MARKING SCHEME

CLASS XII

MATHEMATICS (CODE-041)

SECTION: A (Solution of MCQs of 1 Mark each)

Q no.	ANS	HINTS/SOLUTION
1.	**(D)**	For a square matrix A of order $n\times n$, we have $A.(adj\,A)=\|A\|I_n$, where I_n is the identity matrix of order $n\times n$. So, $A.(adj\,A)=\begin{bmatrix}2025 & 0 & 0\\ 0 & 2025 & 0\\ 0 & 0 & 2025\end{bmatrix}=2025I_3 \quad \Rightarrow \|A\|=2025 \quad \& \quad \|adj\,A\|=\|A\|^{3-1}=(2025)^2$ $\therefore \|A\|+\|adj\,A\|=2025+(2025)^2$.
2.	**(A)**	$P \xrightarrow{\text{Order}} p\times k$ $\quad$ $Y \xrightarrow{\text{Order}} 3\times k$ For PY to exist $k=3$ Order of $PY=p\times k$ $W \xrightarrow{\text{Order}} n\times 3$ $\quad$ $Y \xrightarrow{\text{Order}} 3\times k$ Order of $WY = n\times k$ For $PY+WY$ to exist $\text{order}(PY)=\text{order}(WY)$ $\therefore p=n$
3.	**(C)**	$y=e^x \implies \frac{dy}{dx}=e^x$ In the domain (R) of the function, $\frac{dy}{dx}>0$, hence the function is strictly increasing in $(-\infty,\infty)$
4.	**(B)**	$\|A\|=5,\ \|B^{-1}AB\|^2=\left(\|B^{-1}\|\|A\|\|B\|\right)^2=\|A\|^2=5^2$.
5.	**(B)**	A differential equation of the form $\frac{dy}{dx}=f(x,y)$ is said to be homogeneous, if $f(x,y)$ is a homogeneous function of degree 0. Now, $x^n\frac{dy}{dx}=y\left(\log_e\frac{y}{x}+\log_e e\right)\Rightarrow\frac{dy}{dx}=\frac{y}{x^n}\left(\log_e e.\left(\frac{y}{x}\right)\right)=f(x,y);(Let)$. $f(x,y)$ will be a homogeneous function of degree **0**, if $n=1$.
6.	**(A)**	**Method 1:** (***Short cut***) When the points $(x_1,y_1),(x_2,y_2)$ and (x_1+x_2,y_1+y_2) are collinear in the Cartesian plane then $\begin{vmatrix}x_1-x_2 & y_1-y_2\\ x_1-(x_1+x_2) & y_1-(y_1+y_2)\end{vmatrix}=0\Rightarrow\begin{vmatrix}x_1-x_2 & y_1-y_2\\ -x_2 & -y_2\end{vmatrix}=(-x_1y_2+x_2y_2+x_2y_1-x_2y_2)=0$ $\Rightarrow x_2y_1=x_1y_2$.

		Method 2: When the points $(x_1, y_1),(x_2, y_2)$ and (x_1+x_2, y_1+y_2) are collinear in the Cartesian plane then $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_1+x_2 & y_1+y_2 & 1 \end{vmatrix} = 0$ $\Rightarrow 1.(x_2y_1 + x_2y_2 - x_1y_2 - x_2y_2) - 1(x_1y_1 + x_1y_2 - x_1y_1 - x_2y_1) + (x_1y_2 - x_2y_1) = 0$ $\Rightarrow x_2y_1 = x_1y_2$.
7.	(A)	$A = \begin{bmatrix} 0 & 1 & c \\ -1 & a & -b \\ 2 & 3 & 0 \end{bmatrix}$ When the matrix A is skew symmetric then $A^T = -A \Rightarrow a_{ij} = -a_{ji}$; $\Rightarrow c = -2; a = 0$ and $b = 3$ So , $a+b+c = 0+3-2 = 1$.
8.	(C)	$P(\overline{A}) = \frac{1}{2}; P(\overline{B}) = \frac{2}{3}; P(A \cap B) = \frac{1}{4}$ $\Rightarrow P(A) = \frac{1}{2}; P(B) = \frac{1}{3}$ We have, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{7}{12}$ $P\left(\frac{\overline{A}}{\overline{B}}\right) = \frac{P(\overline{A} \cap \overline{B})}{P(\overline{B})} = \frac{P(\overline{A \cup B})}{P(\overline{B})} = \frac{1 - P(A \cup B)}{P(\overline{B})} = \frac{1 - \frac{7}{12}}{\frac{2}{3}} = \frac{5}{8}$.
9.	(B)	For obtuse angle, $\cos\theta < 0 \Rightarrow \vec{p}.\vec{q} < 0$ $2\alpha^2 - 3\alpha + \alpha < 0 \Rightarrow 2\alpha^2 - 2\alpha < 0 \Rightarrow \alpha \in (0, 1)$
10.	(C)	$\lvert\vec{a}\rvert = 3, \lvert\vec{b}\rvert = 4, \lvert\vec{a}+\vec{b}\rvert = 5$ We have , $\lvert\vec{a}+\vec{b}\rvert^2 + \lvert\vec{a}-\vec{b}\rvert^2 = 2\left(\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2\right) = 2(9+16) = 50 \Rightarrow \lvert\vec{a}-\vec{b}\rvert = 5$.
11.	(B)	<table><tr><td>Corner point</td><td>Value of the objective function $Z = 4x + 3y$</td></tr><tr><td>1. $O(0,0)$</td><td>$z = 0$</td></tr><tr><td>2. $R(40,0)$</td><td>$z = 160$</td></tr><tr><td>3. $Q(30,20)$</td><td>$z = 120 + 60 = 180$</td></tr><tr><td>4. $P(0,40)$</td><td>$z = 120$</td></tr></table> Since , the feasible region is bounded so the maximum value of the objective function $z = 180$ is at $Q(30,20)$.

