JEE–MAIN EXAMINATION – JANUARY, 2023	
(Held On Thursday 31st January, 2023)	TIME : 3 : 00 PM to 6 : 00 PM

Physics

SECTION - A

1. Given below are two statements:

Statement I: In a typical transistor, all three regions emitter, base and collector have same doping level.

Statement II: In a transistor, collector is the thickest and base is the thinnest segment.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and Statement II are correct

(2) Statement I is incorrect but Statement II is correct

(3) Statement I is correct but Statement II is incorrect

(4) Both Statement I and Statement II are incorrect

Sol. **(2)**

Emitter	Base	Collector
Moderate Size	Thin	Thick
Maximum Doping	Minimum Doping	Moderate Doping

2. If the two metals *A* and *B* are exposed to radiation of wavelength 350 nm. The work functions of metals A and B are 4.8eV and 2.2eV. Then choose the correct option.

(1) Both metals A and B will emit photo-electrons

(2) Metal *A* will not emit photo-electrons

(3) Metal B will not emit photo-electrons

(4) Both metals A and B will not emit photo-electrons

Sol. **(2)**

$$E = \frac{hc}{\lambda} = \frac{1240}{350} = 3.54\text{eV}$$

If $E > \phi$, photo electrons will emit.

A will not emit and B will emit.

3. Heat energy of 735 J is given to a diatomic gas allowing the gas to expand at constant pressure. Each gas molecule rotates around an internal axis but do not oscillate. The increase in the internal energy of the gas will be :

(1) 525 J (2) 441 J (3) 572 J (4) 735 J

Sol. **(1)**

At constant Pressure,

$Q = nCpdT = 735J$

$$\Delta U = nCvdT = \frac{735}{\left(Cp/Cv\right)} = \frac{735}{8}$$

$$\Delta U = \frac{735}{\left(7/5\right)} = 525J$$

4. Match List I with List II

LIST I		LIST II	
A.	Angular momentum	I.	$[ML^2 T^{-2}]$
B.	Torque	II.	$[ML^{-2} T^{-2}]$
C.	Stress	III.	$[ML^2 T^{-1}]$
D.	Pressure gradient	IV.	$[ML^{-1} T^{-2}]$

Choose the correct answer from the options given below:

(1) A - III, B - I, C - IV, D – II
(2) A - II, B - III, C - IV, D – I
(3) A - IV, B - II, C - I, D – III
(4) A - I, B - IV, C - III, D – II

Sol. **(1)**

$L = mvr = [M^1L^2T^{-1}]$

$\tau = rF = [M^1L^2T^{-2}]$

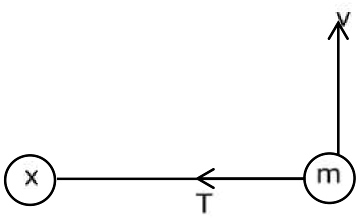
Stress $= \frac{F}{A} = [M^1L^{-1}T^{-2}]$

Pressure Gradient $= \frac{dp}{dx} = [M^1L^{-2}T^{-2}]$

5. A stone of mass 1 kg is tied to end of a massless string of length 1 m. If the breaking tension of the string is 400 N, then maximum linear velocity, the stone can have without breaking the string, while rotating in horizontal plane, is :

(1) 40 ms^{-1}
(2) 400 ms^{-1}
(3) 20 ms^{-1}
(4) 10 ms^{-1}

Sol. **(3)**



$T = \frac{mv^2}{\ell}$

$400 = \frac{1 \times v^2}{1}$

V = 20 m/s

6. A microscope is focused on an object at the bottom of a bucket. If liquid with refractive index $\frac{5}{3}$ is poured inside the bucket, then microscope have to be raised by 30 cm to focus the object again. The height of the liquid in the bucket is :

(1) 12 cm
(2) 50 cm
(3) 18 cm
(4) 75 cm

Sol. **(4)**

$d_{app} = \frac{d}{\mu} = \frac{h}{\left(5/3\right)}$

Shift = h $\frac{-3h}{5}$ = 30

h = 75 cm

7. The number of turns of the coil of a moving coil galvanometer is increased in order to increase current sensitivity by 50%. The percentage change in voltage sensitivity of the galvanometer will be :

(1) 0% (2) 75% (3) 50% (4) 100%

Sol. **(1)**

$\alpha_V = \frac{NAB}{KR} \alpha \frac{N}{R}$

$\alpha_I = \frac{NAB}{K} \alpha\ N$

$N\uparrow, \alpha_I\uparrow, \frac{N}{R} \to \text{Constant}$

$\Delta\alpha_V = 0$

8. A body is moving with constant speed, in a circle of radius 10 m. The body completes one revolution in 4s. At the end of 3[rd] second, the displacement of body (in m) from its starting point is:

(1) 15π (2) $10\sqrt{2}$ (3) 30 (4) 5π

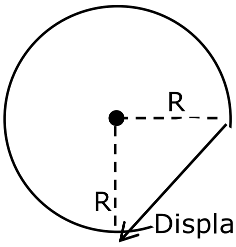
Sol. **(2)**

$w = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}\ \text{rad}/\text{s}$

θ = wt

$\theta = \frac{\pi}{2} \times 3$

$\theta = \frac{3\pi}{2}$ rad



Displacement = $\sqrt{2}R = 10\sqrt{2}m$

9. The H amount of thermal energy is developed by a resistor in 10 s when a current of 4 A is passed through it. If the current is increased to 16 A, the thermal energy developed by the resistor in 10 s will be :

(1) $\frac{H}{4}$ (2) 16H (3) 4H (4) H

Sol. **(2)**

$H = I^2Rt$

$\frac{H_1}{H_2} = \left(\frac{I_1}{I_2}\right)^2 = \left(\frac{4}{16}\right)^2$

$H_2 = 16H_1$

10. A long conducting wire having a current I flowing through it, is bent into a circular coil of N turns. Then it is bent into a circular coil of n turns. The magnetic field is calculated at the centre of coils in both the cases. The ratio of the magnetic field in first case to that of second case is:

(1) n: N (2) n^2: N^2 (3) N^2: n^2 (4) N: n

Sol. **(3)**

Length Remains Same.

$$\ell = N(2\pi r_1) = n(2\pi r_2)$$

$$\frac{B_1}{B_2} = \frac{\left(N\frac{\mu_0 I}{2r_1}\right)}{\left(n\frac{\mu_0 I}{2r_2}\right)} = \frac{N}{n}\left(\frac{r_2}{r_1}\right) = \frac{N}{n}\left(\frac{N}{n}\right)$$

$$\frac{B_1}{B_2} = \left(\frac{N}{n}\right)^2$$

11. A body weight W, is projected vertically upwards from earth's surface to reach a height above the earth which is equal to nine times the radius of earth. The weight of the body at that height will be :

(1) $\frac{W}{100}$ (2) $\frac{W}{91}$ (3) $\frac{W}{3}$ (4) $\frac{W}{9}$

Sol. **(1)**

$$g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

$h = 9R$

$$g_h = \frac{g}{(1+9)^2} = \frac{g}{100}$$

$$w_h = \frac{mg}{100} = \frac{w}{100}$$

12. The radius of electron's second stationary orbit in Bohr's atom is R. The radius of 3rd orbit will be

(1) $\frac{R}{3}$ (2) 3R (3) 2.25R (4) 9R

Sol. **(3)**

$$R \propto \frac{n^2}{z}$$

$$\frac{R_1}{R_2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{2}{3}\right)^2$$

$$R_2 = \frac{9R}{4} = 2.25R$$

13. A hypothetical gas expands adiabatically such that its volume changes from 08 litres to 27 litres. If the ratio of final pressure of the gas to initial pressure of the gas is $\frac{16}{81}$. Then the ratio of $\frac{Cp}{Cv}$ will be.

(1) $\frac{1}{2}$ (2) $\frac{4}{3}$ (3) $\frac{3}{2}$ (4) $\frac{3}{1}$

Sol. **(2)**

For Adiabatic process,

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\left(\frac{8}{27}\right)^{\gamma} = \frac{16}{81}$$

$$\left(\frac{2}{3}\right)^{3\gamma} = \left(\frac{2}{3}\right)^{4}$$

$$3\gamma = 4$$

$$\gamma = \frac{4}{3} = \frac{Cp}{Cv}$$

14. For a solid rod, the Young's modulus of elasticity is $3.2 \times 10^{11} Nm^{-2}$ and density is 8×10^3 kg m^{-3}. The velocity of longitudinal wave in the rod will be.

(1) 145.75×10^3 ms^{-1} (2) 18.96×10^3 ms^{-1}

(3) 3.65×10^3 ms^{-1} (4) 6.32×10^3 ms^{-1}

Sol. **(4)**

$$V = \sqrt{\frac{Y}{\rho}}$$

$$V = \sqrt{\frac{3.2 \times 10^{11}}{8 \times 10^3}} = \sqrt{0.4 \times 10^8}$$

$$V = \sqrt{40 \times 10^6}$$

$$V = 6.32 \times 10^3 \ m/s$$

15. A body of mass 10 kg is moving with an initial speed of 20 m/s. The body stops after 5 s due to friction between body and the floor. The value of the coefficient of friction is: (Take acceleration due to gravity $g = 10$ ms^{-2})

(1) 0.3 (2) 0.5 (3) 0.2 (4) 0.4

Sol. **(4)**

v = u + at

$$0 = 20 - \mu g(5)$$

$$\mu = \frac{2}{5} = 0.4$$

16. Given below are two statements :

Statement I : For transmitting a signal, size of antenna (l) should be comparable to wavelength of signal (at least $l = \frac{\lambda}{4}$ in dimension)

Statement II : In amplitude modulation, amplitude of carrier wave remains constant (unchanged).

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Statement **I** is correct but Statement II is incorrect

(2) Both Statement I and Statement II are correct

(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are incorrect

Sol. **(1)**

Statement –1 is correct.

In Modulation Amplitude of carrier wave is increased.

17. An alternating voltage source $V = 260\sin(628t)$ is connected across a pure inductor of 5mH. Inductive reactance in the circuit is :

(1) 0.318Ω (2) 6.28Ω (3) 3.14Ω (4) 0.5Ω

Sol. **(3)**

$\omega = 628\ \text{rad}/\text{s}$

$X_L = \omega L = 628 \times 5 \times 10^{-3}$

$X_L = 3.14\Omega$

18. Under the same load, wire A having length 5.0 m and cross section $2.5 \times 10^{-5}\ m^2$ stretches uniformly by the same amount as another wire B of length 6.0 m and a cross section of $3.0 \times 10^{-5}\ m^2$ stretches. The ratio of the Young's modulus of wire A to that of wire B will be :

(1) 1: 1 (2) 1: 10 (3) 1: 2 (4) 1: 4

Sol. **(1)**

By Hooke's Law,

$$Y = \frac{FL}{A\Delta L}$$

F, $\Delta L \rightarrow$ Same

$$\frac{Y_1A_1}{L_1} = \frac{Y_2A_2}{L_2}$$

$$\frac{Y_1}{Y_2} = \frac{3\times10^{-5}}{2.5\times10^{-5}}\times\frac{5}{6} = \frac{1}{1}$$

19. Match List I with List II

LIST I		LIST II	
A.	Microwaves	I.	Physiotherapy
B.	UV rays	II.	Treatment of cancer
C.	Infra-red light	III.	Lasik eye surgery
D.	X-ray	IV.	Aircraft navigation

Choose the correct answer from the options given below:

(1) A – IV, B - III, C - I, D – II
(2) A – IV, B – I, C - II, D – III
(3) A - III, B - II, C - I, D – IV
(4) A - II, B - IV, C - III, D – I

Sol. **(1)**

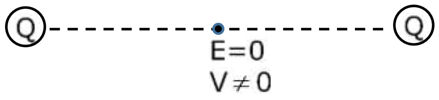
Theoritical

20. Considering a group of positive charges, which of the following statements is correct?

(1) Both the net potential and the net electric field cannot be zero at a point.
(2) Net potential of the system at a point can be zero but net electric field can't be zero at that point.
(3) Net potential of the system cannot be zero at a point but net electric field can be zero at that point.
(4) Both the net potential and the net field can be zero at a point.

