FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The bulk modulus of a liquid is 3×10^{10} Nm^{-2}. The pressure required to reduce the volume of liquid by 2% is :

(A) 3×10^8 Nm^{-2} (B) 9×10^8 Nm^{-2}

(C) 6×10^8 Nm^{-2} (D) 12×10^8 Nm^{-2}

Official Ans. by NTA (C)

Sol. $B = 3 \times 10^{10}$

$$-\frac{\Delta V}{V} = 0.02$$

$$B = \frac{\Delta P}{-\frac{\Delta V}{v}} \Rightarrow \Delta P = -B\left(\frac{\Delta V}{V}\right)$$

$$= (3 \times 10^{10})(0.02)$$

$$= 6 \times 10^8 \, N/m^2$$

2. Given below are two statements : One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : In an uniform magnetic field, speed and energy remains the same for a moving charged particle.

Reason (R) : Moving charged particle experiences magnetic force perpendicular to its direction of motion.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true.

Official Ans. by NTA (A)

Sol. $\vec{F} = q(\vec{v} \times \vec{B})$

$\boxed{\vec{F} \perp \vec{v}}$

Work done $= \vec{F} \cdot \vec{S}$

Work done = 0

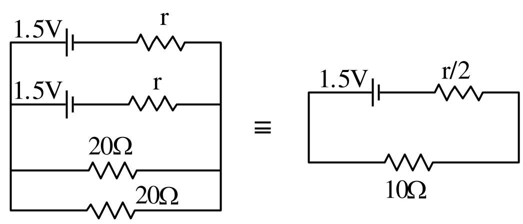
3. Two identical cells each of emf 1.5 V are connected in parallel across a parallel combination of two resistors each of resistance 20Ω. A voltmeter connected in the circuit measures 1.2 V. The internal resistance of each cell is

(A) 2.5Ω (B) 4Ω

(C) 5Ω (D) 10Ω

Official Ans. by NTA (C)

Sol.



$V = E - ir/2$

$$1.2 = 1.5 - i\left(\frac{r}{2}\right)$$

$$i\frac{r}{2} = 0.3$$

$$i = \frac{1.5}{10 + \frac{r}{2}} \Rightarrow 10i + \frac{ir}{2} = 1.5$$

$10i = 1.5 - 0.3$

$i = 0.12$ A

$$\Rightarrow \quad r = \frac{0.6}{0.12} = 5\Omega$$

4. Identify the pair of physical quantities which have different dimensions :

(A) Wave number and Rydberg's constant

(B) Stress and Coefficient of elasticity

(C) Coercivity and Magnetisation

(D) Specific heat capacity and Latent heat

Official Ans. by NTA (D)

Sol. $S = \frac{Q}{m\Delta T} = \frac{J}{Kg°C}$

$L = \frac{Q}{m} = \frac{J}{Kg}$

5. A projectile is projected with velocity of 25 m/s at an angle θ with the horizontal. After t seconds its inclination with horizontal becomes zero. If R represents horizontal range of the projectile, the value of θ will be : [use g = 10 m/s^2]

(A) $\frac{1}{2}\sin^{-1}\left(\frac{5t^2}{4R}\right)$ (B) $\frac{1}{2}\sin^{-1}\left(\frac{4R}{5t^2}\right)$

(C) $\tan^{-1}\left(\frac{4t^2}{5R}\right)$ (D) $\cot^{-1}\left(\frac{R}{20t^2}\right)$

Official Ans. by NTA (D)

Sol. $R = \frac{V^2(2\sin\theta\cos\theta)}{g}$

$t = \frac{V\sin\theta}{g} \Rightarrow V = \frac{gt}{\sin\theta}$

$\Rightarrow R = \frac{g^2t^2}{\sin^2\theta}.\frac{2\sin\theta\cos\theta}{g}$

$\tan\theta = \frac{2gt^2}{R} = \frac{20t^2}{R}$

$\cot\theta = \frac{R}{20t^2}$

6. A block of mass 10 kg starts sliding on a surface with an initial velocity of 9.8 ms^{-1}. The coefficient of friction between the surface and bock is 0.5. The distance covered by the block before coming to rest is : [use g = 9.8 ms^{-2}]

(A) 4.9 m (B) 9.8 m

(C) 12.5 m (D) 19.6 m

Official Ans. by NTA (B)

Sol. $a = -\mu g = -0.5 \times 9.8 = -4.9 m/s^2$

$d = \frac{v^2}{2a} = \frac{9.8 \times 9.8}{2(4.9)}$

= 9.8 m

7. A boy ties a stone of mass 100 g to the end of a 2 m long string and whirls it around in a horizontal plane. The string can withstand the maximum tension of 80 N. If the maximum speed with which the stone can revolve is $\frac{K}{\pi}$ rev. / min. The value of K is : (Assume the string is massless and unstretchable)

(A) 400 (B) 300

(C) 600 (D) 800

Official Ans. by NTA (C)

Sol. $T = M\omega^2R$

T = 80 N M = 0.1 ω = ? R = 2m

$80 = 0.1\ \omega^2(2)$

$\omega^2 = 400$

$\omega = 20$

$2\pi f = 20$

$f = \frac{10}{\pi}\frac{rev}{s}$

$= \frac{600}{\pi}\frac{rev}{min}$

8. A vertical electric field of magnitude 4.9×10^5 N/C just prevents a water droplet of a mass 0.1 g from falling. The value of charge on the droplet will be : (Given g = 9.8 m/s^2)

(A) 1.6×10^{-9} C (B) 2.0×10^{-9} C

(C) 3.2×10^{-9} C (D) 0.5×10^{-9} C

Official Ans. by NTA (B)

Sol. $Mg = qE$

$(0.1\times10^{-3})(9.8) = 4.9\times10^5 q$

$\frac{2\times10^{-4}}{10^5} = q$

$\boxed{q = 2\times10^{-9} C}$

9. A particle experiences a variable force $\vec{F} = (4x\hat{i} + 3y^2\hat{j})$ in a horizontal x-y plane. Assume distance in meters and force is newton. If the particle moves from point (1, 2) to point (2, 3) in the x-y plane, the Kinetic Energy changes by

(A) 50.0 J (B) 12.5 J

(C) 25.0 J (D) 0 J

Official Ans. by NTA (C)

Sol. $F = 4x\hat{i} + 3y^2\hat{j}$

WD = ΔKE

$W = \int \vec{F}.(dx\hat{i} + dy\hat{j})$

$= \int_1^2 4xdx + \int_2^3 3y^2dx$

$= (2x^2)_1^2 + (y^3)_2^3$

$= (8-2) + (27-8)$

$= 6 + 19 = 25J$

10. The approximate height from the surface of earth at which the weight of the body becomes $\frac{1}{3}$ of its weight on the surface of earth is : [Radius of earth R = 6400 km and $\sqrt{3} = 1.732$]

(A) 3840 km (B) 4685 km

(C) 2133 km (D) 4267 km

Official Ans. by NTA (B)

Sol. $Mg' = \frac{M}{3}g$

$g' = \frac{g}{3}$

$g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{3}$

$\frac{R}{R+h} = \frac{1}{\sqrt{3}}.$

$h = (\sqrt{3}-1)R$

$= (1.732-1)6400$

$\boxed{h = 4685km}$

11. A resistance of 40 Ω is connected to a source of alternating current rated 220 V, 50 Hz. Find the time taken by the current to change from its maximum value to rms value :

(A) 2.5 ms (B) 1.25 ms

(C) 2.5 s (D) 0.25 s

Official Ans. by NTA (A)

Sol. Considering sinusoidal AC.

Phase at maximum value $= \frac{\pi}{2}$

Phase at rms value = $\frac{3\pi}{4}$

Thus phase change $= \frac{3\pi}{4} - \frac{\pi}{2} = \frac{\pi}{4}$

Now $\omega = 2\pi f$

$= 2\pi \times 50$

$= 100\pi$

time taken $t = \frac{\theta}{\omega} = \frac{\pi/4}{100\pi} = \frac{1}{400}$s

$t = 2.5 \times 10^{-3} = 2.5$ ms

12. The equations of two waves are given by :

$$y_1 = 5\sin 2\pi(x - vt)cm$$

$$y_2 = 3\sin 2\pi(x - vt + 1.5)cm$$

These waves are simultaneously passing through a string. The amplitude of the resulting wave is

(A) 2 cm (B) 4 cm

(C) 5.8 cm (D) 8 cm

Official Ans. by NTA (A)

Sol. $A_1 = 5 \quad A_2 = 3$

$\Delta\theta = 2\pi(1.5) = 3\pi$

$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(3\pi)}$

$= |A_1 - A_2|$

= 2cm

13. A plane electromagnetic wave travels in a medium of relative permeability 1.61 and relative permittivity 6.44. If magnitude of magnetic intensity is 4.5×10^{-2} Am^{-1} at a point, what will be the approximate magnitude of electric field intensity at that point ?

(Given : permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}, speed of light in vacuum $c = 3 \times 10^8$ ms^{-1})

(A) 16.96 Vm^{-1} (B) 2.25×10^{-2} Vm^{-1}

(C) 8.48 Vm^{-1} (D) 6.75×10^6 Vm^{-1}

Official Ans. by NTA (C)

Sol. $\mu_r = 1.61 \qquad \in_r = 6.44$

$B = 4.5 \times 10^{-2}$

E = ?

$C = \dfrac{1}{\sqrt{\mu_0 \in_0}} \quad V = \dfrac{1}{\sqrt{\mu \in}}$

$\dfrac{C}{V} = \sqrt{\mu_r \in_r} = \sqrt{1.61 \times 6.44}$

$\dfrac{E}{B} = V = \dfrac{3 \times 10^8}{\sqrt{1.61 \times 6.44}} = 9.32 \times 10^7 \, m/s$

$E = 4.5 \times 10^{-2} \times 9.32 \times 10^7$

$= 4.2 \times 10^6$

14. Choose the correct option from the following options given below :

(A) In the ground state of Rutherford's model electrons are in stable equilibrium. While in Thomson's model electrons always experience a net-force.