12.	**(A)**	$\int \frac{dx}{x^3(1+x^4)^{\frac{1}{2}}} = \int \frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{\frac{1}{2}}}$ (Let $1 + x^{-4} = 1 + \frac{1}{x^4} = t, dt = -4x^{-5}dx = -\frac{4}{x^5}dx \Rightarrow \frac{dx}{x^5} = -\frac{1}{4}dt$) $= -\frac{1}{4}\int \frac{dt}{t^{\frac{1}{2}}} = -\frac{1}{4} \times 2 \times \sqrt{t} + c$, where $'c'$ denotes any arbitrary constant of integration. $= -\frac{1}{2}\sqrt{1+\frac{1}{x^4}} + c = -\frac{1}{2x^2}\sqrt{1+x^4} + c$
13.	**(A)**	We know, $\int_0^{2a} f(x)dx = 0,$ **if** $f(2a-x) = -f(x)$ Let $f(x) = \operatorname{cosec}^7 x$. Now, $f(2\pi - x) = \operatorname{cosec}^7(2\pi - x) = -\operatorname{cosec}^7 x = -f(x)$ $\therefore \int_0^{2\pi} \operatorname{cosec}^7 x\, dx = 0;$ Using the property $\int_0^{2a} f(x)dx = 0,$ **if** $f(2a-x) = -f(x)$.
14.	**(B)**	The given differential equation $e^{y'} = x => \frac{dy}{dx} = \log x$ $dy = \log x\, dx \;\; => \int dy = \int \log x\, dx$ $y = x\log x - x + c$ hence the correct option is **(B)**.
15.	**(B)**	The graph represents $y = \cos^{-1} x$ whose domain is $[-1,1]$ and range is $[0,\pi]$.
16.	**(D)**	Since the inequality $Z = 18x + 10y < 134$ has no point in common with the feasible region hence the minimum value of the **objective function** $Z = 18x + 10y$ is 134 at $P(3,8)$.
17.	**(D)**	The graph of the function $f: R \to R$ defined by $f(x) = [x]$; (**where**$[.]$**denotes** $G.I.F$) is a straight line $\forall x \in (2.5-h, 2.5+h)$, $'h'$ is an infinitesimally small positive quantity. Hence, the function is continuous and differentiable at $x = 2.5$.
18.	**(B)**	**The required region is symmetric about the** $y-$ axis. So, required area (in sq units) is $= \left\|2\int_0^4 2\sqrt{y}dy\right\| = 4\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 = \frac{64}{3}$.
19.	**(A)**	**Both (A) and (R) are true and (R) is the correct explanation of (A).**
20.	**(A)**	**Both (A) and (R) are true and (R) is the correct explanation of (A).**

Section –B

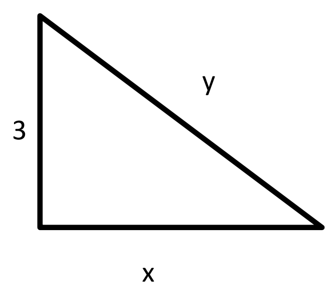
[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]

21	$\cot^{-1}(3x+5) > \frac{\pi}{4} = \cot^{-1} 1$ $\Rightarrow 3x + 5 < 1$ (as $\cot^{-1}x$ is strictly decreasing function in its domain) $\Rightarrow 3x < -4$ $\Rightarrow x < -\frac{4}{3}$ $\therefore x \in \left(-\infty, -\frac{4}{3}\right)$	$\frac{1}{2}$ $\frac{1}{2}$ 1
22.	The marginal cost function is $C'(x) = 0.00039x^2 + 0.004x + 5$. $C'(150) =$ ₹ 14.375 .	1 1
23.(a)	$y = \tan^{-1} x$ and $z = \log_e x$. Then $\frac{dy}{dx} = \frac{1}{1+x^2}$ and $\frac{dz}{dx} = \frac{1}{x}$ So, $\frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}}$ $= \frac{\frac{1}{1+x^2}}{\frac{1}{x}} = \frac{x}{1+x^2}$.	 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
OR 23.(b)	Let $y = (\cos x)^x$. Then, $y = e^{x \log_e \cos x}$ On differentiating both sides with respect to x, we get $\frac{dy}{dx} = e^{x\log_e \cos x} \frac{d}{dx}(x \log_e \cos x)$ $\Rightarrow \frac{dy}{dx} = (\cos x)^x \left\{\log_e \cos x \frac{d}{dx}(x) + x\frac{d}{dx}(\log_e \cos x)\right\}$ $\Rightarrow \frac{dy}{dx} = (\cos x)^x \left\{\log_e \cos x + x.\frac{1}{\cos x}(-\sin x)\right\} \Rightarrow \frac{dy}{dx} = (\cos x)^x(\log_e \cos x - x\tan x)$.	 $\frac{1}{2}$ $\frac{1}{2}$ 1
24.(a)	We have $\vec{b} + \lambda\vec{c} = (-1+3\lambda)\hat{\imath} + (2+\lambda)\hat{\jmath} + \hat{k}$ $(\vec{b} + \lambda\vec{c}).\vec{a} = 0 \Rightarrow 2(-1+3\lambda) + 2(2+\lambda) + 3 = 0$ $\lambda = -\frac{5}{8}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
OR 24.(b)	$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = (4\hat{\imath} + 3\hat{k}) - \hat{k} = 4\hat{\imath} + 2\hat{k}$	$\frac{1}{2}$

