FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Thursday 04th April, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. Let $f : R \to R$ be a function given by

$$f(x)=\begin{cases}\dfrac{1-\cos 2x}{x^2} & , \ x<0 \\ \alpha & , \ x=0 \\ \dfrac{\beta\sqrt{1-\cos x}}{x} & , \ x>0\end{cases}$$

where $\alpha, \beta \in R$. If f is continuous at $x = 0$, then $\alpha^2 + \beta^2$ is equal to :

(1) 48 (2) 12

(3) 3 (4) 6

Ans. (2)

Sol. $f(0^-) = \lim\limits_{x\to 0^-} \dfrac{2\sin^2 x}{x^2} = 2 = \alpha$

$f(0^+) = \lim\limits_{x\to 0^+} \beta \times \sqrt{2}\dfrac{\sin\frac{x}{2}}{2\frac{x}{2}} = \dfrac{\beta}{\sqrt{2}} = 2$

$\Rightarrow \beta = 2\sqrt{2}$

$\alpha^2 + \beta^2 = 4 + 8 = 12$

2. Three urns A, B and C contain 7 red, 5 black; 5 red, 7 black and 6 red, 6 black balls, respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is :

(1) $\dfrac{4}{17}$ (2) $\dfrac{5}{18}$

(3) $\dfrac{7}{18}$ (4) $\dfrac{5}{16}$

Ans. (2)

Sol.

A	B	C
7R, 5B	5R, 7B	6R, 6B

$P(B) = \dfrac{1}{3}.\dfrac{5}{12}+\dfrac{1}{3}.\dfrac{7}{12}+\dfrac{1}{3}.\dfrac{6}{12}$

required probability $= \dfrac{\frac{1}{3}.\frac{5}{12}}{\frac{1}{3}.\left[\frac{5}{12}+\frac{7}{12}+\frac{6}{12}\right]} = \dfrac{5}{18}$

3. The vertices of a triangle are A(–1, 3), B(–2, 2) and C(3, –1). A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to origin is :

(1) $x - y - \left(2+\sqrt{2}\right) = 0$

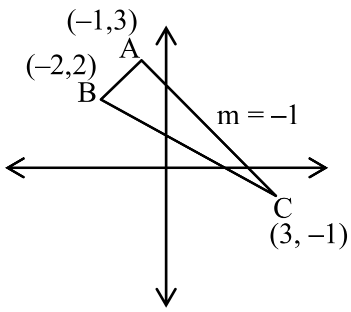
(2) $-x + y - \left(2-\sqrt{2}\right) = 0$

(3) $x + y - \left(2-\sqrt{2}\right) = 0$

(4) $x + y + \left(2-\sqrt{2}\right) = 0$

Ans. (3)

Sol.



equation of AC $\to x + y = 2$

equation of line parallel to AC $x + y = d$

$\left|\dfrac{d-2}{\sqrt{2}}\right| = 1$

$d = 2-\sqrt{2}$

eq^n of new required line

$\boxed{x+y=2-\sqrt{2}}$

4. If the solution $y = y(x)$ of the differential equation $(x^4 + 2x^3 + 3x^2 + 2x + 2)dy - (2x^2 + 2x + 3)dx = 0$ satisfies $y(-1) = -\dfrac{\pi}{4}$, then $y(0)$ is equal to :

(1) $-\dfrac{\pi}{12}$ (2) 0

(3) $\dfrac{\pi}{4}$ (4) $\dfrac{\pi}{2}$

Ans. (3)

Sol. $\int dy = \int \frac{(2x^2+2x+3)}{x^4+2x^3+3x^2+2x+2}dx$

$y = \int \frac{(2x^2+2x+3)}{(x^2+1)(x^2+2x+2)}dx$

$y = \int \frac{dx}{x^2+2x+2} + \int \frac{dx}{x^2+1}$

$y = \tan^{-1}(x+1) + \tan^{-1}x + C$

$y(-1) = \frac{-\pi}{4}$

$\frac{-\pi}{4} = 0 - \frac{\pi}{4} + C \Rightarrow C = 0$

$\Rightarrow y = \tan^{-1}(x+1) + \tan^{-1}x$

$y(0) = \tan^{-1}1 = \frac{\pi}{4}$

5. Let the sum of the maximum and the minimum values of the function $f(x) = \frac{2x^2-3x+8}{2x^2+3x+8}$ be $\frac{m}{n}$, where gcd(m, n) = 1. Then m + n is equal to :