Sol. **(3)**

Electric field is a Vector Quantity.
Electric Potential is a Scalar Quantity.
Eg.



SECTION - B

21. A series LCR circuit consists of $R = 80\Omega, X_L = 100\Omega$, and $X_C = 40\Omega$. The input voltage is 2500 cos(100πt)V. The amplitude of current, in the circuit, is ______A.

Sol. **(25)**

$R = 80\Omega$, $X_L = 100\ \Omega$, $X_c = 40\ \Omega$

$$Z = \sqrt{R^2 + (x_L - X_C)^2}$$

$$Z = \sqrt{80^2 + 60^2} = 100\Omega$$

$$I_0 = \frac{V_0}{Z} = \frac{2500}{100} = 25A$$

22. Two bodies are projected from ground with same speeds 40 ms^{-1} at two different angles with respect to horizontal. The bodies were found to have same range. If one of the body was projected at an angle of 60°, with horizontal then sum of the maximum heights, attained by the two projectiles, is ______m. (Given $g = 10\ ms^{-2}$)

Sol. **(80)**

In Range is same.

$$\alpha + \beta = 90°$$

$$\alpha = 60°$$

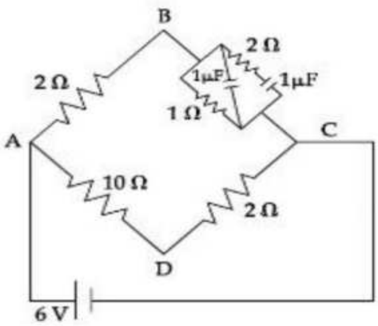
$$\beta = 30°$$

$$H_1 + H_2 = \frac{u_1^2 \sin^2 60°}{2g} + \frac{u_2^2 \sin^2 30°}{2g}$$

$$= \frac{u^2}{2g}\left(\frac{3}{4} + \frac{1}{4}\right) \qquad [u_1 = u_2]$$

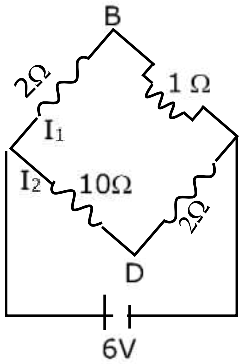
$$H_1 + H_2 = \frac{(40)^2}{20} = 80m$$

23. For the given circuit, in the steady state, $|V_B - V_D| =$ ______V.



Sol. **(1)**

At steady state, C $\rightarrow$ open Circuit



$$I_1 = \frac{6}{3} = 2A$$

$$I_2 = \frac{6}{12} = \frac{1}{2}A$$

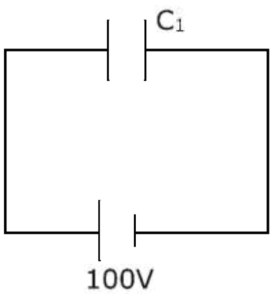
$$V_B + 2I_1 - 10I_2 = V_D$$

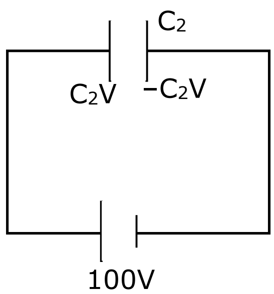
$$V_B - V_D = 5 - 4 = 1V$$

24. Two parallel plate capacitors C_1 and C_2 each having capacitance of $10\mu F$ are individually charged by a 100 V D.C. source. Capacitor C_1 is kept connected to the source and a dielectric slab is inserted between it plates. Capacitor C_2 is disconnected from the source and then a dielectric slab is inserted in it. Afterwards the capacitor C_1 is also disconnected from the source and the two capacitors are finally connected in parallel combination. The common potential of the combination will be _____V. (Assuming Dielectric constant = 10)

Sol. **(55)**

1.





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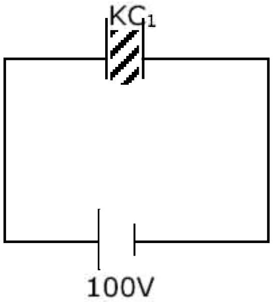
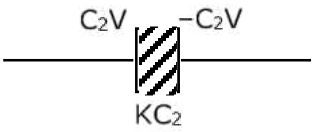
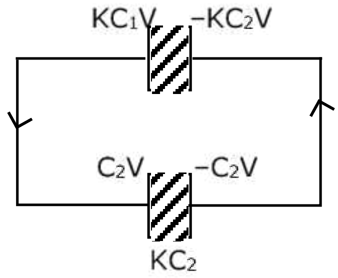




3. 

By charge conservation

$Q_1 = Q_2$

$KC_1V + C_2V = (KC_1+KC_2)\,V_{common}$

$$V_{common} = \frac{(K+1)CV}{2KC} = \frac{K+1}{2K}V$$

$$V_{common} = \frac{11}{20}\times 100 = 55V$$

25. Two light waves of wavelengths 800 and 600 nm are used in Young's double slit experiment to obtain interference fringes on a screen placed 7 m away from plane of slits. If the two slits are separated by 0.35 mm, then shortest distance from the central bright maximum to the point where the bright fringes of the two wavelength coincide will be _______ mm.

Sol. **(48)**

d = 0.35 mm, D = 7m

To Coincide, $n_1\left(\frac{\lambda_1 D}{d}\right) = n_2\left(\frac{\lambda_2 D}{d}\right)$

$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{6}{8} = \frac{3}{4}$$

3rd Maxima of λ_1 and 4th Maxima of λ_2 will coincide.

$$Y = \frac{3\lambda_1 D}{d} = \frac{3\times 800\times 10^{-9}\times 7}{35\times 10^{-5}}$$

$Y = 3 \times 160 \times 10^{-4}$ m

Y = 48mm

26. A ball is dropped from a height of 20 m. If the coefficient of restitution for the collision between ball and floor is 0.5, after hitting the floor, the ball rebounds to a height of ______ m

Sol. **(5)**

$V = \sqrt{2g(20)}$

$eV = \sqrt{2gh}$

$$\frac{1}{e} = \sqrt{\frac{20}{h}}$$

$$h = 20e^2 = 20\left(\frac{1}{2}\right)^2$$

h = 5m

27. If the binding energy of ground state electron in a hydrogen atom is 13.6eV, then, the energy required to remove the electron from the second excited state of Li^{2+} will be : $x \times 10^{-1}$eV. The value of x is____.

Sol. (136)

$$BE = 13.6 \times \frac{z^2}{n^2}$$

$$BE = 13.6 \times \left(\frac{3}{3}\right)^2 = 13.6eV$$

$$BE = 136 \times 10^{-1}\ eV$$

$$x = 136$$

28. A water heater of power 2000 W is used to heat water. The specific heat capacity of water is 4200 J $kg^{-1}\ K^{-1}$. The efficiency of heater is 70%. Time required to heat 2 kg of water from 10°C to 60°C is ______s.

(Assume that the specific heat capacity of water remains constant over the temperature range of the water).

Sol. (300)

$P_{used} = 0.7 \times 2000 = 1400W$

$$P = \frac{ms\Delta T}{t}$$

$$t = \frac{2 \times 4200 \times 50}{1400}$$

$$t = 300\ sec$$

29. Two discs of same mass and different radii are made of different materials such that their thicknesses are 1 cm and 0.5 cm respectively. The densities of materials are in the ratio 3: 5. The moment of inertia of these discs respectively about their diameters will be in the ratio of $\frac{x}{6}$. The value of x is _______.

Sol. (5)

$$M_1 = M_2$$

$$S_1\left(\pi R_1^2 t_1\right) = S_2\left(\pi R_2^2 t_2\right)$$

$$\frac{R_1^2}{R_2^2} = \frac{5}{3} \times \frac{0.5}{1} = \frac{5}{6}$$

$$I = \frac{MR^2}{4}$$

$$\frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^2 = \frac{5}{6}$$

30. The displacement equations of two interfering waves are given by $y_1 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$ cm, $y_2 = 5[\sin \omega t + \sqrt{3}\cos \omega t]$cm respectively. The amplitude of the resultant wave is _______ cm.

Sol. (20)

$$y_1 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$y_2 = 10\left[\sin \omega t \times \frac{1}{2} + \frac{\sqrt{3}}{2}\cos \omega t\right]$$

$$y_2 = 10\sin\left(\omega t + \frac{\pi}{3}\right)$$

y_1 and y_2 are in same phase

$A_r = A_1 + A_2 = 20$ cm

Chemistry

SECTION - A

31. Which one of the following statements is incorrect ?
(1) van Arkel method is used to purify tungsten.
(2) The malleable iron is prepared from cast iron by oxidising impurities in a reverberatory furnace.
(3) Cast iron is obtained by melting pig iron with scrap iron and coke using hot air blast.
(4) Boron and Indium can be purified by zone refining method.

Sol. **1**
Van Arkel method is used for refining of Ti, Zr, Hf, Bi, B

32. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A) : The first ionization enthalpy of 3 d series elements is more than that of group 2 metals
Reason (R) : In 3d series of elements successive filling of d-orbitals takes place.
In the light of the above statements, choose the correct answer from the options given below :
(1) Both (A) and (R) are true but (R) is not the correct explanation of (A)
(2) Both (A) and (R) are true and (R) is the correct explanation of (A)
(3) (A) is true but (R) is false
(4) (A) is false but (R) is true

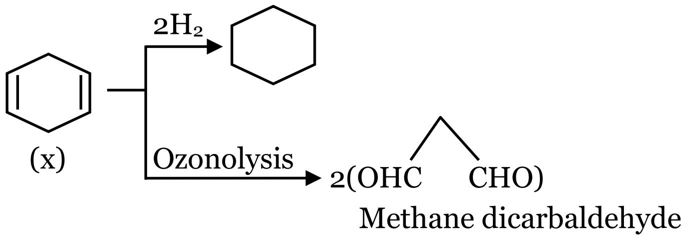
Sol. **2**
d-block elements have more first I.E. than group 2 elements due to poor shielding of d-orbitals

33. Given below are two statements :
Statement I : H_2O_2 is used in the synthesis of Cephalosporin
Statement II : H_2O_2 is used for the restoration of aerobic conditions to sewage wastes.
In the light of the above statements, choose the most appropriate answer from the options given below:
(1) Both Statement I and Statement II are incorrect
(2) Statement I is incorrect but Statement II is correct
(3) Statement I is correct but Statement II is incorrect
(4) Both Statement I and Statement II are correct

Sol. **4**
Fact (NCERT based)

34. A hydrocarbon 'X' with formula C_6H_8 uses two moles H_2 on catalytic hydrogenation of its one mole. On ozonolysis, 'X' yields two moles of methane dicarbaldehyde. The hydrocarbon 'X' is :
(1) cyclohexa-1, 4-diene (2) cyclohexa - 1, 3 – diene
(3) 1-methylcyclopenta-1, 4-diene (4) hexa-1, 3, 5-triene

Sol. **1**



35. Evaluate the following statements for their correctness.

A. The elevation in boiling point temperature of water will be same for $0.1 M NaCl$ and $0.1 M$ urea.

B. Azeotropic mixtures boil without change in their composition.

C. Osmosis always takes place from hypertonic to hypotonic solution.

D. The density of $32\% H_2SO_4$ solution having molarity $4.09 M$ is approximately $1.26\ g\ mL^{-1}$.

E. A negatively charged sol is obtained when KI solution is added to silver nitrate solution.

Choose the correct answer from the options given below :

(1) A, B and D only (2) B and D only

(3) B, D and E only (4) A and C only

Sol. **2**

(A) Value of i is different for both the solutions.