	$\widehat{BA} = \frac{4}{2\sqrt{5}}\hat{\imath} + \frac{2}{2\sqrt{5}}\hat{k} = \frac{2}{\sqrt{5}}\hat{\imath} + \frac{1}{\sqrt{5}}\hat{k}$ So, the angles made by the vector $\overrightarrow{BA}$ with the $\boldsymbol{x}, \boldsymbol{y}$ and the $\boldsymbol{z}$ axes are respectively $cos^{-1}\left(\frac{2}{\sqrt{5}}\right), \frac{\pi}{2}, cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$.	$\frac{1}{2}$ 1
25.	$\overrightarrow{d_1} = \vec{a} + \vec{b} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$, $\overrightarrow{d_2} = \vec{a} - \vec{b} = -6\hat{\jmath} - 8\hat{k}$ Area of the parallelogram = $\frac{1}{2}\left\lvert\overrightarrow{d_1} \times \overrightarrow{d_2}\right\rvert = \frac{1}{2}\left\lvert\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8 \end{vmatrix}\right\rvert = 2\lvert\hat{\imath} + 8\hat{\jmath} - 6\hat{k}\rvert$ Area of the parallelogram = $2\sqrt{101}$ sq. units.	$\frac{1}{2}$ 1 $\frac{1}{2}$

<u>Section –C</u>

[This section comprises of solution short answer type questions (SA) of 3 marks each]

26.	 $x^2 + 3^2 = y^2$ $When\ y = 5\ then\ x = 4, now\ 2x\frac{dx}{dt} = 2y\frac{dy}{dt}$ $4\ (200) = 5\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = 160$ cm/s	$\frac{1}{2}$ $\frac{1}{2}$ 1 1
27.	$A = \frac{1}{3}\sqrt{t} \therefore \frac{dA}{dt} = \frac{1}{6}t^{-\frac{1}{2}} = \frac{1}{6\sqrt{t}}; \forall t \in (5,18)$ $\frac{dA}{dt} = \frac{1}{6\sqrt{t}} \therefore \frac{d^2A}{dt^2} = -\frac{1}{12t\sqrt{t}}$ So, $\frac{d^2A}{dt^2} < 0, \forall t \in (5,18)$ This means that the rate of change of the ability to understand spatial concepts decreases (slows down) with age.	1 1 $\frac{1}{2}$ $\frac{1}{2}$
28(a)	**(i)** $\boldsymbol{\theta} = \cos^{-1}\left(\frac{\overrightarrow{l_1}.\overrightarrow{l_2}}{\lvert\overrightarrow{l_1}\rvert.\lvert\overrightarrow{l_2}\rvert}\right) = \cos^{-1}\left(\frac{(\hat{\imath}-2\hat{\jmath}+3\hat{k}).(3\hat{\imath}-2\hat{\jmath}+\hat{k})}{\lvert(\hat{\imath}-2\hat{\jmath}+3\hat{k})\rvert\lvert(3\hat{\imath}-2\hat{\jmath}+\hat{k})\rvert}\right)$ $= cos^{-1}\left(\frac{3+4+3}{\sqrt{1+4+9}\sqrt{9+4+1}}\right) = cos^{-1}\left(\frac{10}{14}\right) = cos^{-1}\left(\frac{5}{7}\right)$. **(ii)** Scalar projection of $\overrightarrow{l_1}$ on $\overrightarrow{l_2}$ $= \frac{\overrightarrow{l_1}.\overrightarrow{l_2}}{\lvert\overrightarrow{l_2}\rvert} = \frac{(\hat{\imath}-2\hat{\jmath}+3\hat{k}).(3\hat{\imath}-2\hat{\jmath}+\hat{k})}{\lvert(3\hat{\imath}-2\hat{\jmath}+\hat{k})\rvert}$ $= \frac{3+4+3}{\sqrt{9+4+1}} = \frac{10}{\sqrt{14}}$.	1 $\frac{1}{2}$ 1 $\frac{1}{2}$

28(b)	Line perpendicular to the lines $\vec{r} = 2\hat{\imath} + \hat{\jmath} - 3\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 5\hat{k})$ and $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} - 7\hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k})$. has a vector parallel it is given by $\vec{b} = \overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 5 \\ 3 & -2 & 5 \end{vmatrix} = 20\hat{\imath} + 10\hat{\jmath} - 8\hat{k}$ $\therefore$ equation of line in vector form is $\vec{r} = -\hat{\imath} + 2\hat{\jmath} + 7\hat{k} + a(10\hat{\imath} + 5\hat{\jmath} - 4\hat{k})$ And equation of line in cartesian form is $\frac{x+1}{10} = \frac{y-2}{5} = \frac{z-7}{-4}$	1 1 1
29.(a)	$\int \left\{\frac{1}{log_e x} - \frac{1}{(log_e x)^2}\right\} dx$ $= \int \frac{dx}{log_e x} - \int \frac{1}{(log_e x)^2} dx = \frac{1}{log_e x}\int dx - \int \left\{\frac{d}{dx}\left(\frac{1}{log_e x}\right)\int dx\right\} dx - \int \frac{1}{(log_e x)^2} dx$ $= \frac{x}{log_e x} + \int \frac{1}{(log_e x)^2}\frac{1}{x}.x.dx - \int \frac{1}{(log_e x)^2} dx$ $= \frac{x}{log_e x} + \int \frac{1}{(log_e x)^2} dx - \int \frac{dx}{(log_e x)^2} = \frac{x}{log_e x} + c;$ where $'c'$ is any arbitary constant of integration.	**1** **1** **1**
OR **29.(b)**	$\int_0^1 x(1-x)^n dx$ $= \int_0^1 (1-x)\{1-(1-x)\}^n dx, \left(as, \int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$ $= \int_0^1 x^n (1-x) dx$ $= \int_0^1 x^n dx - \int_0^1 x^{n+1} dx$ $= \frac{1}{n+1}[x^{n+1}]_0^1 - \frac{1}{n+2}[x^{n+2}]_0^1$ $= \frac{1}{n+1} - \frac{1}{n+2} = \frac{1}{(n+1)(n+2)}$.	**1** $\frac{1}{2}$ $\frac{1}{2}$ **1**
30.	The feasible region determined by the constraints, $2x + y \geq 3,\ x + 2y \geq 6,\ x \geq 0, y \geq 0$ is as shown.	

