrt{3}}{3} \pi = 0$$

29. Equation of line through $(3, -4, -5)$ and $(2, -3, 1)$ is

$$\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} \quad \dots(i) \quad 1$$

Eqn. of plane through the three given points is

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 3 & 0 & -6 \\ -1 & 2 & 0 \end{vmatrix} = 0 \Rightarrow (x-1)(12) - (y-2)(-6) + (z-3)(6) = 0$$

$$\text{or } 2x + y + z - 7 = 0 \quad \dots(ii) \quad 2$$

Any point on line (i) is $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$ 1

If this point lies on plane, then $2(-\lambda + 3) + (\lambda - 4) + (6\lambda - 5) - 7 = 1$

$$\Rightarrow \lambda = 2 \quad 1$$

Required point is $(1, -2, 7)$ 1

OR

Equation of plane cutting intercepts (say, a, b, c) on the axes is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \text{ with } A(a, 0, 0), B(0, b, 0) \text{ and } C(0, 0, c) \quad 1$$

$$\text{distance of this plane from origin is } 3p = \frac{|-1|}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2 + \left(\frac{1}{c}\right)^2}} \quad 1 \frac{1}{2}$$

$$\Rightarrow \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{9p^2} \quad \dots(i) \quad 1$$

$$\text{Centroid of } \triangle ABC \text{ is } \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right) = (x, y, z) \quad 1$$

$$\Rightarrow a = 3x, b = 3y, c = 3z, \text{ we get from (i)} \quad \frac{1}{2}$$

$$\frac{1}{9x^2} + \frac{1}{9y^2} + \frac{1}{9z^2} = \frac{1}{9p^2} \text{ or } \frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2} \quad 1$$