(B) True

(C) Osmosic takes place from hypotonic to hypertonic solution.

(D) $d = \frac{100}{\frac{1000}{4.09} \times \frac{32}{98}} \cong$ 1.26 gm/ml

(E) Positively charged sol will be form.

36. The Lewis acid character of boron tri halides follows the order :

(1) $BI_3 > BBr_3 > BCl_3 > BF_3$ (2) $BBr_3 > BI_3 > BCl_3 > BF_3$

(3) $BCl_3 > BF_3 > BBr_3 > BI_3$ (4) $BF_3 > BCl_3 > BBr_3 > BI_3$

Sol. **1**

Due to back bonding Lewis acidic strength of Boron halides is $BI_3 > BBr_3 > BCl_3 > BF_3$

37. When a hydrocarbon A undergoes complete combustion it requires 11 equivalents of oxygen and produces 4 equivalents of water. What is the molecular formula of A ?

(1) C_5H_8 (2) $C_{11}H_4$ (3) C_9H_8 (4) $C_{11}H_8$

Sol. **3**

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

$x + \frac{y}{4} = 11$ $\frac{y}{2} = 4$

x = 9 y = 8 (C_9H_8)

38. Arrange the following orbitals in decreasing order of energy.

A. $n = 3, l = 0, m = 0$ B. $n = 4, l = 0, m = 0$

C. $n = 3, l = 1, m = 0$ D. $n = 3, l = 2, m = 1$

The correct option for the order is :

(1) $D > B > C > A$ (2) $D > B > A > C$ (3) $A > C > B > D$ (4) $B > D > C > A$

Sol. **1**

According to (n+l) rule Orbital has more value of (n+l) has more energy. If value of some then orbital has more value of n has more energy

39. The element playing significant role in neuromuscular function and interneuronal transmission is :

(1) Li (2) Mg (3) Be (4) Ca

Sol. **4**

Fact (NCERT based)

40. Given below are two statements :

Statement I : Upon heating a borax bead dipped in cupric sulphate in a luminous flame, the colour of the bead becomes green

Statement II : The green colour observerd is due to the formation of copper(I) metaborate

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are true
(2) Statement I is true but Statement II is false
(3) Statement I is false but Statement II is true
(4) Both Statement I and Statement II are false

Sol. **4**

Due to formation of Cu (II) met borate it gives blue colour

41. Which of the following compounds are not used as disinfectants ?

A. Chloroxylenol B. Bithional C. Veronal D. Prontosil
E. Terpineol

Choose the correct answer from the options given below :

(1) C, D (2) B, D, E (3) A, B (4) A, B, E

Sol. **1**

* Vernonal is a tranquilizer
* Prontosil is a antibiotic drug.

42. Incorrect statement for the use of indicators in acid-base titration is :

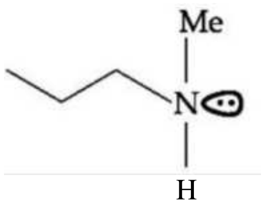
(1) Methyl orange may be used for a weak acid vs weak base titration.
(2) Phenolphthalein is a suitable indicator for a weak acid vs strong base titration.
(3) Methyl orange is a suitable indicator for a strong acid vs weak base titration.
(4) Phenolphthalein may be used for a strong acid vs strong base titration.

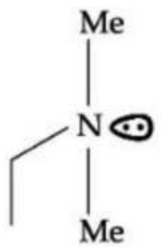
Sol. **1**

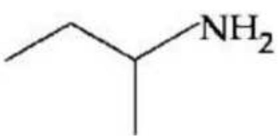
Weak acid – weak base :-
Neither phenolphthalein nor methyl orange is suitable.

43. An organic compound [A]($C_4H_{11}N$), shows optical activity and gives N_2 gas on treatment with HNO_2. The compound [A] reacts with $PhSO_2Cl$ producing a compound which is soluble in KOH.

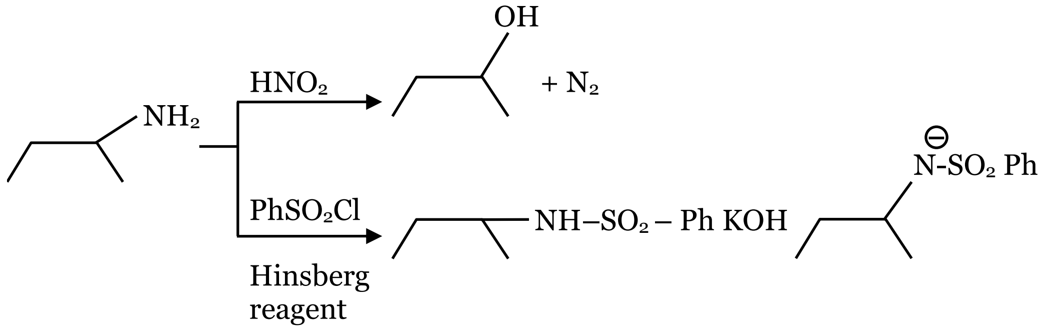
(1) 

(2) 

(3) 

(4) 

Sol. 4



44. The normal rain water is slightly acidic and its pH value is 5.6 because of which one of the following?

(1) $CO_2 + H_2O \rightarrow H_2CO_3$
(2) $2SO_2 + O_2 + 2H_2O \rightarrow 2H_2SO_4$
(3) $4NO_2 + O_2 + 2H_2O \rightarrow 4HNO_3$
(4) $N_2O_5 + H_2O \rightarrow 2HNO_3$

Sol. 1

Due to presence of CO_2 in air normal rain water is slightly acidic

45. Match List I with List II

LIST I		LIST II	
A.	Physisorption	I.	Single Layer Adsorption
B.	Chemisorption	II.	$20-40\ kJ\ mol^{-1}$
C.	$N_2(g) + 3H_2(g) \overset{Fe(s)}{\longrightarrow} 2NH_3(g)$	III.	Chromatography
D.	Analytical Application or Adsorption	IV.	Heterogeneous catalysis

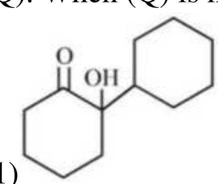
Choose the correct answer from the options given below:

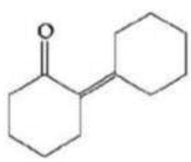
(1) A - II, B - I, C - IV, D – III
(2) A - IV, B - II, C - III, D – I
(3) A - II, B - III, C - I, D – IV
(4) A - III, B - IV, C - I, D – II

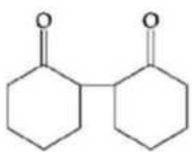
Sol. 1

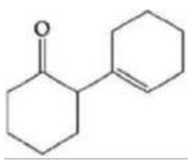
Theory based

46. Cyclohexylamine when treated with nitrous acid yields (P). On treating (P) with PCC results in (Q). When (Q) is heated with dil. NaOH we get (R) The final product (R) is :

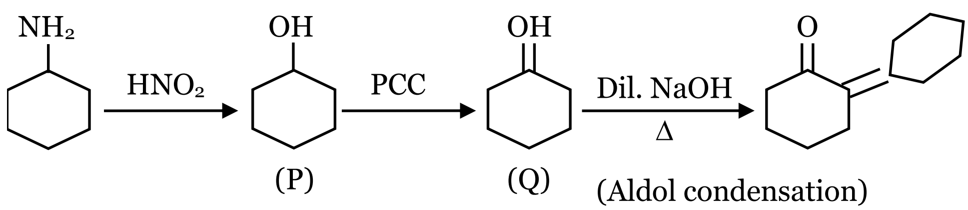
(1) 

(2) 

(3) 

(4) 

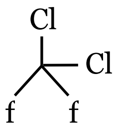
Sol. 2

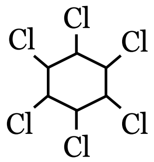


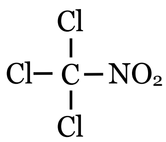
47. In the following halogenated organic compounds the one with maximum number of chlorine atoms in its structure is :

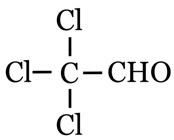
(1) Freon-12 (2) Gammaxene (3) Chloropicrin (4) Chloral

Sol. **2**

1. Freon – 12 

2. Gammaxene  or $C_6H_6Cl_6$

3. Chloropicrin 

4. Chloral 

48. In Dumas method for the estimation of N_2, the sample is heated with copper oxide and the gas evolved is passed over :

(1) Copper oxide (2) Ni (3) Pd (4) Copper gauze

Sol. **2**

Duma's method. The nitrogen containing organic compound, when heated with CuO in a atmosphere of CO_2, yields free N_2 in addition to CO_2 and H_2O.

$$C_xH_yN_z + (2x + \frac{y}{2})CuO \rightarrow$$

$$xCO_2 + \frac{y}{2}H_2O + \frac{z}{2}N_2 + (2x + \frac{y}{2})Cu$$

Traces of nitrogen oxides formed, if any, are reduced to nitrogen by passing the gaseous mixture over heated copper gauze.

49. Which of the following elements have half-filled f-orbitals in their ground state ?

(Given : atomic number $Sm = 62; Eu = 63; Tb = 65; Gd = 64, Pm = 61$)

A. Sm B. B. EuC. Tb D. Gd E. Pm

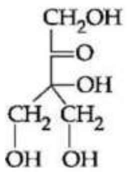
Choose the correct answer from the options given below :

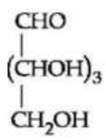
(1) A and B only (2) A and E only (3) C and D only (4) B and D only

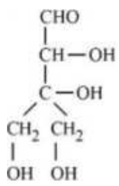
Sol. **4**

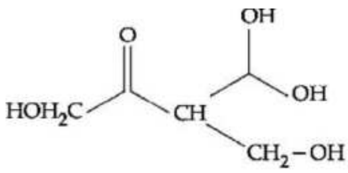
Fact (NCERT based)

50. Compound A, $C_5H_{10}O_5$, given a tetraacetate with AC_2O and oxidation of A with $Br_2 - H_2O$ gives an acid, $C_5H_{10}O_6$. Reduction of A with HI gives isopentane. The possible structure of A is :