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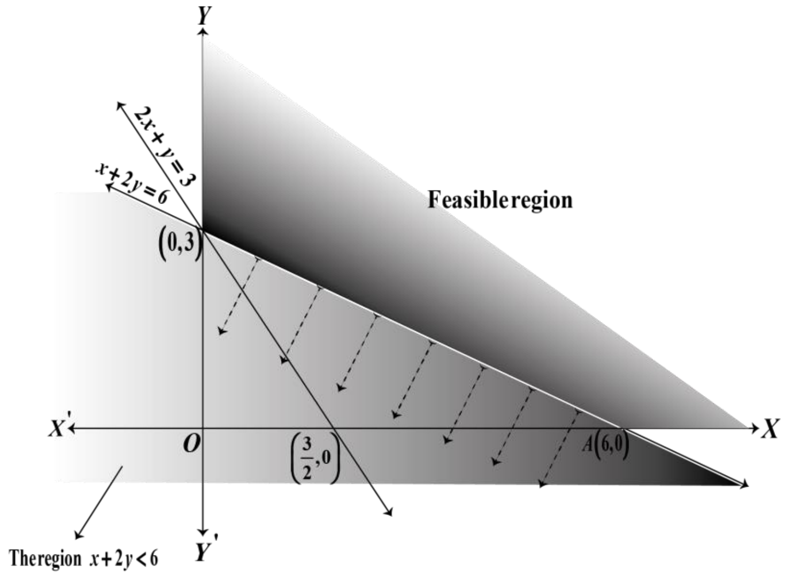


The corner points of the **unbounded** feasible region are $A(6,0)$ and $B(0,3)$.

The values of Z at these corner points are as follows:

Corner point	Value of the objective function $Z = x + 2y$
$A(6,0)$	6
$B(0,3)$	6

1

We observe the region $x+2y<6$ have no points in common with the unbounded feasible region. Hence the minimum value of $z=6$. $\frac{1}{2}$

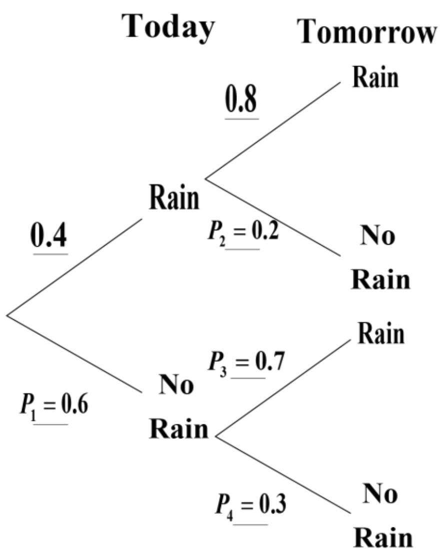
It can be seen that the value of Z at points **A** and **B** is same. If we take any other point on the line $x+2y=6$ such as (2,2) on line $x+2y=6,$ then $Z=6$.

Thus, the minimum value of Z occurs for more than **2** points, and is equal to 6. $\frac{1}{2}$

31.(a) Since the event of raining today and not raining today are complementary events so if the probability that it rains today is 0.4 then the probability that it does not rain today is $1-0.4=0.6 \Rightarrow P_1 = 0.6$

If it rains today, the probability that it will rain tomorrow is 0.8 then the probability that it will not rain tomorrow is $\mathbf{1-0.8=0.2}$.

If it does not rain today, the probability that it will rain tomorrow is 0.7 then the probability that it will not rain tomorrow is $\mathbf{1-0.7=0.3}$



(i) $\boldsymbol{P_1 \times P_4 - P_2 \times P_3 = 0.6 \times 0.3 - 0.2 \times 0.7 = 0.04.}$ **1**

(ii) Let $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$ be the events that it will rain today and it will not rain today respectively. **1**

$\boldsymbol{P(E_1) = 0.4 \;\&\; P(E_2) = 0.6}$

$\boldsymbol{A}$ be the event that it will rain tomorrow. $P\left(\frac{A}{E_1}\right) = 0.8 \;\&\; P\left(\frac{A}{E_2}\right) = 0.7$ **$\frac{1}{2}$**

We have, $P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) = 0.4 \times 0.8 + 0.6 \times 0.7 = 0.74.$

The probability of rain tomorrow is $\mathbf{0.74}$. **$\frac{1}{2}$**

OR

31.(b)

Given $P(X = r)\alpha \frac{1}{5^r}$ **$\frac{1}{2}$**

$P(X = r) = k\frac{1}{5^r},$ (where $\boldsymbol{k}$ is a non-zero constant)