(B) An atom has a nearly continuous mass distribution in a Rutherford's model but has a highly non-uniform mass distribution in Thomson's model

(C) A classical atom based on Rutherford's model is doomed to collapse.

(D) The positively charged part of the atom possesses most of the mass in Rutherford's model but not in Thomson's model.

Official Ans. by NTA (C)

Sol. According to Rutherford, e^- revolves around nucleus in circular orbit. Thus e^- is always accelerating (centripetal acceleration). An accelerating change emits EM radiation and thus e^- should loose energy and finally should collapse in the nucleus.

15. Nucleus A is having mass number 220 and its binding energy per nucleon is 5.6 MeV. It splits in two fragments 'B' and 'C' of mass numbers 105 and 115. The binding energy of nucleons in 'B' and 'C' is 6.4 MeV per nucleon. The energy Q released per fission will be :

(A) 0.8 MeV (B) 275 MeV

(C) 220 MeV (D) 176 MeV

Official Ans. by NTA (D)

Sol. $Q = (B.E)_P - (B.E)_R$

$= (105 + 115)(6.4) - (220)(5.6)$

= 176 MeV

16. A baseband signal of 3.5 MHz frequency is modulated with a carrier signal of 3.5 GHz frequency using amplitude modulation method. What should be the minimum size of antenna required to transmit the modulated signal ?

(A) 42.8 m (B) 42.8 mm

(C) 21.4 mm (D) 21.4 m

Official Ans. by NTA (C)

Sol. $f_c = 3.5\text{GHz} \quad f_m = 3.5\text{MHz}$

Side band frequencies are $f_c - f_m$ & $f_c + f_m$. which are almost f_c

$$\lambda = \frac{c}{f_c}$$

Minimum length of antenna =

$$\frac{c}{f_c 4} = \frac{\lambda}{4} = \frac{3\times10^8}{3.5\times10^9\times4}$$

= 21.4 mm

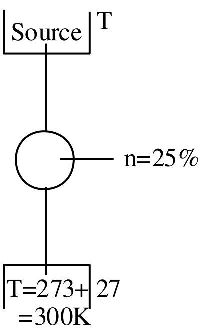
17. A Carnot engine whose heat sinks at 27°C, has an efficiency of 25%. By how many degrees should the temperature of the source be changed to increase the efficiency by 100% of the original efficiency ?

(A) Increases by 18°C (B) Increase by 200°C

(C) Increase by 120°C (D) Increase by 73°

Official Ans. by NTA (B)

Sol.



$$1 - \frac{300}{T} = 0.25$$

$$\frac{300}{T} = 0.75$$

T = 400K

If efficiency increased by 100% then new efficiency ⇒ n' = 50%

$$1 - \frac{300}{T'} = 0.5$$

$$T' = 600\text{K}$$

Increase in temp = 600 – 400

= 200 K or 200°C

18. A parallel plate capacitor is formed by two plates each of area $30\pi\ \text{cm}^2$ separated by 1 mm. A material of dielectric strength $3.6 \times 10^7\ \text{Vm}^{-1}$ is filled between the plates. If the maximum charge that can be stored on the capacitor without causing any dielectric breakdown is 7×10^{-6} C, the value of dielectric constant of the material is :

$$\left\{\text{Use}: \frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\text{Nm}^2\text{C}^{-2}\right\}$$

(A) 1.66 (B) 1.75

(C) 2.25 (D) 2.33

Official Ans. by NTA (D)

Sol. $$K = \frac{q}{A\in_0 E} = \frac{7\times10^{-6}}{30\pi\times10^{-4}\times\frac{1}{4\pi\times9\times10^9}\times3.6\times10^7}$$

$$K = \frac{36\times7}{30\times3.6} = 2.33$$

19. The magnetic field at the centre of a circular coil of radius r, due to current I flowing through it, is B. The magnetic field at a point along the axis at a distance $\frac{r}{2}$ from the centre is :

(A) B/2 (B) 2B

(C) $\left(\frac{2}{\sqrt{5}}\right)^3 B$ (D) $\left(\frac{2}{\sqrt{3}}\right)^3 B$

Official Ans. by NTA (C)

Sol. $$B_C = \frac{\mu_0 I}{2r}, B_a = \frac{\mu_0 I r^2}{2(x^2+r^2)^{3/2}}$$

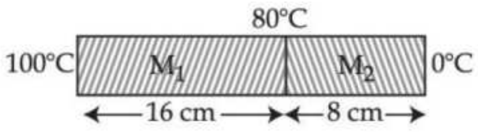
At $x = \frac{r}{2}$

$$B_a = \frac{\mu_0 I r^2}{2\left(\frac{r^2}{4}+r^2\right)^{3/2}}$$

$$= \frac{\mu_0 I r^2}{2\left(\frac{5}{4}r^2\right)^{3/2}} = \frac{\mu_0 I}{2r}\left(\frac{4}{5}\right)^{3/2}$$

$$= \frac{\mu_0 I}{2r}\left(\frac{2}{\sqrt{5}}\right)^3$$

20. Two metallic blocks M_1 and M_2 of same area of cross-section are connected to each other (as shown in figure). If the thermal conductivity of M_2 is K then the thermal conductivity of M_1 will be : [Assume steady state heat conduction]



(A) 10 K (B) 8 K

(C) 12.5 K (D) 2 K

Official Ans. by NTA (B)

Sol. $\Delta T \ \alpha \ R \ \alpha \ \frac{\ell}{k}$,

$$\frac{\Delta T_1}{\Delta T_2} = \frac{\ell_1}{k_1} \times \frac{k_2}{\ell_2} = \frac{16}{k_1} \times \frac{k}{8}$$

$$\frac{20}{80} = \frac{16}{k_1} \times \frac{k}{8} \rightarrow k_1 = 8k$$

SECTION-B

1. 0.056 kg of Nitrogen is enclosed in a vessel at a temperature of 127°C. The amount of heat required to double the speed of its molecules is _____ k cal. (Take R = 2 cal $mole^{-1}K^{-1}$)

Official Ans. by NTA (12)

Sol. 0.056 kg N_2 = 56 gm of N_2 = 2 mole of N_2

$T_1 = 400$ K, $v \alpha \sqrt{T}$ so $T_2 = 4T_1 = 1600K$

$$Q = \frac{f}{2} nR\Delta T$$

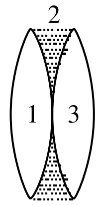
f = 5

Q = 12 k cal

2. Two identical thin biconvex lenses of focal length 15 cm and refractive index 1.5 are in contact with each other. The space between the lenses is filled with a liquid of refractive index 1.25. The focal length of the combination is ______ cm.

Official Ans. by NTA (10)

Sol.



$$\frac{1}{f_1} = \frac{1}{15} = \left(\frac{3}{2} - 1\right)\left[\frac{2}{R}\right]$$

$$\frac{1}{R} = \frac{1}{15}$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$= \frac{1}{15} + \left(\frac{5}{4} - 1\right)\left[\frac{-2}{R}\right] + \frac{1}{15}$$

$$= \frac{1}{15} - \frac{1}{30} + \frac{1}{15}$$

$$= \frac{2 - 1 + 2}{30}$$

$$= \frac{3}{30} = \frac{1}{10}$$

= 10

3. A transistor is used in common-emitter mode in an amplifier circuit. When a signal of 10 mV is added to the base-emitter voltage, the base current changes by 10 μA and the collector current changes by 1.5 mA. The load resistance is 5 kΩ. The voltage gain of the transistor will be _____ .

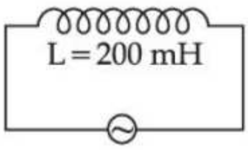
Official Ans. by NTA (750)

Sol. $r_i = \frac{10mV}{10\mu A} = 10^3\Omega$

$$\beta = \frac{1.5mA}{10\mu A} = 150$$

$$A_V = \left(\frac{R_0}{r_i}\right)\beta = \left(\frac{5000}{1000}\right) \times 150 = 750$$

4. As shown in the figure an inductor of inductance 200 mH is connected to an AC source of emf 220 V and frequency 50 Hz. The instantaneous voltage of the source is 0 V when the peak value of current is $\frac{\sqrt{a}}{\pi}$ A. The value of a is ______.



Official Ans. by NTA (242)

Sol. $f = 50Hz$

$X_L = 2\pi fL$

$= 2\pi(50)(200\times10^{-3})$

$= 20\pi\Omega$

$i_0 = \frac{V_0}{X_L} \Rightarrow \frac{V_{rms}\sqrt{2}}{X_L}$

$= \frac{(220)\sqrt{2}}{20\pi} = \frac{11\sqrt{2}}{\pi}$

$\boxed{i_o = \frac{\sqrt{242}}{\pi}}$

5. Sodium light of wavelengths 650 nm and 655 nm is used to study diffraction at a single slit of aperture 0.5 mm. The distance between the slit and the screen is 2.0 m. The separation between the positions of the first maxima of diffraction pattern obtained in the two cases is ______ $\times 10^{-5}$ m.