(1) 182 (2) 217

(3) 195 (4) 201

Ans. (4)

Sol. $y = \frac{2x^2-3x+8}{2x^2+3x+8}$

$x^2(2y-2) + x(3y+3) + 8y - 8 = 0$

use $D \geq 0$

$(3y+3)^2 - 4(2y-2)(8y-8) \geq 0$

$(11y-5)(5y-11) \leq 0$

$\Rightarrow y \in \left[\frac{5}{11}, \frac{11}{5}\right]$

y = 1 is also included

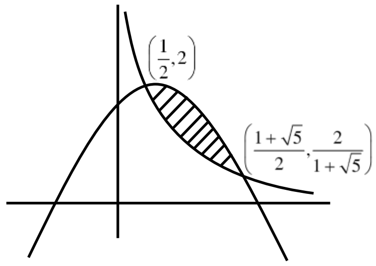
6. One of the points of intersection of the curves $y = 1 + 3x - 2x^2$ and $y = \frac{1}{x}$ is $\left(\frac{1}{2}, 2\right)$. Let the area of the region enclosed by these curves be $\frac{1}{24}\left(\ell\sqrt{5}+m\right) - n\log_e\left(1+\sqrt{5}\right)$, where ℓ, m, n $\in$ N. Then ℓ + m + n is equal to

(1) 32 (2) 30

(3) 29 (4) 31

Ans. (2)

Sol.



$A = \int_{\frac{1}{2}}^{\frac{1+\sqrt{5}}{2}} \left(1+3x-2x^2-\frac{1}{x}\right)dx$

$A = \left[x + \frac{3x^2}{2} - \frac{2x^3}{3} - \ell n\, x\right]_{\frac{1}{2}}^{\frac{1+\sqrt{5}}{2}}$

$A = \frac{1+\sqrt{5}}{2} + \frac{3}{2}\left(\frac{1+\sqrt{5}}{2}\right)^2 - \frac{2}{3}\left(\frac{1+\sqrt{5}}{2}\right)^3 - \ell n\left(\frac{1+\sqrt{5}}{2}\right)$

$-\frac{1}{2} - \frac{3}{2}\left(\frac{1}{4}\right) + \frac{2}{3}\left(\frac{1}{8}\right) + \ell n\left(\frac{1}{2}\right)$

$A = \frac{1}{2} + \frac{\sqrt{5}}{2} + \frac{3}{8} + \frac{3}{4}\sqrt{5} + \frac{15}{8} - \frac{4}{3} - \frac{2}{3}\sqrt{5}$

$-\frac{1}{2} - \frac{3}{8} + \frac{1}{12} - \ell n\left(1+\sqrt{5}\right)$

$= \sqrt{5}\left(\frac{1}{2} + \frac{3}{4} - \frac{2}{3}\right) + \frac{15}{8} - \frac{4}{3} + \frac{1}{12} - \ell n\left(1+\sqrt{5}\right)$

$= \frac{14}{24}\sqrt{5} + \frac{15}{24} - \ell n\left(1+\sqrt{5}\right)$

7. If the system of equations

$x + \left(\sqrt{2}\sin\alpha\right)y + \left(\sqrt{2}\cos\alpha\right)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$

$x + (\sin\alpha)y - (\cos\alpha)z = 0$

has a non-trivial solution, then $\alpha \in \left(0, \frac{\pi}{2}\right)$ is equal to :

(1) $\frac{3\pi}{4}$ (2) $\frac{7\pi}{24}$

(3) $\frac{5\pi}{24}$ (4) $\frac{11\pi}{24}$

Ans. (3)

Sol. $\begin{vmatrix} 1 & \sqrt{2}\sin\alpha & \sqrt{2}\cos\alpha \\ 1 & \sin\alpha & -\cos\alpha \\ 1 & \cos\alpha & \sin\alpha \end{vmatrix} = 0$

$\Rightarrow 1 - \sqrt{2}\sin\alpha(\sin\alpha + \cos\alpha) + \sqrt{2}\cos\alpha(\cos\alpha - \sin\alpha) = 0$