$P(r = 0) = k.\frac{1}{5^0}$

$P(r = 1) = k.\frac{1}{5^1}$

$P(r = 2) = k.\frac{1}{5^2}$ **$\frac{1}{2}$**

$P(r = 3) = k.\frac{1}{5^3}$

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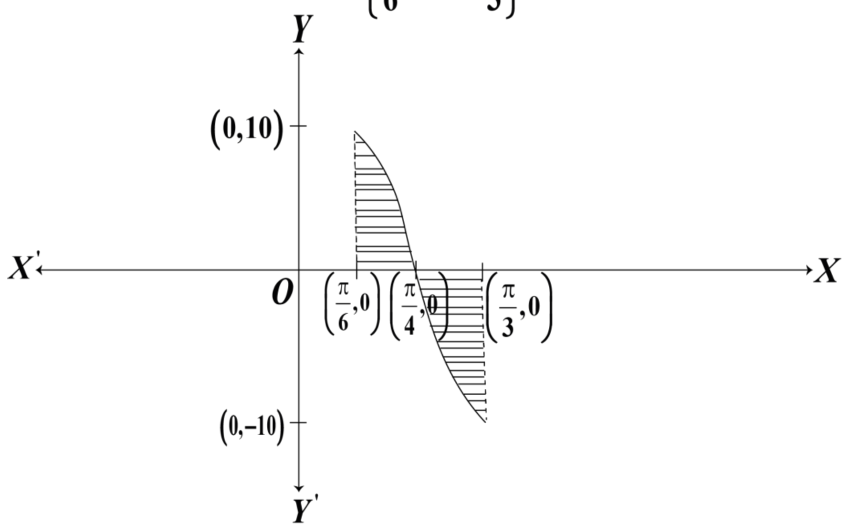
We have, $P(X = 0) + P(X = 1) + P(X = 2) + \ldots\ldots\ldots\ldots\ldots = 1$ **$\frac{1}{2}$**

	$$\Rightarrow k\left(1+\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\ldots\ldots\ldots\ldots\right)=1$$ $$\Rightarrow k\left(\frac{1}{1-\frac{1}{5}}\right)=1 \Rightarrow k=\frac{4}{5}$$ So, $P(X<3)=P(X=0)+P(X=1)+P(X=2)$ $$=\frac{4}{5}\left(1+\frac{1}{5}+\frac{1}{5^2}\right)=\frac{4}{5}\left(\frac{25+5+1}{25}\right)=\frac{124}{125}.$$	$\frac{1}{2}$ 1

Section –D

[This section comprises of solution of long answer type questions (LA) of 5 marks each]

32.	$$y=20\cos 2x\ ;\ \left\{\frac{\pi}{6}\le x\le\frac{\pi}{3}\right\}$$  Required area $=20\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} cos2x\ dx+\left\|20\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} cos2x\ dx\right\|$ $=20\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}+\left\|20\left[\frac{\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}\right\|$ $=10\left(1-\frac{\sqrt{3}}{2}\right)+\ 10\left(1-\frac{\sqrt{3}}{2}\right)=20\left(1-\frac{\sqrt{3}}{2}\right)$ sq. units.	1 1+1 1 1
33.	$\boldsymbol{y=ax^2+bx+c}$ $\mathbf{15=4a+2b+c}$ $\mathbf{25=16a+4b+c}$ $\mathbf{15=196a+14b+c}$ The set of equations can be represented in the matrix form as $\boldsymbol{AX=B}$, where $A=\begin{bmatrix}4&2&1\\16&4&1\\196&14&1\end{bmatrix}$, $X=\begin{bmatrix}a\\b\\c\end{bmatrix}$ and $B=\begin{bmatrix}15\\25\\15\end{bmatrix}\Rightarrow\begin{bmatrix}4&2&1\\16&4&1\\196&14&1\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix}=\begin{bmatrix}15\\25\\15\end{bmatrix}$. $\|A\|=4(4-14)-2(16-196)+(224-784)=-40+360-560=-240\neq 0.$ Hence $\boldsymbol{A^{-1}}$ exists.	1 $\frac{1}{2}$ $\frac{1}{2}$