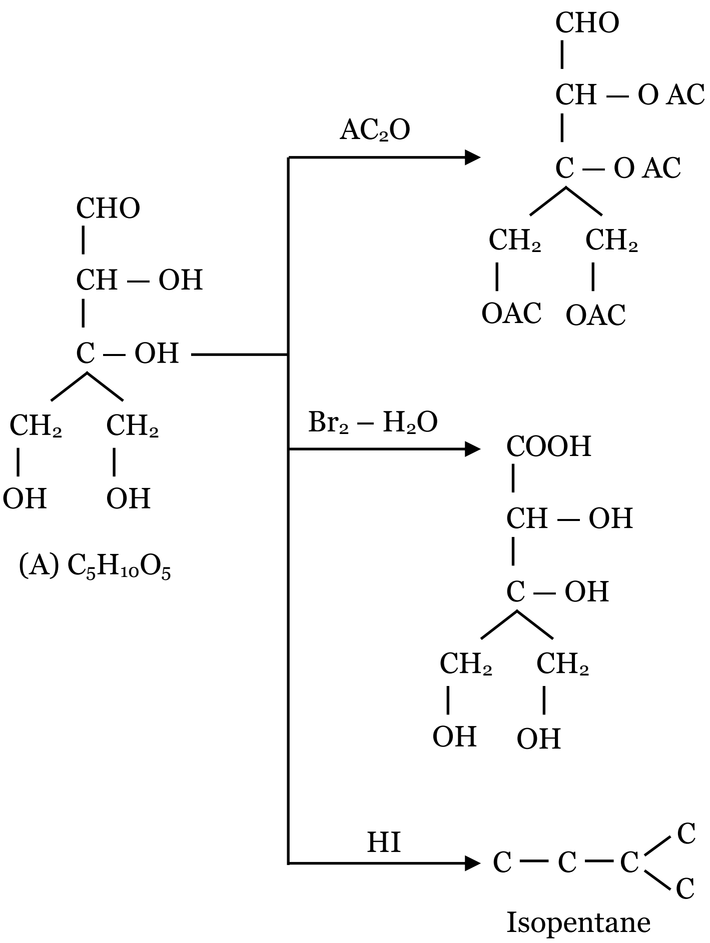
(1) 

(2) 

(3) 

(4) 

Sol. **3**



SECTION B

51. The rate constant for a first order reaction is 20 min^{-1}. The time required for the initial concentration of the reactant to reduce to its $\frac{1}{32}$ level is ______ 10^{-2} min. (Nearest integer)

(Given : ln 10 = 2.303
log 2 = 0.3010)

Sol. **17**

$$t = \frac{1}{20} \ln 32$$

$$= \frac{2.303 \times 5 \times 0.3010}{20} = 17.33 \times 10^{-2}$$

$$\approx 17 \times 10^{-2}$$

52. Enthalpies of formation of $CCl_4(g), H_2O(g), CO_2(g)$ and $HCl(g)$ are $-105, -242, -394$ and -92 kJ mol^{-1} respectively. The magnitude of enthalpy of the reaction given below is kJmol^{-1}. (nearest integer)

$$CCl_4(g) + 2H_2O(g) \rightarrow CO_2(g) + 4HCl(g)$$

Sol. **173**

$$\Delta H_r = (\Delta H_f)CO_2 + (\Delta H_f)_{HCl} - (\Delta H_f)_{CCl_4} - 2(\Delta H_f)H_2O$$

$$= -173$$

53. A sample of a metal oxide has formula $M_{0.83}O_{1.00}$. The metal M can exist in two oxidation states +2 and +3. In the sample of $M_{0.83}O_{1.00}$, the percentage of metal ions existing in +2 oxidation state is %. (nearest integer)

Sol. **59**

$M^{2+} \rightarrow x \quad M^{3+} \rightarrow (0.83 - x)$

$2x + 3(0.83 - x) = 2$

$x = 2.49 - 2 = 0.49$

$$\% \text{ of } M^{2+} = \frac{0.49}{0.83} \times 100 = 59\%$$

54. The resistivity of a 0.8M solution of an electrolyte is $5 \times 10^{-3} \Omega\text{cm}$. Its molar conductivity is $\times 10^4 \Omega^{-1} \text{ cm}^2 \text{ mol}^{-1}$
(Nearest integer)

Sol. **25**

$$K = \frac{1}{5 \times 10^{-3}}$$

$$\wedge_m = K \times \frac{1000}{M} = \frac{1}{5 \times 10^{-3}} \times \frac{1000}{0.8}$$

$$= \frac{1000}{40} \times 10^4 = 25 \times 10^4$$

55. At 298 K, the solubility of silver chloride in water is 1.434×10^{-3} g L^{-1}. The value of $-\log K_{sp}$ for silver chloride is (Given mass of Ag is 107.9 g mol^{-1} and mass of Cl is 35.5 g mol^{-1})

Sol. 10

1.434×10^{-3} gm/L

$$= \frac{1.434 \times 10^{-3}}{107.9 + 35.5} M = 10^{-5} m$$

$Ksp = S^2 = 10^{-10} \Rightarrow -\log Ksp = +10$

56. If the CFSE of $[Ti(H_2O)_6]^{3+}$ is -96.0 kJ/mol, this complex will absorb maximum at wavelength nm. (nearest integer)

Assume Planck's constant $(h) = 6.4 \times 10^{-34}$ Js, Speed of light $(c) = 3.0 \times 10^8$ m/s and Avogadro's Constant $(N_A) = 6 \times 10^{23}$/mol

Sol. **480**

$$CFSE = \left(-\frac{2}{5}x + \frac{3}{5}y\right)\Delta_0$$

$$-96 = \frac{-2}{5} \times 1 \times \Delta_0$$

$$\Delta_0 = 240 \text{ kJ / mole} = \frac{240 \times 10^3}{NA / \text{molecule}}$$

$$\Delta_0 = \frac{hc}{\lambda abs}$$

$$\frac{240 \times 10^3}{6 \times 10^{23}} = \frac{6.4 \times 10^{-34} \times 3 \times 10^8}{\lambda abs}$$

$$\lambda ab = \frac{6.4 \times 3 \times 6 \times 10^{-3}}{240 \times 10^3} m$$

$= 4.8 \times 10^{-7}$ m

$= 4.8 \times 10^{-7} \times 10^9$ nm

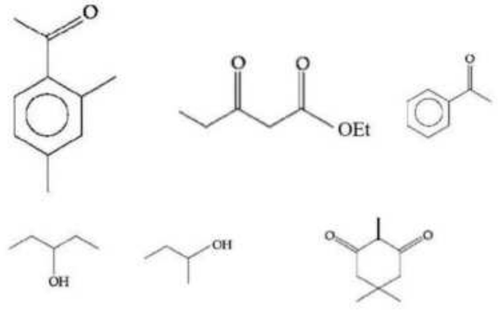
$= 480$ nm

57. The number of alkali metal(s), from Li, K, Cs, Rb having ionization enthalpy greater than 400 kJ mol^{-1} and forming stable super oxide is

Sol. **2**

K, Rb and Cs form stable super oxides but Cs has ionisation enthalpy less than 400 kJ.

58. The number of molecules which gives haloform test among the following molecules is



Sol. **3**

59. Assume carbon burns according to following equation :

$2C_{(s)} + O_{2(g)} \rightarrow 2CO(g)$

when 12 g carbon is burnt in 48 g of oxygen, the volume of carbon monoxide produced is $\times 10^{-1}$ L at STP [nearest integer]

[Given: Assume co as ideal gas, Mass of c is 12 g mol^{-1}, Mass of O is 16 g mol^{-1} and molar volume of an ideal gas STP is 22.7 L mol^{-1}]

Sol. **227**

$2C + O_2 \rightarrow 2CO$

12g 48 gm

1 mole 1.5 mole

"C" is LR.

Moles of CO formed = 1

Volume of CO = 1 × 22.7

= 227×10^{-1} L

60. Amongst the following, the number of species having the linear shape is

$XeF_2, I_3^+, C_3O_2, I_3^-, CO_2, SO_2, BeCl_2$ and $BCl_2^{\ominus}$

Sol. **5**

XeF_2, I_3^-, C_3O_2, CO_2, $BeCl_2$

Mathematics

Section A

61. The equation $e^{4x}+8e^{3x}+13e^{2x}-8e^{x}+1=0, x\in\mathbb{R}$ has :
(1) four solutions two of which are negative
(2) two solutions and only one of them is negative
(3) two solutions and both are negative
(4) no solution

Sol. **3**

$e^{4x} + 8e^{3x} + 13e^{2x} + 13e^{2x} - 8e^{x} + 1 = 0, x \in R$

Let $e^x = t > 0$ & $x = \ln t$

$t^4 + 8t^3 + 13t^2 - 8t + 1 = 0$

Dividing by t^2,

$t^2 + 8t + 13 - \frac{8}{t} + \frac{1}{t^2} = 0$

$t^2 + \frac{1}{t^2} + 8\left(t - \frac{1}{t}\right) + 13 = 0$

Let $t - \frac{1}{t} = u \Rightarrow t^2 + \frac{1}{t^2} - 2\ u^2$

$\Rightarrow t^2 + \frac{1}{t^2} = u^2 + 2$

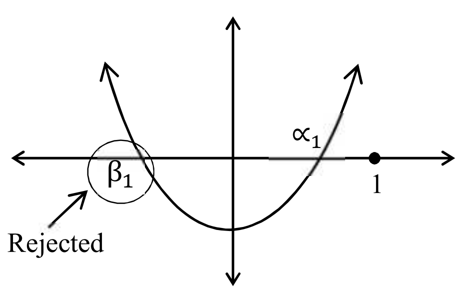
$u^2 + 2 + 8u + 13 = 0$

$(u + 3)(u + 5) = 0$

$u = -3$ & $u = -5$

$t - \frac{1}{t} = -3$

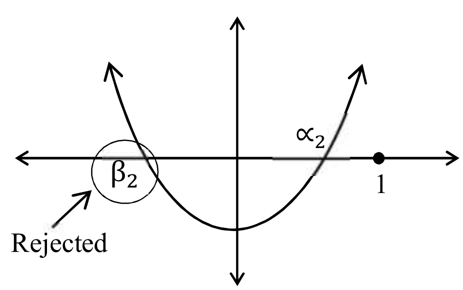
$t^2 + 3t - 1 = 0$



$0 < \alpha_1 < 1$

$t - \frac{1}{t} = -5$

$t^2 + 5t - 1 = 0$



$0 < \alpha_2 < 1$

$\Rightarrow x_1 = \ell n\ \alpha_1 < 0$ $\Rightarrow x_2 = \ln \alpha_2 < 0$

62. Among the relations

$S = \left\{(a,b): a,b \in \mathbb{R} - \{0\}, 2 + \frac{a}{b} > 0\right\}$ and $T = \{(a,b): a,b \in \mathbb{R}, a^2 - b^2 \in \mathbb{Z}\}$,

(1) neither S nor T is transitive (2) S is transitive but T is not

(3) T is symmetric but S is not (4) both S and T are symmetric

Sol. **3**

$S = \{(a,b) \mid a,b \in R - \{0\}, 2 + \frac{a}{b} > 0\}$ &

$T = \left\{(a,b) \mid a,b \in R, a^2 - b^2 \in Z\right\}$

For S, $2 + \frac{a}{b} > 0 \Rightarrow \frac{a}{b} > -2$

Let $(-1,2) \in S\left(\therefore -\frac{1}{2} > -2\right)$

$\&(2,-1) \notin s\left(\therefore \frac{2}{-1} \text{ not greater than } -2\right)$

So, S is not symmetric

For T,

If $(a,b) \in T \Rightarrow a^2 - b^2 \in Z$

$\Rightarrow -(a^2 - b^2) \in Z$

$\Rightarrow b^2 - a^2 \in Z$

$\Rightarrow (b,a) \in T$

So, T is symmetric

63. Let $\alpha > 0$. If $\int_0^\alpha \frac{x}{\sqrt{x+\alpha} - \sqrt{x}} dx = \frac{16 + 20\sqrt{2}}{15}$, then α is equal to :

(1) 4 (2) $2\sqrt{2}$ (3) $\sqrt{2}$ (4) 2

Sol. **3**

$\alpha > 0$

$I = \int_0^\alpha \frac{x}{\sqrt{x+2} - \sqrt{x}} dx = \frac{16 + 2\sqrt{2}}{15}$