Official Ans. by NTA (3)

Sol. $a\sin\theta = \frac{3\lambda}{2}$

$\frac{y}{L} = \theta = \frac{3\lambda}{2a}$ $\quad$ L = 2m

$y_1 = \frac{3\lambda_1 L}{2a}$ $\quad$ $\lambda_2 = 655$ nm

$y_2 = \frac{3\lambda_2 L}{2a}$ $\quad$ $\lambda_1 = 650$ nm

a = 0.5 nm

$\Delta y = y_2 - y_1 = \frac{3(\lambda_2 - \lambda_1)}{2a}L$

$= \frac{3(655-650)}{2\times0.5\times10^{-3}}\times2\times10^{-9}$

$= \frac{3\times5\times2}{1\times10^{-3}}\times10^{-9}$

$= 3\times10^{-5}$

6. When light of frequency twice the threshold frequency is incident on the metal plate, the maximum velocity of emitted election is v_1. When the frequency of incident radiation is increased to five times the threshold value, the maximum velocity of emitted electron becomes v_2. If $v_2 = x\ v_1$, the value of x will be ______.

Official Ans. by NTA (2)

Sol. $hv = hv_{th} + \frac{1}{2}mv^2$

$v = 2v_{th}$

$2hv_{th} = hv_{th} + \frac{1}{2}mv_1^2$ ..... (1)

$v = 5v_{th}$

$5hv_{th} = hv_{th} + \frac{1}{2}mv_2^2$ ....(2)

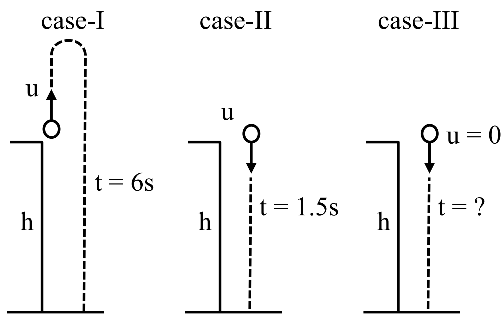
$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{hv_{th}}{4hv_{th}}$

$\left(\frac{v_1}{v_2}\right)^2 = \frac{1}{4} \Rightarrow \boxed{v_2 = 2v_1}$

7. From the top of a tower, a ball is thrown vertically upward which reaches the ground in 6 s. A second ball thrown vertically downward from the same position with the same speed reaches the ground in 1.5 s. A third ball released, from the rest from the same location, will reach the ground in ________ s.

Official Ans. by NTA (3)

Sol. Let height of tower be h and speed of projection in first two cases be u.



For case-I : 2nd equation $s = ut + \frac{1}{2}at^2$

$h = -u(6) + \frac{1}{2}g(6)^2$

$H = -6u + 18g$ …. (i)

For case-II : $h = u(1.5) + \frac{1}{2}g(1.5)^2$

$h = 1.5u + \frac{2.25g}{2}$ … (ii)

Multiplying equation (ii) by 4 we get

$4h = 6u + 4.5g$ …. (iii)

equation (i) + equation (iii) we get $5h = 22.5g$

$h = 4.5g$ … (iv)

For case-III :

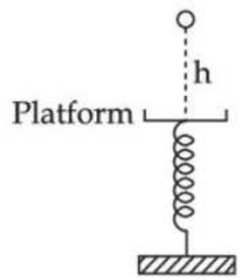
$h = 0 + \frac{1}{2}gt^2$ … (v)

Using equation (4) & equation (5)

$4.5g = \frac{1}{2}gt^2$

$t^2 = 9 \Rightarrow t = 3s$

8. A ball of mass 100 g is dropped from a height h = 10 cm on a platform fixed at the top of vertical spring (as shown in figure). The ball stays on the platform and the platform is depressed by a distance $\frac{h}{2}$. The spring constant is ________ Nm^{-1}. (Use $g = 10\ ms^{-2}$)



Official Ans. by NTA (120)

Sol. By energy conservation

PE = KE

$mg\left(H + \frac{H}{2}\right) = \frac{1}{2}kx^2 \quad \left(x = \frac{H}{2}\right)$

$0.100 \times 10 \times \frac{3}{2}(0.10) = \frac{1}{2}k(0.05 \times 0.05)$

$k = \frac{3 \times 0.10}{0.05 \times 0.05}$

$= \frac{3 \times 1000}{25} = 120\ N/m$

9. In a potentiometer arrangement, a cell gives a balancing point at 75 cm length of wire. This cell is now replaced by another cell of unknown emf. If the ratio of the emf's of two cells respectively is 3 : 2, the difference in the balancing length of the potentiometer wire in above two cases will be ______ cm.

Official Ans. by NTA (25)

Sol. $\frac{\varepsilon_1}{\varepsilon_2} = \frac{\ell_1}{\ell_2}$

$\frac{3}{2} = \frac{75cm}{\ell_2}$

$\ell_2 = 50cm$

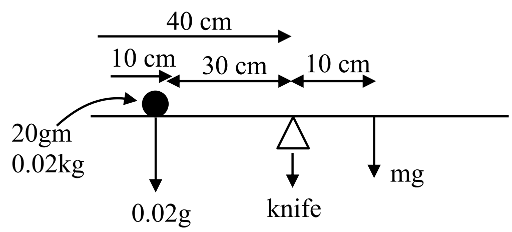
$\ell_1 - \ell_2 = 75 - 50$

= 25 cm

10. A metre scale is balanced on a knife edge at its centre. When two coins, each of mass 10 g are put one on the top of the other at the 10.0 cm mark the scale is found to be balanced at 40.0 cm mark. The mass of the metre scale is found to be $x \times 10^{-2}$ kg. The value of x is

Official Ans. by NTA (6)

Sol. Let mass of meter scale be m.



Balancing torque about knife edge

$(0.02g) \times (30 \times 10^{-2}) = mg \times (10 \times 10^{-2})$

$m = 0.06\ kg = 6 \times 10^{-2}\ kg$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. If a rocket runs on a fuel ($C_{15}H_{30}$) and liquid oxygen, the weight of oxygen required and CO_2 released for every litre of fuel respectively are: (Given: density of the fuel is 0.756 g/mL)

(A) 1188 g and 1296 g (B) 2376 g and 2592 g

(C) 2592g and 2376 g (D) 3429 g and 3142 g

Official Ans. by NTA (C)

Sol. $C_{15}H_{30} + \frac{45}{2}O_2 \rightarrow 15CO_2 + 15H_2O$

Mass of fuel = 0.756 × 1000 g

No. of moles of fuel = $\frac{0.756 \times 1000}{210}$

Wt. of oxygen = $\frac{0.756 \times 1000}{210} \times \frac{45}{2} \times 32$ = 2592g

Wt of CO_2 = $\frac{0.756 \times 1000}{210} \times 15 \times 44$ = 2376 g

2. Consider the following pairs of electrons

(A) (a) $n = 3, 1 = 1, m_1 = 1, m_s = +\frac{1}{2}$

(b) $n = 3, 1 = 2, m_1 = 1, m_s = +\frac{1}{2}$

(B) (a) $n = 3, 1 = 2, m_1 = -2, m_s = -\frac{1}{2}$

(b) $n = 3, 1 = 2, m_1 = -1, m_s = -\frac{1}{2}$

(C) (a) $n = 4, 1 = 2, m_1 = 2, m_s = +\frac{1}{2}$

(b) $n = 3, 1 = 2, m_1 = 2, m_s = +\frac{1}{2}$

The pairs of electron present in degenerate orbitals is/are:

(A) Only A

(B) Only B

(C) Only C

(D) (B) and (C)

Official Ans. by NTA (B)

Sol. Based on "n + l" rule only (B) has pair of electron in degenerate orbitals

3. Match List – I with List - II

List – I		List – II	
(A)	$[PtCl_4]^{2-}$	**(I)**	sp^3d
(B)	BrF_5	**(II)**	d^2sp^3
(C)	PCl_5	**(III)**	dsp^2
(D)	$[Co(NH_3)_6]^{3+}$	**(IV)**	sp^3d^2

(A) (A)→(II), (B)→(IV), (C)→(I), (D)→(III)

(B) (A)→(III), (B)→(IV), (C)→(I), (D)→(II)

(C) (A)→(III), (B)→(I), (C)→(IV), (D)→(II)

(D) (A)→(II), (B)→(I), (C)→(IV), (D)→(III)

Official Ans. by NTA (B)

Sol. Answer (B)

List – I		List – II	
(A)	$[PtCl_4]^{2-}$	(III)	dsp^2
(B)	BrF_5	(IV)	sp^3d^2
(C)	PCl_5	(I)	sp^3d
(D)	$[Co(NH_3)_6]^{3+}$	(II)	d^2sp^3

4. For a reaction at equilibrium

$$A(g) \rightleftharpoons B(g) + \frac{1}{2}C(g)$$

the relation between dissociation constant (K), degree of dissociation (α) and equilibrium pressure (p) is given by :

(A) $K = \dfrac{\alpha^{\frac{1}{2}} p^{\frac{3}{2}}}{\left(1+\frac{3}{2}\alpha\right)^{\frac{1}{2}}(1-\alpha)}$

(B) $K = \dfrac{\alpha^{\frac{3}{2}} p^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$

(C) $K = \dfrac{(\alpha p)^{\frac{3}{2}}}{\left(1+\frac{3}{2}\alpha\right)^{\frac{1}{2}}(1-\alpha)}$

(D) $K = \dfrac{(\alpha p)^{\frac{3}{2}}}{(1+\alpha)(1-\alpha)^{\frac{1}{2}}}$

Official Ans. by NTA (B)

Sol. $$A(g) \rightleftharpoons B(g) + \frac{1}{2}C(g)$$

Initial : P_i 0   0

At eq.: $P_i(1-\alpha)$ $P_i.\alpha$ $P_i\frac{\alpha}{2}$

Now, equilibrium pressure (p) ,

$$P = P_i \times \left(1+\frac{\alpha}{2}\right)$$

$$\therefore P_A = \left(\frac{1-\alpha}{1+\frac{\alpha}{2}}\right)P$$

$$P_B = \left(\frac{\alpha}{1+\frac{\alpha}{2}}\right)P$$

$$P_C = \left(\frac{\frac{\alpha}{2}}{1+\frac{\alpha}{2}}\right)P$$

$$\therefore K = \frac{P_c^{\frac{1}{2}} \times P_B}{P_A}$$

$$K = \frac{\alpha^{\frac{3}{2}} p^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$$

5. Given below are two statements :

Statement I : Emulsions of oil in water are unstable and sometimes they separate into two layers on standing.