	Now, $adj(A) = \begin{bmatrix} -10 & 180 & -560 \\ 12 & -192 & 336 \\ -2 & 12 & -16 \end{bmatrix}^T = \begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}$ $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = -\frac{1}{240}\begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}\begin{bmatrix} 15 \\ 25 \\ 15 \end{bmatrix} = -\frac{5}{240}\begin{bmatrix} -10 & 12 & -2 \\ 180 & -192 & 12 \\ -560 & 336 & -16 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix} = -\frac{5}{240}\begin{bmatrix} 24 \\ -384 \\ -48 \end{bmatrix}$ $\therefore \boldsymbol{a = -\frac{1}{2}, b = 8, c = 1}$ So, the equation becomes $\boldsymbol{y = -\frac{1}{2}x^2 + 8x + 1}$	1 1 $\frac{1}{2}$ $\frac{1}{2}$
34.(a)	We have, $f(x) = \|x\|^3, \begin{cases} x^3, \text{if } x \geq 0 \\ (-x)^3 = -x^3, \text{if} x < 0 \end{cases}$ Now, $(LHD\ at\ x = 0) = \lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \left(\frac{-x^3 - 0}{x}\right) = \lim_{x \to 0^-}(-x^2) = 0$ $(RHD\ at x = 0) \lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \left(\frac{x^3 - 0}{x}\right) = \lim_{x \to 0}(-x^2) = 0$ $\therefore (LHD\ of\ f(x)\ at\ x = 0) = (RHD\ of\ f(x)\ at\ x = 0)$ So, $\boldsymbol{f(x)}$ is differentiable at $\boldsymbol{x = 0}$ and the derivative of $\boldsymbol{f(x)}$ is given by $f'(x) = \begin{cases} 3x^2, \text{if} x \geq 0 \\ -3x^2, \text{if} x < 0 \end{cases}$ Now, $(LHD of f'(x) at x = 0) = \lim_{x \to 0^-} \frac{f'(x) - f'(0)}{x - 0} = \lim_{x \to 0^-} \left(\frac{-3x^2 - 0}{x}\right) = \lim_{x \to 0^-}(-3x) = 0$ $(RHD\ of f'(x)\ at\ x = 0) = \lim_{x \to 0^+} \frac{f'(x) - f'(0)}{x - 0} = \lim_{x \to 0^+} \left(\frac{3x^2 - 0}{x - 0}\right) = \lim_{x \to 0^+}(3x) = 0$ $\therefore (LHD\ of f'(x) at\ x = 0) = (RHD of f'(x) at x = 0)$ So, $f'(x)$ is differentiable at $\boldsymbol{x = 0.}$ Hence, $f''(x) = \begin{cases} 6x, \text{if} x \geq 0 \\ -6x, \text{if} x < 0. \end{cases}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
OR **34 .(b)**	Given relation is $(x - a)^2 + (y - b)^2 = c^2, c > 0.$ Let $\boldsymbol{x - a = c\cos}\theta$ and $y - b = c\sin\theta.$ Therefore, $\frac{dx}{d\theta} = -c\sin\theta$ And $\frac{dy}{d\theta} = c\cos\theta$ $\therefore \frac{dy}{dx} = -\cot\theta$ Differentiate both sides with respect to θ, we get $\frac{d}{d\theta}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}(-\cot\theta)$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$

Or, $\frac{d}{dx}\left(\frac{dy}{dx}\right)\frac{dx}{d\theta} = cos\,e\,c^2\theta$

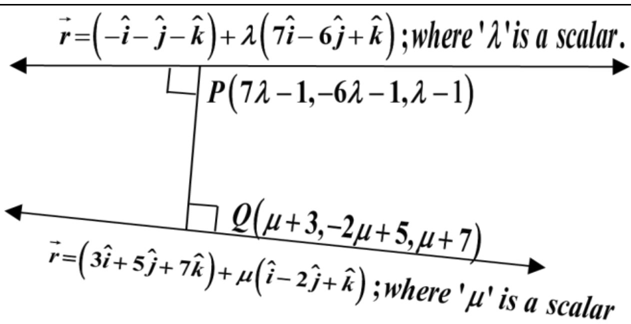
Or, $\frac{d^2y}{dx^2}(-c\,sin\,\theta) = \text{cosec}^2\theta$ **1/2**

$$\frac{d^2y}{dx^2} = -\frac{cosec^3\theta}{c}$$ **1/2**

$$\therefore \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{c[1+cot^2\theta]^{\frac{3}{2}}}{-cos\,ec^3\theta} = \frac{-c(cos\,ec^2\theta)^{\frac{3}{2}}}{\text{cosec}^3\theta} = -c,$$ **1**

Which is constant and is independent of $\boldsymbol{a}$ and $\boldsymbol{b}$. **1/2**

35.(a)



Given that equation of lines are

$\vec{r} = \left(-\hat{\imath}-\hat{\jmath}-\hat{k}\right)+\lambda\left(7\hat{\imath}-6\hat{\jmath}+\hat{k}\right)$................$(i)$ and

$\vec{r} = \left(3\hat{\imath}+5\hat{\jmath}+7\hat{k}\right)+\mu\left(\hat{\imath}-2\hat{\jmath}+\hat{k}\right)$..................$(ii)$

The given lines are non-parallel lines as vectors $7\hat{\imath}-6\hat{\jmath}+\hat{k}$ and $\hat{\imath}-2\hat{\jmath}+\hat{k}$ are not parallel. There is a unique line segment $\boldsymbol{PQ}$ ($\boldsymbol{P}$ lying on line $(\boldsymbol{i})$ and $\boldsymbol{Q}$ on the other line$(\boldsymbol{ii})$), which is at right angles to both the lines $\boldsymbol{PQ}$ is the shortest distance between the lines.

Hence, the shortest possible distance between the lines $= \boldsymbol{PQ}$.

Let the position vector of the point $\boldsymbol{P}$ lying on the line $\vec{r} = \left(-\hat{\imath}-\hat{\jmath}-\hat{k}\right)+\lambda\left(7\hat{\imath}-6\hat{\jmath}+\hat{k}\right)$ where '$\boldsymbol{\lambda}$' is a scalar, is $(7\lambda-1)\hat{\imath}-(6\lambda+1)\hat{\jmath}+(\lambda-1)\hat{k}$, for some $\boldsymbol{\lambda}$ and the position vector of the point $\boldsymbol{Q}$ lying on the line $\vec{r} = \left(3\hat{\imath}+5\hat{\jmath}+7\hat{k}\right)+\mu\left(\hat{\imath}-2\hat{\jmath}+\hat{k}\right)$ where '$\boldsymbol{\mu}$' is a scalar, is **1/2** **1/2**

$(\mu+3)\hat{\imath}+(-2\mu+5)\hat{\jmath}+(\mu+7)\hat{k}$, for some $\boldsymbol{\mu}$. Now, the vector

$$\overrightarrow{PQ} = \overrightarrow{OQ}-\overrightarrow{OP} = (\mu+3-7\lambda+1)\hat{\imath}+(-2\mu+5+6\lambda+1)\hat{\jmath}+(\mu+7-\lambda+1)\hat{k}$$