$I = \int_0^\alpha \frac{x\left(\sqrt{x+\alpha} + \sqrt{x}\right)}{\alpha} dx$

$= \frac{1}{\alpha}\left[\int_0^\alpha x\sqrt{x+\alpha}\sqrt{x+\alpha}\, dx + \int_0^\alpha x^{3/2} dx\right]$

$$I_1 = \int_0^{\alpha} (x+\alpha-\alpha)\sqrt{x+\alpha}\,dx$$

$$= \int_0^{\alpha} (x+\alpha)^{3/2} - \alpha\int_0^{\alpha} (x+\alpha)^{1/2} dx$$

$$= \frac{2}{5}\left[(x+\alpha)^{5/2}\right]^{\alpha} - \frac{\alpha(2)}{3}\left[(x+\alpha)^{3/2}\right]_0^{\alpha}$$

$$= \frac{2}{5}\left[(2\alpha)^{5/2} - \alpha^{5/2}\right] - \frac{2\alpha}{3}\left[(2\alpha)^{3/2} - \alpha^{3/2}\right]$$

$$= \frac{2}{5}(2\alpha)^{5/2} - \frac{2}{5}\alpha^{5/2} - \frac{(2\alpha)^{5/2}}{3} + \frac{2\alpha^{5/2}}{3}$$

$$= (2\alpha)^{5/2}\left[\frac{2}{5} - \frac{1}{3}\right] + 2\alpha^{5/2}\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$= (2\alpha)^{5/2}\left[\frac{1}{15}\right] + 2\alpha^{5/2}\left[\frac{2}{15}\right]$$

$$= \frac{(2\alpha)^{5/2}}{15} + \frac{4\alpha^{5/2}}{15}$$

$$= \frac{4\alpha^{5/2}}{15}\left[\sqrt{2}+1\right]$$

$$I_2 = \int_0^{\alpha} x^{3/2} dx = \frac{2}{5}\left[x^{5/2}\right]_0^{\alpha} = \frac{2}{5}\alpha^{5/2}$$

$$I = \frac{1}{\alpha}(I_1 + I_2)$$

$$I = \frac{1}{\alpha}\left[\frac{4\alpha^{5/2}\left(\sqrt{2}+1\right)}{15} + \frac{2}{5}\alpha^{5/2}\right]$$

$$= \frac{2\alpha^{5/2}}{15\alpha}\left[2\left(\sqrt{2}+1\right)+3\right]$$

$$= \frac{2}{15}\alpha^{3/2}\left[2\sqrt{2}+5\right]$$

$$\frac{16+20\sqrt{2}}{15} = \frac{2}{15}\alpha^{3/2}\left[2\sqrt{2}+5\right]$$

$$\alpha^{3/2} = 2\sqrt{2}$$

$$\alpha^3 = 8$$

$$\alpha = 2$$

64. The complex number $z = \frac{i-1}{\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}}$ is equal to :

(1) $\sqrt{2}i\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$ (2) $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

(3) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$ (4) $\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}$

Sol. **3**

$$Z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$$

$$i-1 = \sqrt{2}\left(\frac{-1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) = \sqrt{2}\cdot e^{i\cdot\frac{3\pi}{4}}$$

$$z = \frac{\sqrt{2}\cdot e^{i\cdot\frac{3\pi}{4}}}{e^{i\cdot\frac{3\pi}{4}}}$$

$$= \sqrt{2}\cdot e^{i\left(\frac{3\pi}{4}-\frac{\pi}{3}\right)}$$

$$= \sqrt{2}\, e^{\frac{5\pi}{12}i}$$

$$= \sqrt{2}\left(\cos\left(\frac{5\pi}{12}\right) + i\sin\left(\frac{5\pi}{12}\right)\right)$$

65. Let $y = y(x)$ be the solution of the differential equation $(3y^2 - 5x^2)y\,dx + 2x(x^2 - y^2)dy = 0$ such that $y(1) = 1$. Then $|(y(2))^3 - 12y(2)|$ is equal to :

(1) $16\sqrt{2}$ (2) $32\sqrt{2}$ (3) 32 (4) 64

Sol. **2**

$$\left(3y^2 - 5x^2\right)ydx + 2x\left(x^2 - y^2\right)dy = 0$$

$$2x\left(x^2 - y^2\right)dy = \left(5x^2 - 3y^2\right)y\,dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{\left(5x^2 - 3y^2\right)y}{2x\left(x^2 - y^2\right)}$$

Let y = tx

$$\frac{dy}{dx} = t + x\frac{dt}{dx}$$

$$t + x\frac{dt}{dx} = \frac{\left(5 - 3t^2\right)t}{2\left(1 - t^2\right)}$$

$$x\frac{dt}{dx} = t\left[\frac{5 - 3t^2 - 2 + 2t^2}{2\left(1 - t^2\right)}\right]$$

$$=\frac{t}{2}\left[\frac{3-t^2}{1-t^2}\right]$$

$$\frac{1}{3}\int\frac{3\left(1-t^2\right)dt}{3t-t^3}=\int\frac{dx}{2x}$$

$$\frac{1}{3}\ell n\left|3t-t^3\right|=\frac{1}{2}\ln|x|+c$$

$$2\ell n\left|3t-t^3\right|=3\ln|x|+6c$$

$$\left(3t-t^3\right)^2=x^3\lambda$$

$$\left(\frac{3y}{x}-\frac{y^3}{x^3}\right)^2=x^3\lambda$$

$$\left(3yx^2-y^3\right)^2=x^9\lambda$$

$$x=1\Rightarrow y=1$$

$$(3-1)^2=1\times\lambda$$

$$\lambda=4$$

$$\left(3yx^2-y^3\right)^2=4x^9$$

Let x =2

$$\left(3y(2)\times4-\left(y(2)^3\right)^2\right)=4(2)^9$$

Taking square root both the sides,

$$\left|(y(2))^3-12y(2)\right|=2(2)^{9/2}$$

$$=2(2)^4\sqrt{2}=32\sqrt{2}$$

66. $\lim\limits_{x\to\infty}\frac{(\sqrt{3x+1}+\sqrt{3x-1})^6+(\sqrt{3x+1}-\sqrt{3x-1})^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$

(1) does not exist (2) is equal to 27 (3) is equal to $\frac{27}{2}$ (4) is equal to 9

Sol. **2**

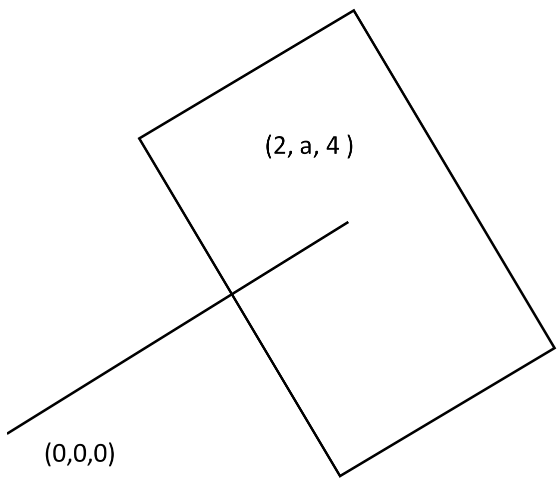
$$\lim_{x\to\infty}\frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}$$

Taking height power common

$$\lim_{x\to\infty}\frac{x^6\left[\left(\sqrt{3+\frac{1}{x}}+\sqrt{3-\frac{1}{x}}\right)^6+\left(\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right)^6\right]}{x^6\left[\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6\right]}$$

$$=\frac{\left(\sqrt{3}+\sqrt{3}\right)^6+\left(\sqrt{3}-\sqrt{3}\right)^6}{(1+1)^6+(1-1)^6}$$

$$=\frac{\left(2\sqrt{3}\right)^6}{2^6}=\left(\sqrt{3}\right)^6=27$$



67. The foot of perpendicular from the origin O to a plane P which meets the co-ordinate axes at the points A, B, C is $(2, a, 4), a \in N$. If the volume of the tetrahedron OABC is 144unit^3, then which of the following points is NOT on P ?

(1) (0,6,3) (2) (0,4,4) (3) (2,2,4) (4) (3,0,4)

Sol. **4**

$\vec{n}=(2,a,4)$

Plane is

$2x+ay+4z=4+a^2+16$

$=20+a^2$

$A\left(\frac{20+a^2}{2},0,0\right)$

$B\left(0,\frac{20+a^2}{a},0\right)$

$C\left(0,0,\frac{20+a^2}{4}\right)$

$\frac{1}{6}\times\frac{\left(20+a^2\right)^3}{8a}=144=2^4\times 3^2$

$\left(20+a^2\right)^3=2^8 3^3 a$

$20+a^2=(4a)^{\frac{1}{3}}(12)$

a = 2 satisfies above equation

So, 2x + 2y + 4z = 24

X + Y + 2z = 12

(A) (0, 6, 3)

(B) (0, 4, 4)

(C) (2,2,4)

(D) (3,0,4)

68. Let $(a,b)\subset(0,2\pi)$ be the largest interval for which $\sin^{-1}(\sin\theta)-\cos^{-1}(\sin\theta)>0,\theta\in(0,2\pi)$, holds. If $\alpha x^2+\beta x+\sin^{-1}(x^2-6x+10)+\cos^{-1}(x^2-6x+10)=0$ and $\alpha-\beta=b-a$, then α is equal to :

(1) $\frac{\pi}{16}$ (2) $\frac{\pi}{48}$ (3) $\frac{\pi}{12}$ (4) $\frac{\pi}{8}$

Sol. **3**

$x^2-6x+10=(x-3)^2+1\geq 1$

So, x = 3 is the only element in the Domain

So, $\alpha x^2+\beta x+\sin^{-1}\left(x^2-6x+10\right)+\cos^{-1}\left(x^2-6x+10\right)=0$

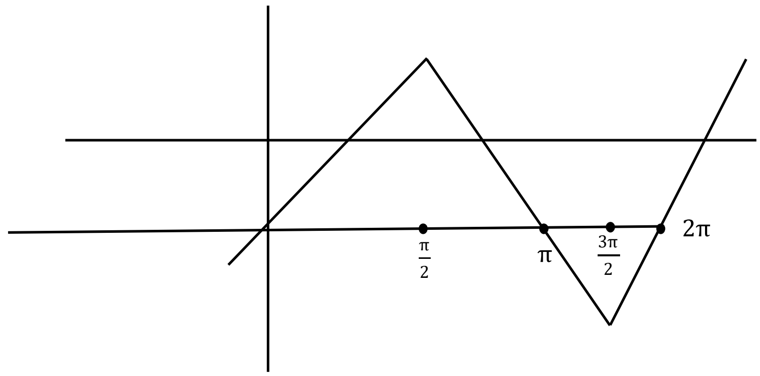
$9\alpha+3\beta+\frac{\pi}{2}=0$

$\text{Sin}^{-1}(\sin\theta)-\cos^{-1}(\sin\theta)>0$

$\sin^{-1}(\sin\theta)-\left(\frac{\pi}{2}-\sin^{-1}(\sin\theta)\right)>0$

$2\sin^{-1}(\sin\theta)>\frac{\pi}{2}$

$\sin^{-1}(\sin\theta)>\frac{\pi}{4}$



So, $\theta \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$\alpha - \beta = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}$ ……(1)

& $9\alpha + 3\beta = \frac{-\pi}{2}$

$3\alpha + \beta = \frac{-\pi}{6}$ ……(2)

Adding (1) & (2)

$4\alpha = \frac{\pi}{2} - \frac{\pi}{6} = \frac{2\pi}{6}$

$\alpha = \frac{\pi}{12}$

69. Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and α (>0), and the mean and standard deviation of marks of class B of n students be respectively 55 and $30-\alpha$. If the mean and variance of the marks of the combined class of $100+$ n studants are respectively 50 and 350 , then the sum of variances of classes A and B is :