Statement II :For stabilisation of an emulsion, excess of electrolyte is added.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (C)

Sol. Statement I : Fact

Statement II: The principle emulsifying agents for O/W emulsions are proteins, gums natural and synthetic soaps etc…

6. Given below are the oxides:

Na_2O, As_2O_3, N_2O, NO and Cl_2O_7

Number of amphoteric oxides is:

(A) 0   (B) 1

(C) 2   (D) 3

Official Ans. by NTA (B)

Sol. Na_2O = Basic As_2O_3 = Amphoteric

N_2O = Neutral   NO = Neutral

Cl_2O_7 = Acidic

7. Match List – I with List – II

	List - I		List - II
(A)	Sphalerite	(I)	$FeCO_3$
(B)	Calamine	(II)	PbS
(C)	Galena	(III)	$ZnCO_3$
(D)	Siderite	(IV)	ZnS

Choose the most appropriate answer from the options given below:

(A) (A) - (IV), (B) – (III), (C) – (II), (D) – (I)

(B) (A) - (IV), (B) – (I), (C) – (II), (D) – (III)

(C) (A) - (II), (B) – (III), (C) – (I), (D) – (IV)

(D) (A) - (III), (B) – (IV), (C) – (II), (D) – (I)

Official Ans. by NTA (A)

Sol.

	List - I		List - II
(A)	Sphalerite	(IV)	ZnS
(B)	Calamine	(III)	$ZnCO_3$
(C)	Galena	(II)	PbS
(D)	Siderite	(I)	$FeCO_3$

8. The highest industrial consumption of molecular hydrogen is to produce compounds of element:

(A) Carbon (B) Nitrogen

(C) Oxygen (D) Chlorine

Official Ans. by NTA (B)

Sol. Nitrogen . Around 55% of hydrogen around would goes to ammonia production

9. Which of the following statements are correct ?

(A) Both LiCl and $MgCl_2$ are soluble in ethanol.

(B) The oxides Li_2O and MgO combine with excess of oxygen to give superoxide.

(C) LiF is less soluble in water than other alkali metal fluorides.

(D) Li_2O is more soluble in water than other alkali metal oxides.

Choose the most appropriate answer from the options given below:

(A) (A) and (C) only (B) (A), (C) and (D) only

(C) (B) and (C) only (D) (A) and (C) only

Official Ans. by NTA (A)

Sol. (A) Both LiCl and $MgCl_2$ are soluble in ethanol

(B) Li and Mg do not form superoxide

(C) LiF has high lattice energy

(D) Li_2O is least soluble in water than other alkali metal oxides

10. Identify the correct statement for B_2H_6 from those given below.

(A) In B_2H_6, all B-H bonds are equivalent.

(B) In B_2H_6 there are four 3-centre-2-electron bonds.

(C) B_2H_6 is a Lewis acid.

(D) B_2H_6 can be synthesized form both BF_3 and $NaBH_4$.

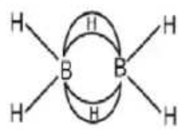
(E) B_2H_6 is a planar molecule.

Choose the most appropriate answer from the options given below :

(A) (A) and (E) only (B) (B), (C) and (E) only

(C) (C) and (D) only (D) (C) and (E) only

Official Ans. by NTA (C)



Sol. (A) (B)

Two 3 centre – 2 – electron bonds

(C) B_2H_6 is e^- deficient species

(E) B_2H_6 is non – Planar molecule

(D) $BF_3 + LiAlH_4 \rightarrow 2B_2H_6 + 3LiF + 3AlF_3$

$NaBH_4 + I_2 \rightarrow B_2H_6 + 2NaI + H_2$

11. The most stable trihalide of nitrogen is:

(A) NF_3 (B) NCl_3

(C) NBr_3 (D) NI_3

Official Ans. by NTA (A)

Sol. Order of stability: -

$NF_3 > NCl_3 > NBr_3 > NI_3$

12. Which one of the following elemental forms is not present in the enamel of the teeth?

(A) Ca^{2+} (B) P^{3+}

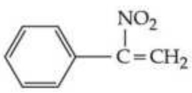
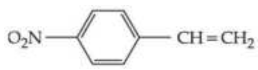
(C) F^{-} (D) P^{5+}

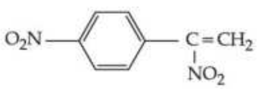
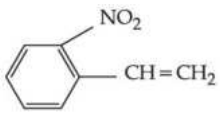
Official Ans. by NTA (B)

Sol. Calcium and phosphate are the major components of teeth enamel

13. In the given reactions sequence, the major product 'C' is :

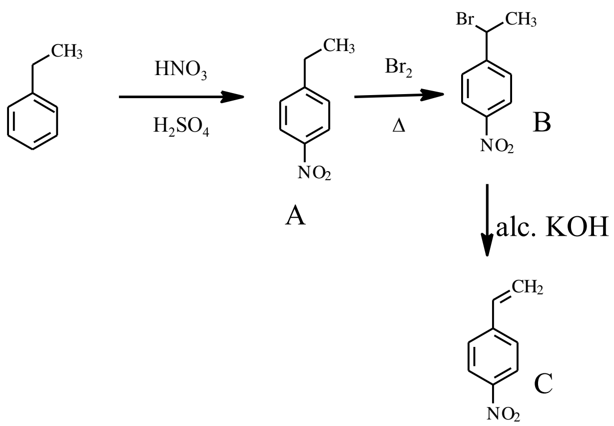
$$C_8H_{10} \xrightarrow[H_2SO_4]{HNO_3} A \xrightarrow[\Delta]{Br_2} B \xrightarrow[KOH]{alcoholic} C$$

(A)  (B) 

(C)  (D) 

Official Ans. by NTA (B)

Sol. C_8H_{10} DU = 9 – 5 = 4



14. Two statements are given below :

Statement I: The melting point of monocarboxylic acid with even number of carbon atoms is higher than that of with odd number of carbon atoms acid immediately below and above it in the series.

Statement II : The solubility of monocarboxylic acids in water decreases with increase in molar mass.

Choose the most appropriate option:

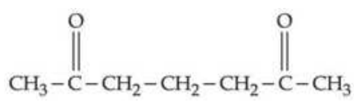
(A) Both Statement I and Statement II are correct.
(B) Both Statement I and Statement II are incorrect.
(C) Statement I is correct but Statement II is incorrect.
(D) Statement I is incorrect but Statement II is correct.

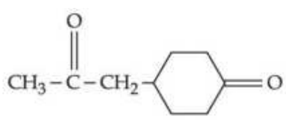
Official Ans. by NTA (A)

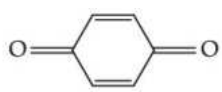
Sol. I . Better packing efficiency of monocarboxylic acids with even number of carbon atoms results in higher M.P

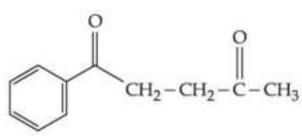
II. As molar mass increases hydrophobic part size increase hence solubility decreases.

15. Which of the following is an example of conjugated diketone?

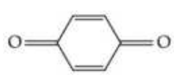
(A) 

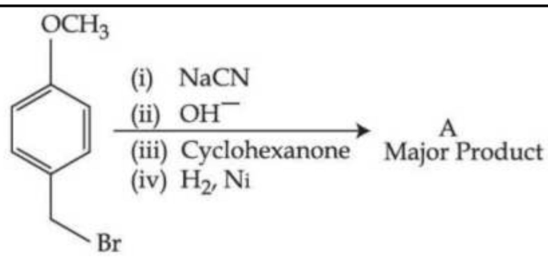
(B) 

(C) 

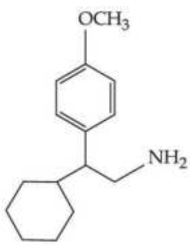
(D) 

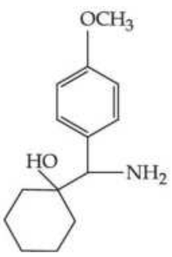
Official Ans. by NTA (C)

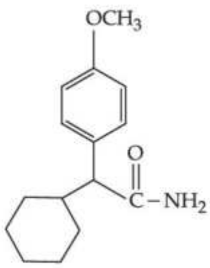
Sol.  **is a conjugated diketone**

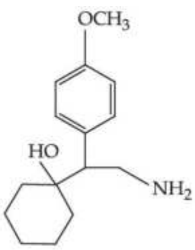
16. 

The major product of the above reaction is

(A) 

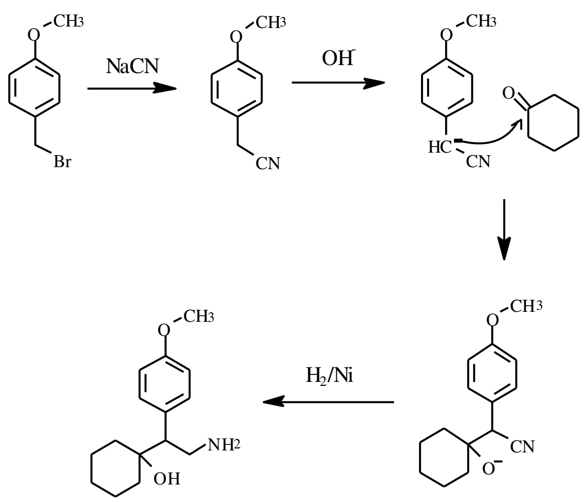
(B) 

(C) 

(D) 

Official Ans. by NTA (D)

Sol.