$i.e., \overrightarrow{PQ} = (\mu-7\lambda+4)\hat{\imath}+(-2\mu+6\lambda+6)\hat{\jmath}+(\mu-\lambda+8)\hat{k}$; (where '$\boldsymbol{O}$' is the origin), is perpendicular to both the lines, so the vector $\overrightarrow{PQ}$ is perpendicular to both the vectors $7\hat{\imath}-6\hat{\jmath}+\hat{k}$ and $\hat{\imath}-2\hat{\jmath}+\hat{k}$. **1**

$\Rightarrow (\mu-7\lambda+4).7+(-2\mu+6\lambda+6).(-6)+(\mu-\lambda+8).1 = 0$

$\&(\mu-7\lambda+4).1+(-2\mu+6\lambda+6).(-2)+(\mu-\lambda+8).1=0$

$\Rightarrow \mathbf{20\mu-86\lambda=0} \;=> \mathbf{10\mu-43\lambda=0}\&6\mu-20\lambda=0 \Rightarrow 3\mu-10\lambda=0$ **1**

On solving the above equations, we get $\boldsymbol{\mu=\lambda=0}$ **$\frac{1}{2}$**

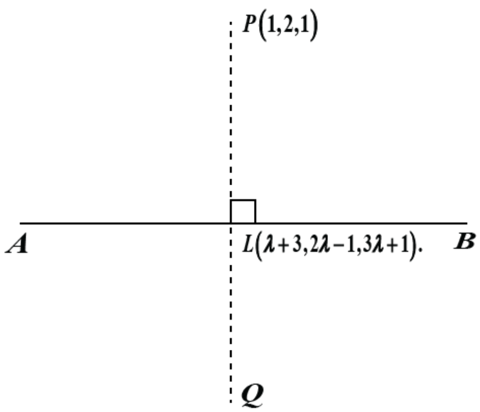
So, the position vector of the points ***P*** and ***Q*** are $-\hat{\imath}-\hat{\jmath}-\hat{k}$ and $3\hat{\imath}+5\hat{\jmath}+7\hat{k}$ respectively.

$\overrightarrow{PQ}=4\hat{\imath}+6\hat{\jmath}+8\hat{k}$ and **$\frac{1}{2}$**

$|\overrightarrow{PQ}|=\sqrt{4^2+6^2+8^2}=\sqrt{116}=2\sqrt{29}\,units.$ **1**

OR

35.(b)



Let $\boldsymbol{P(1,2,1)}$ be the given point and ***L*** be the foot of the perpendicular from ***P*** to the given line ***AB*** $(as\ shown\ in\ the\ figure\ above)$.

Let's put $\boldsymbol{\frac{x-3}{1}=\frac{y+1}{2}=\frac{z-1}{3}=\lambda}$. Then, $\boldsymbol{x=\lambda+3, y=2\lambda-1, z=3\lambda+1}$ **$\frac{1}{2}$**

Let the coordinates of the point ***L*** be $\boldsymbol{(\lambda+3,2\lambda-1,3\lambda+1)}$.

So, direction ratios of ***PL*** are $(\lambda+3-1,2\lambda-1-2,3\lambda+1-1)$ $i.e., (\lambda+2,2\lambda-3,3\lambda)$ **$\frac{1}{2}$**

Direction ratios of the given line are **1, 2 and 3,** which is perpendicular to ***PL***. Therefore, we have,

$$(\lambda+2).1+(2\lambda-3).2+3\lambda.3=0 \Rightarrow 14\lambda=4 \Rightarrow \lambda=\frac{2}{7}$$ **$\frac{1}{2}$**

Then, $\lambda+3=\frac{2}{7}+3=\frac{23}{7};\; 2\lambda-1=2\left(\frac{2}{7}\right)-1=-\frac{3}{7};\; 3\lambda+1=3\left(\frac{2}{7}\right)+1=\frac{13}{7}$ **$\frac{1}{2}$**

Therefore, coordinates of the point ***L*** are $\left(\frac{23}{7},-\frac{3}{7},\frac{13}{7}\right)$.

Let $Q(x_1,y_1,z_1)$ be the image of $\boldsymbol{P(1,2,1)}$ with respect to the given line. Then, ***L*** is the mid-point of ***PQ***. **1**

Therefore, $\frac{1+x_1}{2}=\frac{23}{7}, \frac{2+y_1}{2}=-\frac{3}{7}, \frac{1+z_1}{2}=\frac{13}{7} \Rightarrow x_1=\frac{39}{7}, y_1=-\frac{20}{7}, z_1=\frac{19}{7}$

Hence, the image of the point $\boldsymbol{P(1,2,1)}$ with respect to the given line $Q\left(\frac{39}{7},-\frac{20}{7},\frac{19}{7}\right)$. **1**

The equation of the line joining $\boldsymbol{P(1,2,1)}$ and $Q\left(\frac{39}{7},-\frac{20}{7},\frac{19}{7}\right)$ is

	$\frac{x-1}{32/7}=\frac{y-2}{-34/7}=\frac{z-1}{12/7} \Rightarrow \frac{x-1}{16}=\frac{y-2}{-17}=\frac{z-1}{6}$.	1

Section –E

[This section comprises solution of 3 case- study/passage based questions of 4 marks each with two sub parts. Solution of the first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. Solution of the third case study question has two sub parts of 2 marks each.)