(1) 650 (2) 450 (3) 900 (4) 500

Sol. **4**

$m_A = 40$ $S.d_A = \alpha > 0$ $n_A = 100$

$m_B = 55$ $S.d_B = 30 - \alpha$ $n_B = n$

$m_{AVB} = 50$ $Variance_{AVB} = 350$ $n_{AVB} = 100 + n$

$A = \{x_1, \ldots\ldots., x_{100}\}$ $B = \{y_1 \ldots., y_n\}$

$\sum x_i = 4000$

$\sum y_i = 55n$

$\sum (x_i + y_i) = 50(100 + n)$

$4000 + 55n = 5000 + 50n$

Using formula of standard deviation

$5n = 1000 \; n = 200$

$\alpha^2 = \frac{\sum x_i^2}{100} - (40)^2$ | $(30-\alpha)^2 \frac{\sum y_i^2}{200} - (55)^2$

$\sum x_i^2 = 100(1000 + \alpha^2)$

$\sum y_i^2 = 200((55)^2 + (30-\alpha)^2)$

$350 = \frac{\sum (x_i^2 + y_i^2)}{300} - (50)^2$

$\sum x_i^2 + \sum y_i^2 = ((50)^2 + 350)300$

$160000 + 100\alpha^2 + 200(55)^2 + 200(30-\alpha)^2$

$(50)^2 300 + 350 \times 300$

$1600 + \alpha^2 + 6050 + 2(30-\alpha)^2 = 7500 + 1050$

$\alpha^2 + 1800 - 120\alpha + 2\alpha^2 - 900 = 0$

$3\alpha^2 - 120\alpha + 900 = 0$

$\alpha^2 - 40\alpha + 300 = 0$

$(\alpha - 10)(\alpha - 30) = 0$

$\alpha = 10 \text{ or } \alpha = 30$

if $\alpha = 10$ VarA = 100 & VarB = 400

$\text{Var}_A + \text{V}_{arB} = 500$

70. The absolute minimum value, of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where $[t]$ denotes the greatest integer function, in the interval $[-1,2]$, is:

(1) $\frac{1}{4}$ (2) $\frac{3}{2}$ (3) $\frac{5}{4}$ (4) $\frac{3}{4}$

Sol. **4**

$f(x) = |x^2 - x + 1| + [x^2 - x + 1]$

$x \in [-1,2]$ Here $x^2 - x + 1 > 0, \forall x \in R$

Minimum value of $x^2 - x + 1$ occurs at $a = \frac{1}{2} \in [-1,2]$

So, $\text{Min} f(x) = f\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \left[\frac{3}{4}\right] = \frac{3}{4}$

71. Let H be the hyperbola, whose foci are $(1 \pm \sqrt{2}, 0)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is

(1) $\frac{3}{2}$ (2) 2 (3) 3 (4) $\frac{5}{2}$

Sol. **2**

$F_1F_2 = 2ae = \left(1+\sqrt{2}\right) - \left(1-\sqrt{2}\right) = 2\sqrt{2}$

$ae = \sqrt{2}$

$e = \sqrt{2}$

$\Rightarrow a = 1 \quad \Rightarrow b = 1 \left(\because e = \sqrt{2}\right)$

L.L.R. $= \dfrac{2b^2}{a} = \dfrac{2(1)^2}{1} = 2$

72. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $a_7 = 3$, the product $a_1 a_4$ is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to :

(1) 9 (2) $\frac{33}{4}$ (3) $\frac{381}{4}$ (4) 24

Sol. **4**

$a_7 = 3 \; a_1a_4$ minimum

$a + 6d = 3$

$a(a+3d) \rightarrow$ minimum

$S_n = 0 \quad \Rightarrow \dfrac{n}{2}\left[na_1 + (n-1)d\right] = 0$

$2a_1 + (n-1)d = 0 \qquad \ldots\ldots(1)$

Let $a(a+3d)$ is minimum

$f(d) = (3-6d)(3-6d+3d)$

$f(d) = (3-6d)(3-3d)$

$= 18d^2 - 27d + 9$ is minimum at $d = \dfrac{27}{2\times18} = \dfrac{9\times3}{2\times9\times2} = \dfrac{3}{4}$

So, $d = \dfrac{3}{4}$

$a_1 + 6d = 3$

$a_1 = 3 - 6\left(\dfrac{3}{4}\right) = 3 - \dfrac{9}{2} = -\dfrac{3}{2}$

Putting $a_1 = \dfrac{-3}{2}$ & $d = \dfrac{3}{4}$ in (1)

$$2\left(\frac{-3}{2}\right)+(n-1)\left(\frac{3}{4}\right)=0$$

$$\frac{3}{4}(n-1)=3$$

$$n-1=4$$

$$n=5$$

$$ni-4a_{n(n+2)}$$

$n=5$ so $n!=5!=120$

& $a_{5(7)}=a_{35}=\frac{-3}{2}+(34)\left(\frac{3}{4}\right)$

$$=\frac{-3}{2}+\frac{51}{2}$$

$$=\frac{48}{2}=24$$

$$5!-4(24)=24$$

73. If a point $P(\alpha,\beta,\gamma)$ satisfying

$$(\alpha\beta\gamma)\begin{pmatrix}2 & 10 & 8\\ 9 & 3 & 8\\ 8 & 4 & 8\end{pmatrix}=(000)$$

lies on the plane $2x+4y+3z=5$, then $6\alpha+9\beta+7\gamma$ is equal to :

(1) -1 (2) $\frac{11}{5}$ (3) $\frac{5}{4}$ (4) 11

Sol. **4**

$2\alpha+9\beta+8\gamma=0$ …….(1)

$10\alpha+3\beta+4\gamma=0$ …….(2)

$8\alpha+8\beta+8\gamma=0$ …….(3)

$\alpha+\beta+\gamma=0$

$\gamma=-\alpha-\beta$

$2\alpha+9\beta-8\alpha-8\beta=0$

$\beta=6\alpha$

$\gamma=-\alpha-6\alpha=-7\alpha$

$(\alpha,6\alpha,-7\alpha)$ Satisfies the above system of equation

$2\alpha+4(6\alpha)+3(-7\alpha)=5$

$5\alpha=5$

$\alpha=1$

$\beta=6$

$\gamma = -7$

$6\alpha + 9\beta + 7\gamma = 6 + 54 - 49 = 11$

74. Let : $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \vec{b} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{c} = 5\hat{\imath} - 3\hat{\jmath} + 3\hat{k}$ be there vectors. If $\vec{r}$ is a vector such that, $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then $25|\vec{r}|^2$ is equal to

(1) 560 (2) 449 (3) 339 (4) 336

Sol. **3**

$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$

$\vec{r} \times \vec{b} - \vec{c} \times \vec{b} = \vec{0}$

$(\vec{r} - \vec{c}) \times \vec{b} = \vec{0}$

$\vec{r} - \vec{c} = \lambda \vec{b}$

$\vec{r} = \lambda \vec{b} + \vec{c} = (\lambda + 5)\hat{i} - (\lambda + 3)\hat{j} + (2\lambda + 3)\hat{k}$

$\vec{r}.\vec{a} = 0$

$1(\lambda + 5) - 2(\lambda + 3) + 3(2\lambda + 3) = 0$

$5\lambda + 8 = 0 \Rightarrow \lambda = \frac{-8}{5}$

$\vec{r} = \frac{17}{5}\hat{i} - \frac{7}{5}\hat{j} + \frac{1}{5}\hat{k}$

$25|\vec{r}|^2 = 17^2 + 7^2 + 1^2 = 339$

75. Let the plane $P: 8x + \alpha_1 y + \alpha_2 z + 12 = 0$ be parallel to the line L: $\frac{x+2}{2} = \frac{y-3}{3} = \frac{z+4}{5}$. If the intercept of P on the y-axis is 1 , then the distance between P and L is :

(1) $\sqrt{\frac{7}{2}}$ (2) $\sqrt{\frac{2}{7}}$ (3) $\frac{6}{\sqrt{14}}$ (4) $\sqrt{14}$

Sol. **4**

$y - \text{intercept} = \frac{-12}{\alpha_1} = 1$

$\alpha_1 = -12$ & $\vec{n} = (8, \alpha_1, \alpha_2)$

$\vec{\ell} = (2, 3, 5)$

$\vec{n}.\vec{\ell} = 0$ ($\because$ plane P & line L are parallel)

$16 + 3\alpha_1 + 5\alpha_2 = 0$

$16 - 36 + 5\alpha_2 = 0$

$5\alpha_2 = 20$

$\alpha_2 = 4$

$8x - 12y + 4z + 12 = 0$

$\Rightarrow 2x - 3y + z + 3 = 0$

(–2, 3, – 4) is a point on the line L distance bet^n the point (–2, 3, –4) and the plane P is :-

$d = \frac{|-4-9-4+3|}{\sqrt{2^2+3^2+1^2}}$

$= \frac{14}{\sqrt{14}} = \sqrt{14}$

76. Let P be the plane, passing through the point $(1, -1, -5)$ and perpendicular to the line joining the points $(4, 1, -3)$ and $(2, 4, 3)$. Then the distance of P from the point $(3, -2, 2)$ is

(1) 5 (2) 4 (3) 7 (4) 6

Sol. **1**

Let A(4, 1, –3) & B(2, 4, 3)

$\vec{n} = \overrightarrow{AB} = (-2, 3, 6)$

Plane P is :

$-2(x-1)+3(y+1)+6(z+5) = 0$

$-2x + 2 + 3y + 3 + 6z + 30 = 0$

$2x - 3y - 6z = 35$

Distance of P from the point (3, –2, 2) is

$= \frac{|6+6-12-35|}{\sqrt{2^2+3^2+6^2}}$

$= \frac{35}{7} = 5$ **Ans. (1)**

77. The number of values of $r \in \{p, q, \sim p, \sim q\}$ for which $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$ is a tautology, is:

(1) 3 (2) 4 (3) 1 (4) 2

Sol. **4**

$((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$

$(p \wedge q) \Rightarrow (r \vee q)$

$\sim (p \wedge q) \vee (r \vee q)$

$\sim p \vee \sim q \vee r \vee q$

& $(p \wedge r) \Rightarrow q$

$\sim (p \wedge r) \vee q$

$\sim p \vee \sim r \vee q$

$(\sim p \vee \sim q \vee r \vee q) \wedge (\sim p \vee \sim r \vee q)$

$\equiv \sim p \vee \sim r \vee q$

If $r = p$

$\sim p \vee \sim p \vee q \rightarrow$ Not tautology

If $r = \sim p$

$\sim p \vee p \vee q \quad \rightarrow \quad$ tautology

If $\quad r = q$

$\sim p \vee \sim q \vee q \quad \rightarrow \quad$ tautology

If $\quad r = \sim q$

$\sim p \vee q \vee q \quad \rightarrow \quad$ Not tautology

Ans. 2 (D)

78. The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle $2x^2 + 2y^2 - (1+a)x - (1-a)y = 0$, is equal to :

(1) $(0,4]$ (2) $(4, \infty)$ (3) $(2,12]$ (4) $(8, \infty)$

Sol. **4**

$$x^2 + y^2 - \left(\frac{1+a}{2}\right)x - \left(\frac{1-a}{2}\right)y = 0$$

$$x\left(x - \left(\frac{1+a}{2}\right)\right) + y\left(y - \left(\frac{1-a}{2}\right)\right) = 0$$

$$y - y_1 = m(x - x_1) \qquad (x_1, y_1) = \left(\frac{1+a}{2}, \frac{1-a}{2}\right)$$