$\Rightarrow 1 + \sqrt{2}\cos 2\alpha - \sqrt{2}\sin 2\alpha = 0$

$\cos 2\alpha - \sin 2\alpha = -\frac{1}{\sqrt{2}}$

$\cos\left(2\alpha + \frac{\pi}{4}\right) = -\frac{1}{2}$

$2\alpha + \frac{\pi}{4} = 2n\pi \pm \frac{2\pi}{3}$

$\alpha + \frac{\pi}{8} = n\pi \pm \frac{\pi}{3}$

$n = 0,$

$x = \frac{\pi}{3} - \frac{\pi}{8} = \frac{5\pi}{24}$

8. There are 5 points P_1, P_2, P_3, P_4, P_5 on the side AB, excluding A and B, of a triangle ABC. Similarly there are 6 points $P_6, P_7, \ldots, P_{11}$ on the side BC and 7 points $P_{12}, P_{13}, \ldots, P_{18}$ on the side CA of the triangle. The number of triangles, that can be formed using the points $P_1, P_2, \ldots, P_{18}$ as vertices, is :

(1) 776 (2) 751

(3) 796 (4) 771

Ans. (2)

Sol. ${}^{18}C_3 - {}^{5}C_3 - {}^{6}C_3 - {}^{7}C_3$

$= 751$

9. Let $f(x) = \begin{cases} -2, & -2 \le x \le 0 \\ x-2, & 0 < x \le 2 \end{cases}$ and $h(x) = f(|x|) + |f(x)|$.

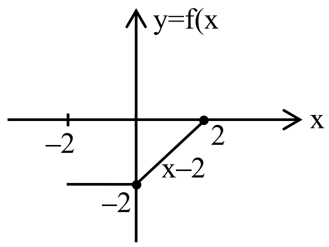
Then $\int_{-2}^{2} h(x)dx$ is equal to :

(1) 2 (2) 4

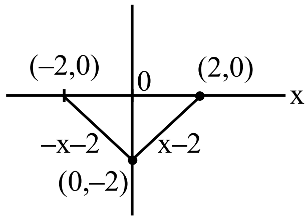
(3) 1 (4) 6

Ans. (1)

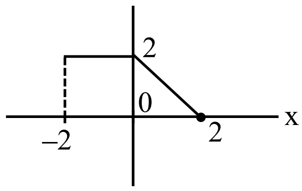
Sol.



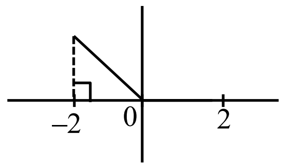
f(|x|) →



|f(x)|



$h(x) = \begin{cases} x - 2 + 2 - x = 0, & 0 \le x \le 2 \\ -x - 2 + 2 = -x & -2 \le x < 0 \end{cases}$



$\Rightarrow \int_{0}^{2} h(x)dx = 0$ and $\int_{-2}^{0} h(x)dx = 2$

10. The sum of all rational terms in the expansion of $\left(2^{\frac{1}{5}} + 5^{\frac{1}{3}}\right)^{15}$ is equal to :

(1) 3133 (2) 633

(3) 931 (4) 6131

Ans. (1)

Sol. $T_{r+1} = {}^{15}C_r \left(5^{\frac{1}{3}}\right)^r \left(2^{\frac{1}{5}}\right)^{15-r}$

$= {}^{15}C_r\, 5^{\frac{r}{3}} . 2^{\frac{15-r}{5}}$

$R = 3\lambda, 15\mu$

$\Rightarrow r = 0, 15$

2 rational terms

$\Rightarrow {}^{15}C_0 2^3 + {}^{15}C_{15}(5)^5$

$= 8 + 3125 = 3133$

11. Let a unit vector which makes an angle of 60° with $2\hat{i}+2\hat{j}-\hat{k}$ and an angle of 45° with $\hat{i}-\hat{k}$ be $\vec{C}$. Then $\vec{C}+\left(-\frac{1}{2}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{\sqrt{2}}{3}\hat{k}\right)$ is :

(1) $-\frac{\sqrt{2}}{3}\hat{i}+\frac{\sqrt{2}}{3}\hat{j}+\left(\frac{1}{2}+\frac{2\sqrt{2}}{3}\right)\hat{k}$

(2) $\frac{\sqrt{2}}{3}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{1}{2}\hat{k}$

(3) $\left(\frac{1}{\sqrt{3}}+\frac{1}{2}\right)\hat{i}+\left(\frac{1}{\sqrt{3}}-\frac{1}{3\sqrt{2}}\right)\hat{j}+\left(\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{3}\right)\hat{k}$