17. Which of the following is an example of polyester?

(A) Butadiene-styrene copolymer

(B) Melamine polymer

(C) Neoprene

(D) Poly-β-hydroxybutyrate-co-β-hydroxy valerate

Official Ans. by NTA (D)

Sol. **Factual**

18. A polysaccharide 'X' on boiling with dil H_2SO_4 at 393 K under 2-3 atm pressure yields 'Y'.

'Y' on treatment with bromine water gives gluconic acid. 'X' contains β-glycosidic linkages only. Compound 'X' is :

(A) starch (B) cellulose

(C) amylose (D) amylopectin

Official Ans. by NTA (B)

Sol. Cellulose contains β – glycosidic linkages only

19. Which of the following is not a broad spectrum antibiotic?

(A) Vancomycin (B) Ampicillin

(C) Ofloxacin (D) Penicillin G

Official Ans. by NTA (D)

Sol. Penicillin G following is a narrow spectrum antibiotic

20. During the qualitative analysis of salt with cation y^{2+} , addition of a reagent (X) to alkaline solution of the salt gives a bright red precipitate. The reagent (X) and the cation (y^{2+}) present respectively are:

(A) Dimethylglyoxime and Ni^{2+}

(B) Dimethylglyoxime and Co^{2+}

(C) Nessler's reagent and Hg^{2+}

(D) Nessler's reagent and Ni^{2+}

Official Ans. by NTA (A)

Sol. $Ni^{2+} + DMG^{-} \rightarrow [Ni(DMG)_2]\downarrow$

(Bright red precipitate)

SECTION-B

1. Atoms of element X form hcp lattice and those of element Y occupy $\frac{2}{3}$ of its tetrahedral voids. The percentage of element X in the lattice is__________ (Nearest integer)

Official Ans. by NTA (43)

Sol. $X \rightarrow 6 \qquad Y \rightarrow \frac{2}{3} \times 2 \times 6 = 8$

$\% X = \frac{6}{14} \times 100 = 42.8 \simeq 43\%$

2. $2O_3(g) \rightleftharpoons 3O_2(g)$

At 300 K, ozone is fifty percent dissociated. The standard free energy change at this temperature and 1 atm pressure is (–) __J mol $^{-1}$ (Nearest integer)

[Given: ln 1.35 = 0.3 and R = 8.3 J K^{-1} mol^{-1}]

Official Ans. by NTA (747)

Sol. $2O_3 \rightleftharpoons 3O_2(g)$

$\frac{2}{5} \qquad \frac{3}{5}$

$k_p = \frac{P^3_{O_2}}{P^2_{O_3}}$

$k_p = 1.35$

$\Delta G° = -RT \ln k_p$

$= -8.3 \times 300 \times \ln 1.35$

$= -747$ J/mol

3. The osmotic pressure of blood is 7.47 bar at 300 K. To inject glucose to a patient intravenously, it has to be isotonic with blood. The concentration of glucose solution in gL^{-1} is _____(Molar mass of glucose = 180 g mol^{-1}

R = 0.083 L bar K^{-1} mol^{-1}) (Nearest integer)

Official Ans. by NTA (54)

Sol. $\pi = C.R.T$

7.47= C × 0.083 × 300

C = 0.3 M

= 0.3 ×180 gL^{-1}

= 54 gL^{-1}

4. The cell potential for the following cell

$Pt | H_2(g) | H^+(aq) || Cu^{2+}(0.01M) | Cu(s)$

is 0.576 V at 298 K. The pH of the solution is ___.

(Nearest integer)

Official Ans. by NTA (5)

Sol.

Anode : $H_2 \rightarrow 2H^+ + 2e^-$

Cathode : $Cu^{2+} + 2e^- \rightarrow Cu$

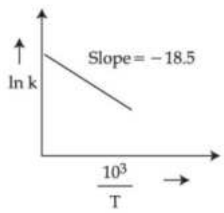
$Cu^{2+} + H_2 \rightarrow 2H^+ + Cu$

$$E_{cell} = E^0_{cell} - \frac{0.06}{2} \log \frac{[H^+]^2}{[Cu^{2+}]}$$

$$0.576 = 0.34 - \frac{0.06}{2} \log \left\{ \frac{[H^+]^2}{(0.01)} \right\}$$

$+3.93 \; -\log(H^+) + \log 0.1 \Rightarrow pH = 4.93 \simeq 5$

5. The rate constants for decomposition of acetaldehyde have been measured over the temperature range 700 –1000 K. The data has been analysed by plotting In k vs $\frac{10^3}{T}$ graph.The value of activation energy for the reaction is___ kJ mol^{-1}. (Nearest integer) (Given : R = 8.31 J K^{-1} mol^{-1})



Official Ans. by NTA (154)

Sol. $\ln k = \ln A - \frac{Ea}{10^3 RT} \times 10^3 = \ell nA + \frac{10^3}{T}\left[-\frac{Ea}{10^3 RT}\right]$

From the graph

$\frac{-Ea}{10^3 \times R} = -18.5$

Ea = 153.735 kJ/mol

~154

6. The difference in oxidation state of chromium in chromate and dichromate salts is _______

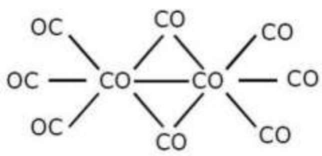
Official Ans. by NTA (0)

Sol. $\overset{+6}{Cr}O_4^{2-}, \overset{+6}{Cr_2}O_7^{2-}$ difference is zero

7. In the cobalt-carbonyl complex: $[Co_2(CO)_8]$, number of Co-Co bonds is "X" and terminal CO ligands is "Y". X + Y =____

Official Ans. by NTA (7)

Sol.



X = 1

Y = 6

8. A 0.166 g sample of an organic compound was digested with cone. H_2SO_4 and then distilled with NaOH. The ammonia gas evolved was passed through 50.0 mL of 0.5 N H_2SO_4. The used acid required 30.0 mL of 0.25 N NaOH for complete neutralization. The mass percentage of nitrogen in the organic compound is____.

Official Ans. by NTA (63)

Sol. m_{eq} of NaOH used = 30 ×0.25

m_{eq} of H_2SO_4 taken = 50 × 0.5

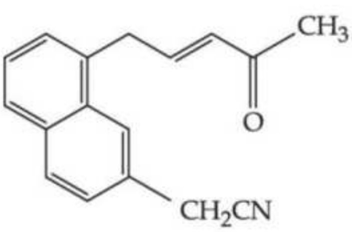
∴ m_{eq} of H_2SO_4 used

= 50 × 0.25 ×30 ×0.25 = 17.5 m mol of NH_3

$$\therefore \% N = \frac{17.5 \times 10^{-3} \times 14}{0.166} \times 100 = 147.59\%$$

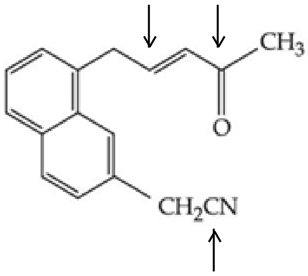
(Not possible)

9. Number of electrophilic centre in the given compound is ____



Official Ans. by NTA (3)

Sol.



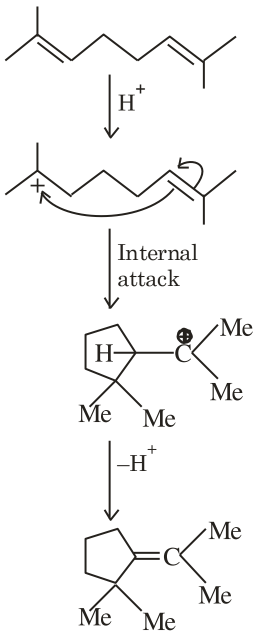
10. The major product 'A' of the following given reaction has _____ sp^2 hybridized carbon atoms.

2,7 – Dimethyl1 – 2, 6 – octadiene

$\xrightarrow{H^+} \underset{\text{Mojor Product}}{A}$

Official Ans. by NTA (2)

Sol. Answer (2)



FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let $A = \{z \in C : 1 \le |z-(1+i)| \le 2\}$ and $B = \{z \in A : |z-(1-i)| = 1\}$. Then, B :

(A) is an empty set

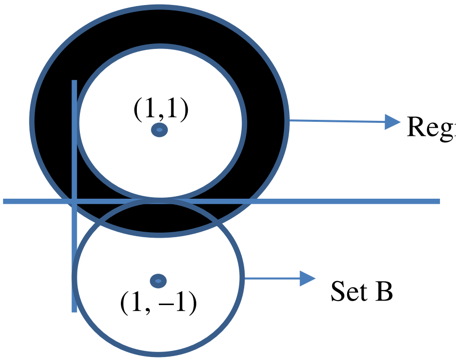
(B) contains exactly two elements

(C) contains exactly three elements

(D) is an infinite set

Official Ans. by NTA (D)

Sol. $A = \{z \in C : 1 \le |z-(1+i)| \le 2\}$



$B = \{z \in A : |z-(1-i)| = 1\}$.

$A \cap B$ has infinite set.