$\&\ x + y = 0$

$-\ x - y_1 = mx - mx_1$

$mx_1 - y_1 = (1+m)x$

$$x = \frac{mx_1 - y_1}{1+m} \quad \& \quad y = \frac{y_1 - mx_1}{1+m}$$

$$\frac{\frac{y_1}{2} - y}{\frac{x_1}{2} - x} = -\frac{1}{m}$$

$$\frac{\frac{y_1}{2} - \left(\frac{y_1 - mx_1}{1+m}\right)}{\frac{x_1}{2} - \left(\frac{mx_1 - y_1}{1+m}\right)} = -\frac{1}{m}$$

$$= \frac{(1+m)y_1 - 2y_1 + 2mx_1}{(1+m)x_1 - 2mx_1 + 2y_1} = -\frac{1}{m}$$

$$\frac{m(y_1 + 2x_1) - y_1}{-mx_1 + x_1 + 2y_1} = -\frac{1}{m}$$

$m^2(y_1+2x_1) - my_1 = mx_1 - x_1 - 2y_1$

$m^2(y_1+2x_1) - (y_1 + x_2)m + x_1 + 2y_1 = 0$

$D > 0$

$(y_1+x_1)^2 - 4(y_1 + 2x_1)(x_2 + 2y_1) > 0$

$x_1 = \frac{1+a}{2}$, $y_1 = \frac{1-a}{2}$

$x_1 + y_1 = 1$

$y_1 + 2x_1 = \frac{1-a}{2} + 1 + a = \frac{3}{2} - \frac{a}{2} = \frac{3-a}{2}$

$x_1 + 2y_1 = \frac{1+a}{2} + 1 - a = \frac{3}{2} + \frac{a}{2} = \frac{3+a}{2}$

$1 - 4\left(\frac{3-a}{2}\right)\left(\frac{3+a}{2}\right) > 0$

$1 - (9 - a^2) > 0$

$a^2 - 8 > 0$

$a^2 > 8 \qquad \rightarrow (8, \infty)$

Ans. 4

79. If $\phi(x) = \frac{1}{\sqrt{x}}\int_{\frac{\pi}{4}}^{x}\left(4\sqrt{2}\sin t - 3\phi'(t)\right)dt, x > 0$, then $\emptyset'\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\frac{8}{6+\sqrt{\pi}}$ (2) $\frac{4}{6+\sqrt{\pi}}$ (3) $\frac{8}{\sqrt{\pi}}$ (4) $\frac{4}{6-\sqrt{\pi}}$

Sol. **1**

$\sqrt{x}\ \phi(x) = \int_{\frac{\pi}{4}}^{x}(4\sqrt{2}\sin t - 3\phi'(t))dt$

Differentiating w.r.t. x,

$\frac{1}{2\sqrt{x}}\phi(x) + \sqrt{x}\,\phi'(x) = 4\sqrt{2}\sin x - 3\phi'(x)$

$(\sqrt{x}+3)\phi'(x) + \frac{1}{2\sqrt{x}}\phi(x) = 4\sqrt{2}\sin x$

$\phi'(x) + \frac{1}{2\sqrt{x}(\sqrt{x}+3)}\phi(x) = \frac{4\sqrt{2}\sin x}{\sqrt{x}+3}$

Put $x = \left(\frac{\pi}{4}\right)$

$\phi'\left(\frac{\pi}{4}\right) + 0 = \frac{4\sqrt{2}\times\frac{1}{\sqrt{2}}}{\sqrt{\frac{\pi}{4}}+3} \quad \left(\because \phi\left(\frac{\pi}{4}\right) = 0\right)$

$\phi'\left(\frac{\pi}{4}\right) = \frac{4\times 2}{\sqrt{\pi}+6} = \frac{8}{\sqrt{\pi}+6}$ **Ans. 1**

80. Let $f:\mathbb{R} - \{2,6\} \rightarrow \mathbb{R}$ be real valued function defined as $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$. Then range of f is

(1) $\left(-\infty,-\frac{21}{4}\right)\cup(0,\infty)$ (2) $\left(-\infty,-\frac{21}{4}\right]\cup[1,\infty)$

(3) $\left(-\infty,-\frac{21}{4}\right]\cup\left[\frac{21}{4},\infty\right)$ (4) $\left(-\infty,-\frac{21}{4}\right]\cup[0,\infty)$

Sol. **4**

$y=\frac{x^2+2x+1}{x^2-8x+12}$

$x^2y-8xy+12y=x^2+2x+1$

$x^2(y-1)-(8y+2)x+12y-1=0$

$D\geq 0$

$(8y+2)^2-4(y-1)(12y-1)\geq 0$

$4(4y+1)^2-4(y-1)(12y-1)\geq 0$

$16y^2+8y+1-(12y^2-13y+1)\geq 0$

$4y^2+21y\geq 0$

$4y\left[y+\frac{21}{4}\right]\geq 0$

$\left(-\infty,\frac{-21}{4}\right]\cup[0,\infty)$ **Ans. 4**

Section B

81. Let $A=[a_{ij}], a_{ij}\in Z\cap[0,4], 1\leq i,j\leq 2$. The number of matrices A such that the sum of all entries is a prime number $p\in(2,13)$ is

Sol. **204**

$A=[a_{ij}]$, aij $\in Z\cap[0,4]$, so $a_{ij}=\{0,1,2,3,4\}$

$A=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}_{2\times2}$

$a_{11}+a_{12}+a_{21}+a_{22}=P$ where P is a prime number

$a_{11}+a_{12}+a_{21}+a_{22}=3, 5, 7, 11$

$(1+x+x^2+x^3+x^4)^4$

$(1-x^5)^4(1-x)^{-4}$

$(1-4x^5+6x^{10}-4x^{15}+x^{20})(1+{}^4C_1x+{}^5C_2x^2+{}^6C_3x^3+{}^7C_4x^7\ldots.)$

For 3 ${}^6C_3=\frac{6\times5\times4}{6}=20.$

For 5 $(-4)+{}^8C_5=-4+56=52.$

For 7 $(-4)\,{}^5C_2+{}^{10}C_7=-40+120=80.$

For 11 ${}^{14}C_{11}+(-4)\,{}^9C_6+6({}^4C_1)=364-336+24=52$

Total number of matrices = 204

$a_{11}+a_{12}+a_{22}+a_{12}=3$

0	0	0	3	$= \frac{4!}{3!} = 4$
0	0	2	1	$= \frac{4!}{2!} = 12$
0	1	1	1	$= \frac{4!}{3!} = 4$
				20

$a_{11} + a_{12} + a_{21} + a_{22} = 5$

0	0	1	4	$= \frac{4!}{2!} = 12$
0	0	2	3	$= \frac{4!}{2!} = 12$
0	1	1	3	$= \frac{4!}{2!} = 12$
0	1	2	2	$= \frac{4!}{2!} = 12$
1	1	1	2	$= \frac{4!}{3!} = 4$
				52

$a_{11} + a_{12} + a_{21} + a_{22} = 7$

0	0	3	4	$= \frac{4!}{2!} = 12$
0	1	3	3	$= \frac{4!}{2!} = 12$
0	1	2	4	$= 4! = 24$
1	1	1	4	$= \frac{4!}{3!} = 4$
0	2	2	3	$= \frac{4!}{3!} = 12$
1	1	2	3	$= \frac{4!}{2!} = 12$
1	2	2	2	$= \frac{4!}{3!} = 4$
				80

$a_{11} + a_{12} + a_{21} + a_{22} = 11$

0	3	4	4	$= \frac{4!}{2!} = 12$
1	2	4	4	$= \frac{4!}{2!} = 12$

1	3	3	4	$= \frac{4!}{2!} = 12$
2	3	3	4	$= \frac{4!}{3!} = 12$
2	3	3	3	$= \frac{4!}{3!} = 4$
				52

total matrix is = 20 + 52 + 80 + 52 = 204.

82. Let A be a $n \times n$ matrix such that $|A| = 2$. If the determinant of the matrix $\text{Adj}(2 \cdot \text{Adj}(2\,A^{-1})) \cdot$ is 2^{84}, then n is equal to

Sol. **84**

$|A| = 2$

$|\text{Adj}\,(2\,\text{Adj}(2A^{-1}))| = 2^{84}$

$|2\,\text{Adj}\,(2\,A^{-1})|^{n-1} = 2^{84}$

$(2^n\,|\text{Adj}\,(2\,A^{-1})|)^{n-1} = 2^{84}$

$(2^n\,|\,2^{n-1}\,\text{Adj}\,(A^{-1})|)^{n-1} = 2^{84}$

$(2^n \times (2^{n-1})^n\,|\text{Adj}\,(A^{-1})|)^{n-1} = 2^{84}$

$(2^n \times 2^{n(n-1)} \times |A^{-1}|)^{n-1} = 2^{84}$

$(2^n \times 2^{n(n-1)} \times \left(\frac{1}{2}\right)^{n-1})^{n-1} = 2^{84}$

$(2^{n+n^2-n-n+1})^{n-1} = 2^{84}$

$(2^{n^2-n+1})^{n-1} = 2^{84}$

$(n^2 - n + 1)(n + 1) = 84$

83. If the constant term in the binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{x^l}\right)^9$ is -84 and the coefficient of x^{-3l} is $2^\alpha \beta$, where $\beta < 0$ is an odd number, then $|\alpha l - \beta|$ is equal to

Sol. **98**

$$\left(\frac{x^{5/2}}{2} - \frac{4}{x^\ell}\right)^9$$

$$T_{r+1} = {}^9C_r \left(\frac{x^{5/2}}{2}\right)^{9-r} \left(\frac{-4}{x^\ell}\right)^r$$

$$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} (-4)^r \quad x^{\frac{45-5r}{2} - \ell r}$$

For constant term

$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} (-4)^r = -84$

$= {}^9C_r \left(\frac{1}{2}\right)^{9-r} (-1)^r\, 2^{2r} = -84$

$= {}^9C_r\, 2^{r-9}\, 2^{2r}\, (-1)^r = -84$

$= {}^9C_r\, 2^{3r-9}\, (-1)^r = -84$

$\Rightarrow \boxed{r = 3}$

$\frac{45 - 5r}{2} - \ell r = 0$

$\frac{45 - 15}{2} - 3\ell = 0$

$15 = 3\ell$

$\boxed{\ell = 5}$

For coefficient of x^{-15} is

$\frac{45 - 5r}{2} - 5r = -15$

$45 - 5r - 10r = -30$

$75 = 15r$

$\boxed{r = 5}$

For coefficient of x^{-15} is ${}^9C_5 \left(\frac{1}{2}\right)^4 (-4)^5$

$\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times \frac{1}{2^4} \times 2^{10} \times (-1)$

$= 9 \times 2 \times 7 \times 2^6 \times (-1)$

$= 2^7(-63) = 2^\alpha \beta$

$\alpha = 7, \beta = -63$

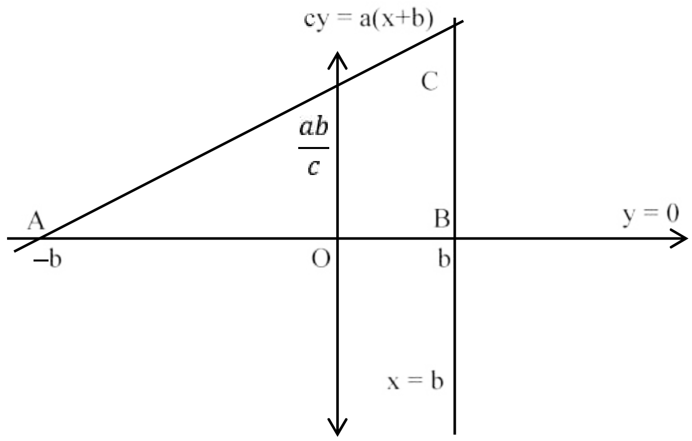
$|\alpha \ell - \beta| = |7(5)+63| = |35 + 63|$