(4) $\frac{\sqrt{2}}{3}\hat{i}-\frac{1}{2}\hat{k}$

Ans. (4)

Sol. $\vec{C}=C_1\hat{i}+C_2\hat{j}+C_3\hat{k}$

$C_1^2+C_2^2+C_3^2=1$

$\vec{C}.\left(2\hat{i}+2\hat{j}-\hat{k}\right)=|C|\sqrt{9}\cos 60°$

$2C_1+2C_2-C_3=\frac{3}{2}$

$C_1-C_3=1$

$C_1+2C_2=\frac{1}{2}$

$C_1=\frac{\sqrt{2}}{3}+\frac{1}{2}$

$C_2=\frac{-1}{3\sqrt{2}}$

$C_3=\frac{\sqrt{2}}{3}-\frac{1}{2}$

12. Let the first three terms 2, p and q, with $q \neq 2$, of a G.P. be respectively the 7^{th}, 8^{th} and 13^{th} terms of an A.P. If the 5^{th} term of the G.P. is the n^{th} term of the A.P., then n is equal to

(1) 151 (2) 169

(3) 177 (4) 163

Ans. (4)

Sol. $p^2=2q$

$2=a+6d$ ...(i)

$p=a+7d$ ...(ii)

$q=a+12d$ ...(iii)

$p-2=d$ ((ii) – (i))

$q-p=5d$ ((iii) – (ii))

$q-p=5(p-2)$

$q=6p-10$

$p^2=2(6p-10)$

$p^2-12p+20=0$

$p=10, 2$

$p=10$; $q=50$

$d=8$

$a=-46$

2, 10, 50, 250, 1250

$ar^4=a+(n-1)d$

$1250=-46+(n-1)8$

$n=163$

13. Let $a, b \in R$. Let the mean and the variance of 6 observations –3, 4, 7, –6, a, b be 2 and 23, respectively. The mean deviation about the mean of these 6 observations is :

(1) $\frac{13}{3}$ (2) $\frac{16}{3}$

(3) $\frac{11}{3}$ (4) $\frac{14}{3}$

Ans. (1)

Sol. $\frac{\Sigma x_i}{6}=2$ and $\frac{\Sigma x_i^2}{N}-\mu^2=23$

$\alpha+\beta=10$

$\alpha^2+\beta^2=52$

solving we get $\alpha=4, \beta=6$

$\frac{\Sigma|x_i-\bar{x}|}{6}=\frac{5+2+5+8+2+4}{6}=\frac{13}{3}$

14. If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$, is :

(1) $2x^2 + 11x + 12 = 0$ (2) $4x^2 + 14x + 12 = 0$

(3) $x^2 + 10x + 16 = 0$ (4) $x^2 + 8x + 12 = 0$

Ans. (4)

Sol. Sum $= 8 = -\frac{b}{a}$

Product $= 12 = \frac{1}{a}$ $\Rightarrow a = \frac{1}{12}$

$b = -\frac{2}{3}$

$2a + b = \frac{2}{12} - \frac{2}{3} = -\frac{1}{2}$

$6a + b = \frac{6}{12} - \frac{2}{3} = -\frac{1}{6}$

sum = –8

P = 12

$x^2 + 8x + 12 = 0$

15. Let α and β be the sum and the product of all the non-zero solutions of the equation $(\bar{z})^2 + |z| = 0$, $z \in C$. Then $4(\alpha^2 + \beta^2)$ is equal to :

(1) 6 (2) 4

(3) 8 (4) 2

Ans. (2)

Sol. $z = x + iy$

$\bar{z} = x - iy$

$\bar{z}^2 = x^2 - y^2 - 2ixy$

$\Rightarrow x^2 - y^2 - 2ixy + \sqrt{x^2 + y^2} = 0$

$\Rightarrow x = 0$ or $y = 0$

$-y^2 + |y| = 0$ $\quad$ $x^2 + |x| = 0$

$|y| = |y|^2$ $\quad$ $\Rightarrow x = 0$

$y = 0, \pm 1$

$\Rightarrow i, -i$ are roots $\quad$ $\Rightarrow \alpha = i - i = 0$

$\beta = i(-i) = 1$

$4(0 + 1) = 4$

16. Let the point, on the line passing through the points P(1, –2, 3) and Q(5, –4, 7), farther from the origin and at a distance of 9 units from the point P, be (α, β, γ). Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