2. The remainder when 3^{2022} is divided by 5 is

(A) 1 (B) 2

(C) 3 (D) 4

Official Ans. by NTA (D)

Sol. $3^{2022} = 9^{1011} = (10-1)^{1011} = 10\text{ m} - 1 = 10\text{ m} - 5 + 4$

$= 5(2m-1) + 4$ (m is integer)

Remainder = 4

3. The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds,, it becomes 7 units, then its radius after 9 seconds is :

(A) 9 (B) 10

(C) 11 (D) 12

Official Ans. by NTA (A)

Sol. Let r be the radius of spherical balloon

S = Surface area

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \times \frac{dr}{dt} = k$ (constant)

$4\pi r^2 = kt + C$ (C is constant of integration)

For $t = 0$, $r = 3 \Rightarrow 36\pi = C$

For $t = 5$, $r = 7 \Rightarrow K = 32\pi$

$4\pi r^2 = 32\pi t + 36\pi$

$r^2 = 8t + 9$

for $t = 9$

$r^2 = 81$

$r = 9$

4. Bag A contains 2 white, 1 black and 3 red balls and bag B contains 3 black, 2 red and n white balls. One bag is chosen at random and 2 balls drawn from it at random, are found to be 1 red and 1 black. If the probability that both balls come from Bag A is $\frac{6}{11}$, then n is equal to ______ .

(A) 13 (B) 6

(C) 4 (D) 3

Official Ans. by NTA (C)

Sol. E_1 = denotes selection for 1st bag

E_2 = denotes selection for 2nd bag

$P(E_1) = \frac{1}{2}$, $P(E_2) = \frac{1}{2}$

A = selected balls are 1 red & 1 black

$P\left(\frac{A}{E_1}\right) = \frac{{}^3C_1 \times {}^1C_1}{{}^6C_2} = \frac{1}{5}$

$$P\left(\frac{A}{E_1}\right)=\frac{{}^3C_1\times{}^2C_1}{(n+5)_{C_2}}=\frac{12}{(n+5)(n+4)}$$

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)\times P\left(\frac{A}{E_1}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right)+P(E_2)\times P\left(\frac{A}{E_2}\right)}$$

$$=\frac{\frac{1}{10}}{\frac{1}{10}+\frac{6}{(n+5)(n+4)}}=\frac{6}{11}$$

$\Rightarrow n=4$

5. Let $x^2 + y^2 + Ax + By + C = 0$ be a circle passing through (0, 6) and touching the parabola $y = x^2$ at (2, 4). Then A + C is equal to______ .

(A) 16 (B) 88/5

(C) 72 (D) –8

Official Ans. by NTA (A)

Sol. $x^2+y^2+Ax+By+C=0$ is passing through (0,6)

$\Rightarrow 6B + C = -36$

The tangent of the parabola $y = x^2$ at (2, 4) is

$4x - y - 4 = 0$ ----(1)

The tangent of circle $x^2+y^2+Ax+By+C=0$ at (2, 4) is

$(4 + A)x + (8 + B)y + 2A + 4B + 2C = 0$ -----(2)

From Equation (1) and (2)

$$\frac{4+A}{4}=\frac{8+B}{-1}=\frac{2A+4B+2C}{-4}$$

$A+4B=-36$ ---(3)

$3A+4B+2C=-4$ ---(4)

From equation (3) and (4)

A+ C = 16

6. The number of values of α for which the system of equations :

$x + y + z = \alpha$

$\alpha x + 2\alpha y + 3z = -1$

$x + 3\alpha y + 5z = 4$

is inconsistent, is

(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (B)

Sol. $x+y+z=\alpha$

$\alpha x+2\alpha y+3z=-1$

$x+3\alpha y+5z=4$

Has inconsistent solution

$$D=\begin{vmatrix}1 & 1 & 1\\ \alpha & 2\alpha & 3\\ 1 & 3\alpha & 5\end{vmatrix}=0$$

$\Rightarrow(\alpha-1)^2=0$

$\alpha=1$

For $\alpha=1$

$$D_1=\begin{vmatrix}1 & 1 & 1\\ -1 & 2 & 3\\ 4 & 3 & 5\end{vmatrix}$$

= (10 – 9) – (–5–12) + (–3–8)

= 1+17 –11 $\neq 0$

For $\alpha=1$ the system of equation has Inconsistent solution

7. If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to :

(A) 18 (B) 24

(C) 36 (D) 96

Official Ans. by NTA (B)

Sol. **Here** α,β roots of equation $3x^2+\lambda x-1=0$

$\alpha+\beta=\frac{-\lambda}{3}$, $\alpha\beta=\frac{-1}{3}$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}=15$$

$\lambda^2=9$

Now $6(\alpha^3+\beta^3)^2=6\left((\alpha+\beta)\left((\alpha+\beta)^2-3\alpha\beta\right)\right)^2$

$$=6\left(\frac{\lambda^2}{9}\right)\left\{\frac{\lambda^2}{9}+1\right\}^2=24$$

8. The set of all values of k for which $(\tan^{-1}x)^3+(\cot^{-1}x)^3=k\pi^3, x\in R,$ is the interval :

(A) $\left[\frac{1}{32},\frac{7}{8}\right)$ (B) $\left(\frac{1}{24},\frac{13}{16}\right)$

(C) $\left[\frac{1}{48},\frac{13}{16}\right]$ (D) $\left[\frac{1}{32},\frac{9}{8}\right)$

Official Ans. by NTA (A)

Sol. **Let S =** $(\tan^{-1}x)^3+(\cot^{-1}x)^3$

$$=\left(\tan^{-1}x+\cot^{-1}x\right)-3\tan^{-1}x.\cot^{-1}x\left(\tan^{-1}x+\cot^{-1}x\right)$$

$$=\frac{\pi^3}{8}-\frac{3\pi}{2}\tan^{-1}x\left(\frac{\pi}{2}-\tan^{-1}x\right)$$

$$=\frac{3\pi}{2}\left(\tan^{-1}x-\frac{\pi}{4}\right)^2+\frac{\pi^3}{32}$$

$$\Rightarrow\frac{\pi^3}{32}\le S<\frac{7}{8}\pi^3$$

$$=\frac{\pi^3}{32}\le K\pi^3<\frac{7}{8}\pi^3$$

$$\frac{1}{32}\le K<\frac{7}{8}$$

9. Let $S=\{\sqrt{n}:1\le n\le 50\text{ and n is odd}\}$

Let $a\in S$ and $A=\begin{bmatrix}1&0&a\\-1&1&0\\-a&0&1\end{bmatrix}$

If $\sum_{a\in S}\det(\text{adj}A)=100\lambda,$ then λ is equal to

(A) 218 (B) 221

(C) 663 (D) 1717

Official Ans. by NTA (B)

Sol. $S=\{\sqrt{n}:1\le n\le 50\text{ and n is odd}\}$

$=\left\{\sqrt{1},\sqrt{3},\sqrt{5}..........\sqrt{49}\right\}$, 25 terms

$|A| = 1 + a^2$

$$\sum_{a\in S}\det(\text{adj}A)=\sum_{a\in S}|A|^2=\sum(1+a^2)^2$$

$= 22100 = 100\lambda$

$\lambda = 221$

10. $f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5, x > 1$, which one of the following is NOT correct ?

(A) f is increasing in (1, 2) and decreasing in $(2, \infty)$

(B) $f(x) = -1$ has exactly two solutions

(C) $f'(e) - f''(2) < 0$

(D) $f(x) = 0$ has a root in the interval (e, e +1)

Official Ans. by NTA (C)

Sol. $f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5, x > 1$

$$f'(x)=\frac{4}{x-1}-4(x-1)$$

For $1<x<2\Rightarrow f'(x)>0$

For $x>2\Rightarrow f'(x)<0$ (option 1 is correct)

$f(x)=-1$ has two solution (option 2 is correct)

$f(e)>0$

$f(e+1)<0$

$f(e).f(e+1)<0$ (option 4 is correct)

$$f'(e)-f''(2)=\frac{4}{e-1}-4(e-1)+8>0$$

(option C is incorrect)

11. the tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin, then (x_1, y_1) does NOT lie on the curve :

(A) $x^2+\frac{y^2}{81}=2$ (B) $\frac{y^2}{9}-x^2=8$

(C) $y=4x^2+5$ (D) $\frac{x}{3}-y^2=2$

Official Ans. by NTA (D)

Sol. **The tangent at (x_1, y_1) to the curve**

$y=x^3+3x^2+5$

$y-y_1=\left(3x_1^2+6x_1\right)\left(x-x_1\right)$ passing through origin

$-y_1=\left(3x_1^3+6x_1\right)\left(-x_1\right)$

$y_1=\left(3x_1^3+6x_1^2\right)$ --------(1)

And (x_1, y_1) lies on the curve

$$y = x^3 + 3x^2 + 5$$

$$y_1 = x_1^3 + 3x_1^2 + 5 \quad ----(2)$$

From equation (1) and (2)

$$2y_1 = 3x_1^2 + \frac{15}{2}$$

Hence the equation of curve $y = \frac{3}{2}x^2 + \frac{15}{2}$

This curve does not intersect $\frac{x}{3} - y^2 = 2$

12. The sum of absolute maximum and absolute minimum values of the function

$f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ in the interval $[0, 1]$ is :

(A) $3 + \frac{\sin(1)\cos^2(1/2)}{2}$ (B) $3 + \frac{1}{2}(1 + 2\cos(1))\sin(1)$

(C) $5 + \frac{1}{2}(\sin(1) + \sin(2))$ (D) $2 + \sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$

Official Ans. by NTA (B)

Sol. $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$

$f(x) = |(2x - 1)(x + 2)| + \sin x \cos x$

$$f'(x) = \begin{cases} 4x + 3 + \frac{\cos 2x}{4}, & \frac{1}{2} < x < 1 \\ -(4x+3) + \frac{\cos 2x}{4}, & 0 \le x < \frac{1}{2} \end{cases}$$

For $0 \le x < \frac{1}{2} \Rightarrow f'(x) < 0$

For $\frac{1}{2} < x \le 1 \Rightarrow f'(x) > 0$

f (x) local minima at $x = \frac{1}{2}$ and

local maxima at $x = 1$

$$f\left(\frac{1}{2}\right) + f(1) = 3 + \frac{1}{2}(1 + 2\cos 1)\sin 1$$

13. If $\{a_i\}_{i=1}^{n}$ where n is an even integer , is an arithmetic progression with common difference 1, and $\sum_{i=1}^{n} a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to:

(A) 48 (B) 96

(C) 92 (D) 104

Official Ans. by NTA (B)

Sol. $\sum_{i=1}^{n} a_i = \frac{n}{2}\{2a_1 + (n+1)\} = 192$

$$\Rightarrow 2a_1 + (n-1) = \frac{384}{n} \quad ----(1)$$

$$\sum_{i=1}^{n/2} a_{2i} = \frac{n}{4}\left[2a_1 + 2 + \left(\frac{n}{2} - 1\right)2\right] = 120$$

$$2a_1 + n = \frac{480}{n} \quad ----(2)$$

From equation (2) and (1)

$$1 = \frac{480}{n} - \frac{384}{n}$$

$n = 480 - 384 = 96$

14. If $x = x(y)$ is the solution of the differential equation $y\frac{dx}{dy} = 2x + y^3(y+1)e^y, x(1) = 0$; then $x(e)$ is equal to :

(A) $e^3(e^e - 1)$ (B) $e^e(e^3 - 1)$

(C) $e^2(e^e + 1)$ (D) $e^e(e^2 - 1)$

Official Ans. by NTA (A)

Sol. $y\frac{dx}{dy} = 2x + y^3(y+1)e^y, x(1) = 0$

$$\frac{dx}{dy} - \frac{2}{y}x = y^2(y+1)e^y$$

$$\text{I.f} = e^{\int \frac{-2}{y}dy} = \frac{1}{y^2}$$

$$x.\frac{1}{y^2} = \int (y+1)e^y dy$$

$$\frac{x}{y^2} = (y+1)e^y - e^y + c = y.e^y + c$$

$$x = y^3e^y + cy^2$$

For $x = 0$, $y = 1 \Rightarrow c = -e$

$$x = y^3e^y - e.y^2$$

$x(e) = e^3(e^e - 1)$

15. Let $\lambda x - 2y = \mu$ be a tangent to the hyperbola $a^2x^2 - y^2 = b^2$. Then $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is equal to:

(A) –2 (B) –4

(C) 2 (D) 4

Official Ans. by NTA (D)

Sol. $\lambda x - 2y = \mu$ is a tangent to the curve

$a^2x^2 - y^2 = b^2$ then

$$a^2x^2 - \left(\frac{\lambda x - \mu}{2}\right)^2 = b^2$$

$$\left(4a^2 - \lambda^2\right)x^2 + 2\lambda\mu x - \mu^2 - 4b^2 = 0$$

Disc. = 0

$$4\lambda^2\mu^2 + 4\left(4a^2 - \lambda^2\right)\left(\mu^2 + 4b^2\right) = 0$$

$$4\lambda^2b^2 - 4a^2\mu^2 = 16a^2b^2$$

$$\frac{\lambda^2}{a^2} - \frac{\mu^2}{b^2} = 4$$

16. Let $\hat{a}, \hat{b}$ be unit vectors. If $\vec{c}$ be a vector such that the angle between $\hat{a}$ and $\vec{c}$ is $\frac{\pi}{12}$, and $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$, then $|6\vec{c}|^2$ is equal to

(A) $6(3 - \sqrt{3})$ (B) $3 + \sqrt{3}$

(C) $6(3 + \sqrt{3})$ (D) $6(\sqrt{3} + 1)$

Official Ans. by NTA (C)

Sol. $|\hat{b}|^2 = |\vec{c} + 2(\vec{c} \times \hat{a})|^2$

$$|\hat{b}|^2 = |\vec{c}|^2 + 4|\vec{c} \times \hat{a}|^2 + 4\vec{c}.(\vec{c} \times \hat{a})$$

$$1 = |c|^2 + 4|c|^2 \sin^2\frac{\pi}{12} + 0$$

$$1 = |c|^2 + 4|c|^2\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$$

$$|c|^2 = \frac{1}{3-\sqrt{3}} = \frac{3+\sqrt{3}}{6}$$

So $6^2|c|^2 = 6(3+\sqrt{3})$

17. If a random variable X follows the Binomial distribution B (33, p) such that 3P(X = 0) = P(X = 1), then the value of $\frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$ is equal to

(A) 1320 (B) 1088

(C) $\frac{120}{1331}$ (D) $\frac{1088}{1089}$

Official Ans. by NTA (A)

Sol. n = 33, let probability of success is p and q = 1 – p

$3p(x=0) = p(x=1)$

$3.\,{}^{33}C_0(q)^{33} = {}^{33}C_1pq^{32}$

$p = \frac{1}{12}$, $q = \frac{11}{12}$, $\frac{q}{p} = 11$

$$\frac{p(x=15)}{p(x=18)} - \frac{p(x=16)}{p(x=17)}$$

$$\frac{{}^{33}C_{15}p^{15}q^{18}}{{}^{33}C_{18}p^{18}q^{15}} - \frac{{}^{33}C_{16}p^{16}q^{17}}{{}^{33}C_{17}p^{17}q^{16}} = \left(\frac{q}{p}\right)^3 - \left(\frac{q}{p}\right)$$

$= (11)^3 - 11$

= 1320

18. The domain of the function

$$f(x) = \frac{\cos^{-1}\left(\frac{x^2-5x+6}{x^2-9}\right)}{\log_e(x^2-3x+2)}$$ is

(A) $(-\infty, 1) \cup (2, \infty)$

(B) $(2, \infty)$

(C) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty)$

(D) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

Official Ans. by NTA (DROP)

Sol. $-1 \le \frac{x^2-5x+6}{x^2-9} \le 1$

$\frac{x^2-5x+6}{x^2-9} - 1 \le 0$

$\frac{1}{x+3} \ge 0$

$x \in (-3, \infty)$ **……(1)**

$\frac{x^2-5x+6}{x^2-9} + 1 \ge 0$

$\frac{2x+1}{x+3} \ge 0$

$x \in (-\infty, -3) \cup \left[-\frac{1}{2}, \infty\right)$ **……(2)**

after taking intersection

$x \in \left[-\frac{1}{2}, \infty\right)$

$x^2 - 3x + 2 > 0$

$x \in (-\infty, 1) \cup (2, \infty)$

$x^2 - 3x + 2 \ne 1$

$x \ne \frac{3 \pm \sqrt{5}}{2}$

after taking intersection of each solution

$\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

19. Let

$S = \left\{\theta \in [-\pi, \pi] - \left\{\pm\frac{\pi}{2}\right\} : \sin\theta\tan\theta + \tan\theta = \sin 2\theta\right\}.$

If $T = \sum_{\theta \in S} \cos 2\theta$, then T + n(S) is equal

(A) $7+\sqrt{3}$ (B) 9

(C) $8+\sqrt{3}$ (D) 10

Official Ans. by NTA (B)

Sol. $\sin\theta\tan\theta + \tan\theta = \sin 2\theta$

$\tan\theta(\sin\theta + 1) = \frac{2\tan\theta}{1+\tan^2\theta}$

$\tan\theta = 0 \Rightarrow \theta = -\pi, 0, \pi$

$(\sin\theta + 1) = 2.\cos^2\theta = 2(1+\sin\theta)(1-\sin\theta)$

$\sin\theta = -1$ which is not possible

$\sin\theta = \frac{1}{2} \quad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

n(s) = 5

$T = \cos 0 + \cos 2\pi + \cos 2\pi + \cos\frac{\pi}{3} + \cos\frac{5\pi}{3}$

T = 4

T + n(s) = 9

20. The number of choices of $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, such that $(p\Delta q) \Rightarrow ((p\Delta \sim q) \vee ((\sim p)\Delta q))$ is a tautology, is

(A) 1 (B) 2

(C) 3 (D) 4

Official Ans. by NTA (B)

Sol. For tautology $((p\Delta \sim q) \vee ((\sim p)\Delta q))$ must be true.

This is possible only when $\Delta = \vee \,\&\, \Rightarrow$

SECTION-B

1. The number of one-one function f : {a, b, c, d} → {0, 1, 2, … ..,10} such that 2f(a) – f(b) + 3f(c) + f(d) = 0 is _____ .

Official Ans. by NTA (31)

Sol. 2f(a) + 3f(c) = f(d) – f(b)

Using fundamental principle of counting

Number of one-one function is 31

2. In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct There are 3 marks for each correct answer, -2 marks for each wrong answer and 0 mark if the question is not attempted. Then, the number of ways a student appearing in the examination gets 5 marks is_.

Official Ans. by NTA

Sol. $x_1 + x_2 + x_3 + x_4 + x_5 = 5$

Only one possibilities 3, 3, 3, -2, -2

Number of ways is = $\frac{5!}{3!2!} \times 2 \times 2 = 40$

3. Let $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right) a > 0$, be a fixed point in the xy-plane. The image of A in y-axis be B and the image of B in x-axis be C. If D(3 cos θ, a sin θ) is a point in the fourth quadrant such that the maximum area of ΔACD is 12 square units, then a is equal to ________ .

Official Ans. by NTA (8)

Sol. $A = \left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$

$B = \left(\frac{-3}{\sqrt{a}}, \sqrt{a}\right)$

$C = \left(-\frac{3}{\sqrt{a}}, -\sqrt{a}\right)$

Area of ACD

$$\frac{1}{2}\begin{vmatrix} \frac{3}{\sqrt{a}} & \sqrt{a} \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} \\ 3\cos\theta & a\sin\theta \\ \frac{3}{\sqrt{a}} & \sqrt{a} \end{vmatrix}$$

$\frac{1}{2}6\sqrt{a}(\cos\theta - \sin\theta)$

$3\sqrt{a}(\cos\theta - \sin\theta)$

max values of function is $3\sqrt{a}\sqrt{2}$

$3\sqrt{a}\sqrt{2} = 12$

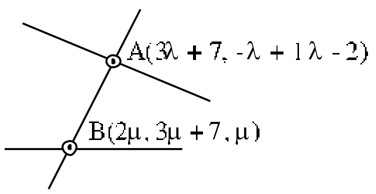
2a = 16

a = 8

4. Let a line having direction ratios 1, -4, 2 intersect the lines $\frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$ and $\frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$ at the point A and B. Then $(AB)^2$ is equal to ____ .