$|\alpha \ell - \beta| = 98.$

84. Let S be the set of all a ∈ N such that the area of the triangle formed by the tangent at the point P(b, c), b, c ∈ ℕ, on the parabola $y^2 = 2ax$ and the lines $x = b, y = 0$ is 16 unit², then $\sum_{a \in s} a$ is equal to

Sol. **146**

tangent at P(b, c) m$y^2 = 2ax$ is



area $= \left|\frac{1}{2} \times 2b \times \frac{2ba}{c}\right| = 16$

$\frac{2b^2a}{C} = 16$

$\frac{b^2a}{C} = 8$

$\because$ P(b, c) lies on $y^2 = 2ax$

$C^2 = 2ab$

$\Rightarrow \quad \frac{b^4a^2}{c^2} = 64$

$\Rightarrow \quad \frac{b^4a^2}{2ab} = 64$

$\Rightarrow \quad b^3a = 128$

$\Rightarrow \quad a = \frac{128}{b^3}$

a can be 128, 16, 2 then

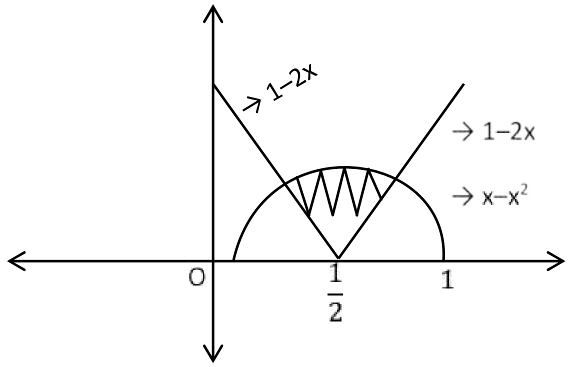
S = {2, 16, 128}

$\sum_{a \in s} a = 146$

85. Let the area of the region $\{(x, y): |2x - 1| \leq y \leq |x^2 - x|, 0 \leq x \leq 1\}$ be A. Then $(6\text{ A} + 11)^2$ is equal to

Sol. **125**

$|2x - 1| \leq y \leq |x^2 - x|, 0 \leq x \leq 1$



$x - x^2 = 1 - 2x$

$x^2 - 3x + 1 = 0$

$x = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$

$x = \frac{3-\sqrt{5}}{2}$ as $0 < x < \frac{1}{2}$

$\text{Area} = 2 \int_{\frac{3-\sqrt{5}}{2}}^{1/2} \left[\left(x - x^2 \right) - \left(1 - 2x \right) \right] dx$

$2 \int_{\frac{3-\sqrt{5}}{2}}^{1/2} \left[3x - x^2 - 1 \right] dx$

$= 2 \left[\frac{3x^2}{2} - \frac{x^3}{3} - x \right]_{\frac{3-\sqrt{5}}{2}}^{1/2}$

For $x = \frac{3-\sqrt{5}}{2}$,

$x^2 = 3x - 1$

$x^3 = 3x^2 - x$

$= 3(3x-1) - x$

$= 8x - 3$

$\frac{3}{2}x^2 - \frac{1}{3}x^3 - x = \frac{3}{2}(3x-1) - \frac{1}{3}(8x-3) - x$

$= \frac{9x-3}{2} - \frac{(8x-3)}{3} - x$

$= \frac{27x - 9 - (16x-6)}{6} - x$

$= \frac{11x-3}{6} - x$

$= \frac{5x-3}{6}$

For $x = \frac{1}{2}$,

$\frac{3}{2}x^2 - \frac{1}{3}x^3 - x = \frac{3}{2}\left(\frac{1}{4}\right) - \frac{1}{3}\left(\frac{1}{8}\right) - \frac{1}{2}$

$= \frac{9-1-12}{24}$

$= \frac{-4}{24} = -\frac{1}{6}$

$$\text{Area} = 2\left[-\frac{1}{6} - \left(\frac{\frac{5(3-\sqrt{5})}{2} - 3}{6}\right)\right]$$

$$= 2\left[-\frac{1}{6} - \left(\frac{15-5\sqrt{5}-6}{12}\right)\right]$$

$$= 2\left[-\frac{1}{6} - \left(\frac{9-5\sqrt{5}}{12}\right)\right]$$

$$= 2\left[\frac{-2-9+5\sqrt{5}}{12}\right]$$

$$= \frac{5\sqrt{5}-11}{6}$$

$(6A+11)^2 = (5\sqrt{5})^2 = 125$

86. The coefficient of x^{-6}, in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$, is

Sol. **5040**

$\left(\frac{4x}{5} \times \frac{5}{2x^2}\right)^9$

General term is

$T_{r+1} = {}^9C_r \left(\frac{4n}{r}\right)^{9-r} \left(\frac{5}{2n^2}\right)^r$

$= {}^9C_r \left(\frac{4}{5}\right)^{9-r} \left(\frac{5}{2}\right)^r x^{9-r-2r}$

For coefficient of term x^{-6} $\quad 9 - r - 2r = -6$

$15 = 3r$

$\boxed{r = 5}$

Coefficient of term x^{-6} $\quad {}^9C_5 \left(\frac{4}{5}\right)^4 \left(\frac{5}{2}\right)^5$

$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times 5 \times 2^3$

$= 9 \times 2 \times 7 \times 5 \times 8$

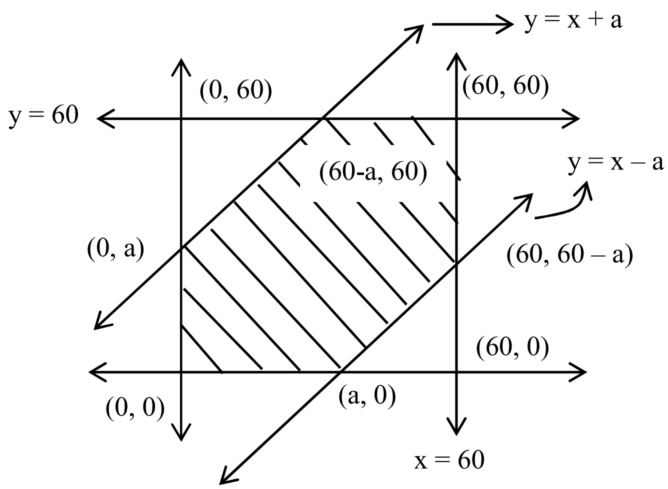
$= 5040$

87. Let A be the event that the absolute difference between two randomly choosen real numbers in the sample space [0,60] is less than or equal to a . If $P(A) = \frac{11}{36}$, then a is equal to

Sol. **10**

$|x - y| < a \rightarrow -a < x - y \quad \& \quad x - y < a$

$x, y \in [0, 60]$



$$P(A) = \frac{\text{Shaded area}}{\text{Total area}} = \frac{(60)^2 - \left[\frac{1}{2}(60 - a)^2 + \frac{1}{2} \times (60 - a)^2\right]}{(60)^2}$$

$$P(A) = \frac{(60)^2 - (60 - a)^2}{(60)^2}$$

$$\frac{11}{36} = \frac{120a - a^2}{3600}$$

$1100 = 120a - a^2$

$a^2 - 120a + 1100 = 0$

$a^2 - 110a - 10a + 1100 = 0$

$a(a - 110) - 10(a - 110) = 0 =$

$(a - 10)(a - 110) = 0$

Ans. $\boxed{a = 10}$

($\because$ for a = 110, P(A) = 1)

88. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11:21$, then $n^2 + n + 15$ is equal to :

Sol. **45**

${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11 : 21$

$$\frac{(2n+1)!}{(n+2)!} \times \frac{(n-1)!}{(2n-1)!} = \frac{11}{21}$$

$$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{11}{21}$$

$$\Rightarrow \frac{2(2n+1)}{(n+2)(n+1)} = \frac{11}{21}$$

$\Rightarrow 42(2n+1) = 11(n^2+3n+2)$

$\Rightarrow 84n + 42 = 11n^2+33n+22$

$\Rightarrow 11n^2 - 51n - 20 = 0$

$\Rightarrow n = 5$

$n^2 + n + 15 = 25 + 5 + 15 = 45$

89. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{31}, 4|\vec{b}| = |\vec{c}| = 2$ and $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a}\times\vec{c}}{\vec{a}\cdot\vec{b}}\right)^2$ is equal to

Sol. **3**

$|\vec{a}| = \sqrt{31} \qquad 4|\vec{b}| = |\vec{c}| = 2$

$2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$

$$\vec{b} \wedge \vec{c} = \frac{2\pi}{3}$$

$\vec{a} \times 2\vec{b} = 3\vec{c} \times \vec{a} = -\vec{a} \times 3\vec{c}$

$\vec{a} \times (2\vec{b} + 3\vec{c}) = \vec{o}$

$\vec{a} = \lambda(2\vec{b} + 3\vec{c})$

$|\vec{a}|^2 = \lambda^2 |2\vec{b} + 3\vec{c}|^2$

$$|\vec{a}|^2 = \lambda^2\left(4|\vec{b}|^2 + 9|\vec{c}|^2 + 12|\vec{b}||\vec{c}|\cos\theta\right)$$

$$31 = \lambda^2 (1 + 9(2)^2 + 12|\vec{b}||\vec{c}| \cos\frac{2\pi}{3})$$

$$31 = \lambda^2 (1 + 36 - 6 \times \frac{1}{2} \times 2)$$

$31 = \lambda^2 (31)$

$\boxed{\lambda^2 = 1}$

$\Rightarrow \lambda = \pm 1$

$\vec{a} = \pm\left(2\vec{b} + 3\vec{c}\right)$

$\vec{a}\times\vec{c} = \pm\left(2\vec{b}+3\vec{c}\right)\times\vec{c}$

$= \pm 2\left(\vec{b}\times\vec{c}\right)$

$\left|\vec{a}\times\vec{c}\right|^2 = 4\left|\vec{b}\times\vec{c}\right|^2 = 3$

$\vec{a}\cdot\vec{b} = \mp 1$

$\left(\dfrac{\left|\vec{a}\times\vec{c}\right|}{\vec{a}\cdot\vec{b}}\right)^2 = 3$

90. The sum $1^2 - 2\cdot 3^2 + 3\cdot 5^2 - 4\cdot 7^2 + 5\cdot 9^2 - \cdots + 15\cdot 29^2$ is

Sol. **6952**

$1^2 - 2.3^2 + 3.5^2 - 4.7^2 + 5.9^2 - \ldots + 15.29^2$

$S = \underbrace{15.29^2 - 14.27^2} + \ldots . + \underbrace{3.5^2 - 2.3^2} + 1^2$

$(n+1)(2n+1)^2 - n(2n-1)^2$

$n(4n^2+4n+1) + 4n^2+4n+1 - n(4n^2-4n+1)$

$= 12n^2 + 4n + 1$

$S = [\Sigma\ 12n^2+4n+1 \text{ for } n = 2, 4, 6, 8, 10, 12, 14] + 1$

$S_1 = \sum_{k=1}^{7} 12(2k)^2 + 4(2k) + 1$

$= \sum_{k=1}^{7} [48k^2 + 8k + 1]$

$= 48\sum_{k=1}^{7} k^2 + 8\sum_{k=1}^{7} k + \sum_{k=1}^{7} 1$

$= \dfrac{48(7)(8)(15)}{6} + \dfrac{8(7)(8)}{2} + 7 = 6951$

$S = \boxed{6952}$