(1) 155 (2) 150

(3) 160 (4) 165

Ans. (1)

Sol. PQ line

$\frac{x-1}{4} = \frac{y+2}{-2} = \frac{z-3}{4}$

pt $(4t + 1, -2t - 2, 4t + 3)$

$\text{distance}^2 = 16t^2 + 4t^2 + 16t^2 = 81$

$t = \pm\frac{3}{2}$

pt (7, –5, 9)

$\alpha^2 + \beta^2 + \gamma^2 = 155$

option (1)

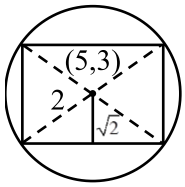
17. A square is inscribed in the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. One side of this square is parallel to $y = x + 3$. If (x_i, y_i) are the vertices of the square, then $\sum(x_i^2 + y_i^2)$ is equal to :

(1) 148 (2) 156

(3) 160 (4) 152

Ans. (4)

Sol.



$y = x + c$ & $x + y + d = 0$

$\left|\frac{5-3+c}{\sqrt{2}}\right| = \sqrt{2}$ $\quad$ $\left|\frac{8+d}{\sqrt{2}}\right| = \sqrt{2}$

$|c + 2| = 2$ $\quad$ $8 + d = \pm 2$

$c = 0, -4$ $\quad$ $d = -10, -6$

pts (5, 5), (3, 3), (7, 3), (5, 1)

$\sum(x_i^2 + y_i^2) = 25 + 25 + 9 + 9 + 49 + 9 + 25 + 1$

= 152

Option (4)

18. If the domain of the function $\sin^{-1}\left(\frac{3x-22}{2x-19}\right)+\log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is $(\alpha, \beta]$, then $3\alpha + 10\beta$ is equal to :

(1) 97 (2) 100

(3) 95 (4) 98

Ans. (1)

Sol. $-1 \le \frac{3x-22}{2x-19} \le 1$ $\qquad \frac{3x^2-8x+5}{x^2-3x-10} > 0$

$x \in \left(5, \frac{41}{5}\right]$

$3\alpha + 10\beta = 97$

Option (1)

19. Let $f(x) = x^5 + 2e^{x/4}$ for all $x \in R$. Consider a function $g(x)$ such that $(gof)(x) = x$ for all $x \in R$. Then the value of $8g'(2)$ is :

(1) 16 (2) 4

(3) 8 (4) 2

Ans. (1)

Sol. $f(x) = 2$

when $x = 0$

$\because g'(f(x))\, f'(x) = 1$

$g'(2) = \frac{1}{f'(0)}$

$\because f'(x) = 5x^4 + \frac{2}{4}e^{x/4}$

$g'(2) = 2$

Ans = 16

Option (1)

20. Let $\alpha \in (0, \infty)$ and $A = \begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

If $\det(\text{adj}(2A - A^T).\text{adj}(A - 2A^T)) = 2^8$, then $(\det(A))^2$ is equal to :

(1) 1 (2) 49

(3) 16 (4) 36

Ans. (3)

Sol. $|\text{adj}(A - 2A^T)\,(2A - A^T)| = 2^8$

$|(A - 2A^T)\,(2A - A^T)| = 2^4$

$|A - 2A^T|\,|2A - A^T| = \pm 16$

$(A - 2A^T)^T = A^T - 2A$

$|A - 2A^T| = |A^T - 2A|$

$\Rightarrow |A - 2A^T|^2 = 16$

$|A - 2A^T| = \pm 4$

$\begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 2 & 0 \\ 4 & 0 & 2 \\ 2\alpha & 2 & 4 \end{bmatrix}$

$\begin{vmatrix} -1 & 0 & \alpha \\ -3 & 0 & -1 \\ -2\alpha & -1 & -2 \end{vmatrix}$

$1 + 3\alpha = 4$

$3\alpha = 3$

$\alpha = 1$

$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix} = -1 - 3 = -4$

$|A|^2 = 16$

SECTION-B

21. If $\lim\limits_{x\to 1}\frac{(5x+1)^{1/3}-(x+5)^{1/3}}{(2x+3)^{1/2}-(x+4)^{1/2}} = \frac{m\sqrt{5}}{n(2n)^{2/3}}$, where $\gcd(m, n) = 1$, then $8m + 12n$ is equal to ______