36.	(i) $V=(40-2x)(25-2x)xcm^3$	**1**
	(ii) $\frac{dV}{dx}=4(3x-50)(x-5)$	**1**
	(iii) (a) For extreme values $\frac{dV}{dx}=4(3x-50)(x-5)=0$	$^1/_2$
	$\Rightarrow x=\frac{50}{3}$ or $x=5$	$^1/_2$
	$\frac{d^2V}{dx^2}=24x-260$	$^1/_2$
	$\therefore \frac{d^2V}{dx^2}$ at $x=5$ is $-140<0$	$^1/_2$
	$\therefore V$ is max $when\ x=5$	
	(iii) **OR**	
	(b) For extreme values $\frac{dV}{dx}=4(3x^2-65x+250)$	$^1/_2$
	$\frac{d^2V}{dx^2}=4(6x-65)$	$^1/_2$
	$\frac{dV}{dx}\ at\ x=\frac{65}{6}$ exists and $\frac{d^2V}{dx^2}\ at\ x=\frac{65}{6}\ is\ 0$.	
	$\frac{d^2V}{dx^2}\ at\ x=\left(\frac{65}{6}\right)^-$ is negative and $\frac{d^2V}{dx^2}\ at\ x=\left(\frac{65}{6}\right)^+$ is positive	$^1/_2$
	$\therefore x=\frac{65}{6}$ is a point of inflection.	$^1/_2$
37.	**(i)** **Number of relations is equal to the number of subsets of the set** $B\times G=2^{n(B\times G)}$	
	$=2^{n(B)\times n(G)}=2^{3\times 2}=2^6$	**1**
	$(\boldsymbol{Where\ n(A)\ denotes\ the\ number\ of\ the\ elements\ in\ the\ finite\ set\ A})$	
	(ii) **Smallest Equivalence relation on G is** $\{(g_1,g_1),(g_2,g_2)\}$	**1**
	(iii) (a) (A) reflexive but not symmetric =	
	$\{(\boldsymbol{b_1},\boldsymbol{b_2}),(\boldsymbol{b_2},\boldsymbol{b_1}),(\boldsymbol{b_1},\boldsymbol{b_1}),(\boldsymbol{b_2},\boldsymbol{b_2}),(\boldsymbol{b_3},\boldsymbol{b_3}),(\boldsymbol{b_2},\boldsymbol{b_3})\}$.	

	So the minimum number of elements to be added are $(b_1,b_1),(b_2,b_2),(b_3,b_3),(b_2,b_3)$ {Note : it can be any one of the pair from, (b_3,b_2), (b_1,b_3), (b_3,b_1) **in place of** (b_2,b_3) ***also***} (B) reflexive and symmetric but not transitive = $\{(b_1,b_2),(b_2,b_1),(b_1,b_1),(b_2,b_2),(b_3,b_3),(b_2,b_3),(b_3,b_2)\}$. **So the minimum number of elements to be added are** $(b_1,b_1),(b_2,b_2),(b_3,b_3),(b_2,b_3),(b_3,b_2)$ **OR (iii) (b) One-one and onto function** $x^2=4y$. let $y=f(x)=\frac{x^2}{4}$ Let $x_1,x_2\in\left[0,20\sqrt{2}\right]$ such that $f(x_1)=f(x_2)\Rightarrow\frac{x_1^2}{4}=\frac{x_1^2}{4}$ $\Rightarrow x_1^2=x_2^2\Rightarrow(x_1-x_2)(x_1+x_2)=0\Rightarrow x_1=x_2$ as $x_1,x_2\in\left[0,20\sqrt{2}\right]$ $\therefore f$ is one-one function Now, $0\le y\le 200$ hence the value of y is non-negative and $f\left(2\sqrt{y}\right)=y$ $\therefore$ for any arbitrary $y\in[0,200]$, the pre-image of y exists in $\left[0,20\sqrt{2}\right]$ hence f is onto function.	1 1 1 1
38.	Let E_1 be the event that one parrot and one owl flew from cage –I E_2 be the event that two parrots flew from Cage-I A be the event that the owl is still in cage-I (i) Total ways for A to happen From cage I 1 parrot and 1 owl flew and then from Cage-II 1 parrot and 1 owl flew back + From cage I 1 parrot and 1 owl flew and then from Cage-II 2 parrots flew back + From cage I 2 parrots flew and then from Cage-II 2 parrots came back. $=(5_{C_1}\times 1_{C_1})(7_{C_1}\times 1_{C_1})+(5_{C_1}\times 1_{C_1})(7_{C_2})+(5_{C_2})(8_{C_2})$ Probability that the owl is still in cage –I = P$(E_1\cap A)$ + P$(E_2\cap A)$ $\frac{(5_{C_1}\times 1_{C_1})(7_{C_1}\times 1_{C_1})+(5_{C_2})(8_{C_2})}{(5_{C_1}\times 1_{C_1})(7_{C_1}\times 1_{C_1})+(5_{C_1}\times 1_{C_1})(7_{C_2})+(5_{C_2})(8_{C_2})}$ $=\frac{35+280}{35+105+280}=\frac{315}{420}=\frac{3}{4}$	$\frac{1}{2}$ 1 $\frac{1}{2}$

	(i) The probability that one parrot and the owl flew from Cage-I to Cage-II given that the owl is still in cage-I is $P\left({E_1}/{A}\right)$	$\frac{1}{2}$
	$P\left({E_1}/{A}\right) = \frac{P(E_1 \cap A)}{P(E_1 \cap A)+P(E_2 \cap A)}$ (by Baye's Theorem)	$\frac{1}{2}$
	$= \frac{\frac{35}{420}}{\frac{315}{420}} = \frac{1}{9}$	**1**