Official Ans. by NTA (84)

Sol.



DR's of AB

$(3\lambda - 2\mu + 7, -\lambda - 3\mu - 6, \lambda - \mu - 2)$

$\frac{3\lambda - 2\mu + 7}{1} = \frac{-\lambda - 3\mu - 6}{-4} = \frac{\lambda - \mu - 2}{2}$

Taking first (2) $-12\lambda + 8\mu - 28 = -\lambda - 3\mu - 6$

$\lambda - \mu + 2 = 0$

Taking second & third $-2\lambda - 6\mu - 12 = -4\lambda + 4\mu + 8$

$\lambda - 5\mu - 10 = 0$

After solving above two equation $\lambda = -5, \mu = -3$

A = (-8, 6, 7)

B = (-6, -2, -3)

$(AB)^2 = 4 + 64 + 16 = 84$

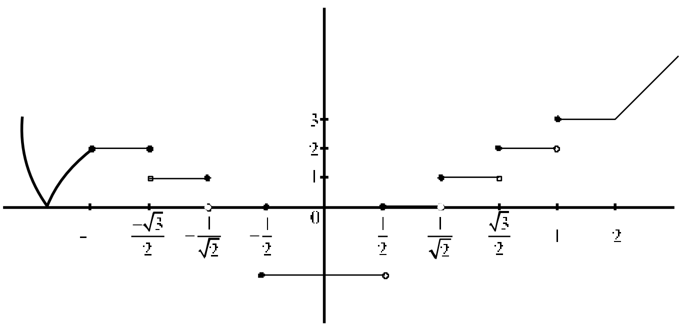
5. The number of points where the function

$$f(x) = \begin{cases} |2x^2 - 3x - 7| & \text{if} \quad x \le -1 \\ [4x^2 - 1] & \text{if} \quad -1 < x < 1 \\ |x+1| + |x-2| & \text{if} \quad x \ge 1 \end{cases}$$

[t] denotes the greatest integer $\le t$, is discontinuous is _______ .

Official Ans. by NTA (7)

Sol.



6. Let $f(\theta) = \sin\theta + \int_{-\pi/2}^{\pi/2} (\sin\theta + t\cos\theta) f(t)dt$. Then the value of $\left|\int_0^{\pi/2} f(\theta)d\theta\right|$ is _______ .

Official Ans. by NTA (1)

Sol. $f(\theta)=\sin\theta+\int_{-\pi/2}^{\pi/2}(\sin\theta+t\cos\theta)\,f(t)\,dt$

$f(\theta)=\sin\theta+\sin\theta\int_{-\pi/2}^{\pi/2}f(t)dt+\cos\theta\int_{-\pi/2}^{\pi/2}tf(t)dt$

Let $A=\int_{-\pi/2}^{\pi/2}f(t)dt,\ \ B=\int_{-\pi/2}^{\pi/2}tf(t)dt$

$f(\theta)=\sin\theta+A\sin\theta+B\cos\theta$

$f(\theta)=(A+1)\sin\theta+B\cos\theta$

$A=\int_{-\pi/2}^{\pi/2}(A+1)\sin t+B\cos t\ dt$

A = 2B ......(1)

$B=\int_{-\pi/2}^{\pi/2}t((A+1)\sin t+B\cos t)$

$B=\int_{-\pi/2}^{\pi/2}t(A+1)\sin t$

$B=(A+1)2\int_{0}^{\pi/2}t\sin t\ \ dt$

B = (A + 1) 2.1

2A + 2 – B = 0 .....(2)

After solving

$B=-\frac{2}{3},A=-\frac{4}{3}$

$\left|\int_0^{\pi/2}f(\theta)d\theta\right|=\left|\int_0^{\pi/2}-\frac{1}{3}\sin\theta-\frac{2}{3}\cos\theta\right|$

= 1

7. Let $\underset{0\le x\le 2}{\text{Max}}\left\{\frac{9-x^2}{5-x}\right\}=\alpha$ and $\underset{0\le x\le 2}{\text{Min}}\left\{\frac{9-x^2}{5-x}\right\}=\beta$

If $\int_{\beta-\frac{8}{3}}^{2\alpha-1}\text{Max}\left\{\frac{9-x^2}{5-x},x\right\}dx=\alpha_1+\alpha_2\log_e\left(\frac{8}{15}\right)$ then

$\alpha_1+\alpha_2$ is equal to _______

Official Ans. by NTA (34)

Sol. $y=\frac{9-x^2}{5-x}=5+x+\frac{16}{x-5}$

$\frac{dy}{dx}=1-\frac{16}{(x-5)^2}$

So critical point is x = 1 in [0, 2]

$y(0)=\frac{9}{5},\ y(1)=2,\ y(2)=\frac{5}{3}$

So $\alpha=2$ and $\beta=\frac{5}{3}$

$I=\int_{-1}^{3}\max\left(\frac{9-x^2}{5-x},x\right)$

$I=\int_{-1}^{9/5}\frac{9-x^2}{5-x}dx+\int_{9/5}^{3}x\,dx$

$I=\int_{-1}^{9/5}5+x+\frac{16}{x-5}dx+\int_{9/5}^{3}x\,dx$

After solving

$I=14+\frac{28}{25}+16\ln\left(\frac{8}{15}\right)+\frac{72}{25}$

$\alpha_1=18$ and $\alpha_2=16$

8. If two tangents drawn from a point (α,β) lying on the ellipse $25x^2+4y^2=1$ to the parabola $y^2=4x$ are such that the slope of one tangent is four times the other, then the value of

$(10\alpha+5)^2+(16\beta^2+50)^2$ equals __________

Official Ans. by NTA (2929)

Sol. $\alpha=\frac{1}{5}\cos\theta,\ \beta=\frac{1}{2}\sin\theta$

Equation of tangent to $y^2=4x$

$y=mx+\frac{1}{m}$

It passes through (α,β)

$\frac{1}{2}\sin\theta=m\frac{1}{5}\cos\theta+\frac{1}{m}$

$m^2\left(\frac{\cos\theta}{5}\right)-m\left(\frac{1}{2}\sin\theta\right)+1=0$

It has two roots m_1 and m_2 where $m_1=4m_2$

$m_1+m_2=\frac{\frac{1}{2}\sin\theta}{\frac{\cos\theta}{5}}$

$m_1m_2=\frac{5}{\cos\theta}$

After eliminating m_1 and m_2

$$\cos\theta = \frac{-5 \pm \sqrt{29}}{2}$$

$$\alpha = \frac{-5 \pm \sqrt{29}}{10} \Rightarrow 10\alpha + 5 = \pm\sqrt{29}$$

$$\beta^2 = \frac{1}{4}\sin^2\theta \Rightarrow 16\beta^2 = -50 \pm 10\sqrt{29}$$

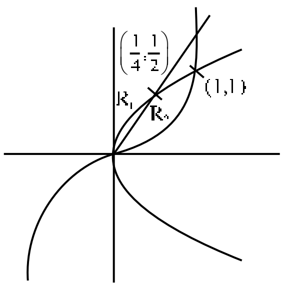
$$(10\alpha + 5)^2 + (16\beta^2 + 50)^2 = 2929$$

9. Let S be the region bounded by the curves $y = x^3$ and $y^2 = x$. The curve $y = 2|x|$ divides S into two regions of areas R_1 and R_2.

If max $\{R_1, R_2\} = R_2$, then $\frac{R_2}{R_1}$ is equal to _____ .

Official Ans. by NTA (19)

Sol.



$$S = \int_0^1 \sqrt{x} - x^3$$

$$= \left[\frac{2x^{3/2}}{3} - \frac{x^4}{4}\right]_1^0$$

$$= \frac{5}{12}$$

$$R_1 = \int_0^{1/4} (\sqrt{x} - 2x)dx$$

$$= \left[\frac{2x^{3/2}}{3} - x^2\right]_0^{1/4} = \frac{1}{48}$$

$$\therefore\ R_2 = \frac{19}{48}$$

So, $\frac{R_2}{R_1} = 19$

10. If the shortest distance between the line $\vec{r} = (-\hat{i} + 3k) + \lambda(\hat{i} - a\hat{j})$ and $\vec{r} = (-\hat{j} + 2k) + \mu(\hat{i} - \hat{j} + k)$ is $\sqrt{\frac{2}{3}}$, then the integral value of a is equal to

Official Ans. by NTA (2)

Sol. $a_1 = (-1, 0, 3)$

$a_2 = (0, -1, 2)$

$b_1 = (1, -a, 0)$ dr's of line (1)

$b_2 = (1, -1, 1)$ dr's of line (2)

$\bar{a}_2 - \bar{a}_1 = (1, -1, -1)$

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix}$$

$$\bar{b}_1 \times \bar{b}_2 = \hat{i}(-a) - \hat{j} + k(a-1)$$

$$\left|\bar{b}_1 \times \bar{b}_2\right| = \sqrt{a^2 + 1 + (a-1)^2}$$

$$a_2 - a_1 . \bar{b}_1 \times \bar{b}_2 = 2 - 2a$$

$$\frac{2(1-a)}{\sqrt{a^2 + 1 + (a-1)^2}} = \sqrt{\frac{2}{3}}$$

Squaring an both the side

After solving $a = 2, \frac{1}{2}$