Ans. (100)

Sol. $\lim\limits_{x\to 1}\frac{\frac{1}{3}(5x+1)^{-2/3}5-\frac{1}{3}(x+5)^{-2/3}}{\frac{1}{2}(2x+3)^{-1/2}.2-\frac{1}{2}(x+4)^{-1/2}}$

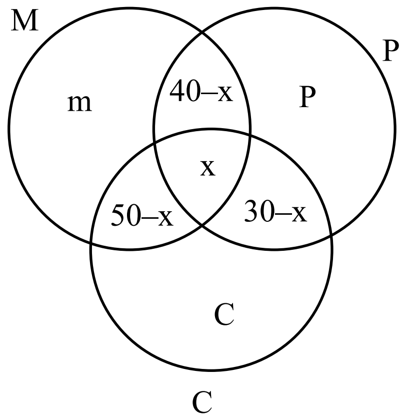
$= \frac{8}{3}\frac{\sqrt{5}}{6^{2/3}}$ $\quad m = 8$, $n = 3$

$8m + 12n = 100$

22. In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then m + n is equal to _____

Ans. (45)

Sol.



$125 \le m + 90 - x \le 130$

$85 \le P + 70 - x \le 95$

$75 \le C + 80 - x \le 90$

$m + P + C + 120 - 2x = 210$

$\Rightarrow 15 \le x \le 45 \ \& \ 30 - x \ge 0$

$\Rightarrow 15 \le x \le 30$

$30 + 15 = 45$

23. Let the solution y = y(x) of the differential equation $\frac{dy}{dx} - y = 1 + 4\sin x$ satisfy $y(\pi) = 1$. Then $y\left(\frac{\pi}{2}\right) + 10$ is equal to _____

Ans. (7)

Sol. $ye^{-x} = \int (e^{-x} + 4e^{-x}\sin x)dx$

$ye^{-x} = -e^{-x} - 2(e^{-x}\sin x \ \ e^{-x}\cos x) + C$

$y = -1 - 2(\sin x + \cos x) + ce^{x}$

$\because y(\pi) = 1 \Rightarrow c = 0$

$y(\pi/2) = -1 - 2 = -3$

Ans = 10 – 3 = 7

24. If the shortest distance between the lines $\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-5}{4}$ and $\frac{x-3}{1} = \frac{y-2}{-3} = \frac{z+4}{2}$ is $\frac{38}{3\sqrt{5}}k$ and $\int_0^k [x^2]dx = \alpha - \sqrt{\alpha}$, where [x] denotes the greatest integer function, then $6\alpha^3$ is equal to _____

Ans. (48)

Sol. $\frac{38}{3\sqrt{5}}\hat{k} = \frac{(5\hat{i} + 5\hat{j} - 9\hat{k})}{\sqrt{5}} \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & -3 & 2 \end{vmatrix}$

$\frac{38}{3\sqrt{5}}\hat{k} = \frac{19}{\sqrt{5}}$

$k = \frac{19}{\sqrt{5}}$

$k = \frac{3}{2}$

$\int_0^{3/2} [x^2] = \int_0^1 0 + \int_1^{\sqrt{2}} 1 + \int_{\sqrt{2}}^{3/2} 2$

$= \sqrt{2} - 1 + 2\left(\frac{3}{2} - \sqrt{2}\right)$

$= 2 - \sqrt{2}$

$\alpha = 2$

$\Rightarrow 6\alpha^3 = 48$

25. Let A be a square matrix of order 2 such that |A| = 2 and the sum of its diagonal elements is –3. If the points (x, y) satisfying $A^2 + xA + yI = 0$ lie on a hyperbola, whose transverse axis is parallel to the x-axis, eccentricity is e and the length of the latus rectum is ℓ, then $e^4 + \ell^4$ is equal to ______

Ans. (Bouns)

NTA Ans. (25)

Sol. Given |A| = 2

trace A = –3

and $A^2 + xA + yI = 0$

$\Rightarrow x = 3, y = 2$

so, information is incomplete to determine eccentricity of hyperbola (e) and length of latus rectum of hyperbola (ℓ)

26. Let $a = 1 + \frac{^2C_2}{3!} + \frac{^3C_2}{4!} + \frac{^4C_2}{5!} + \ldots,$

$b = 1 + \frac{^1C_0 + {}^1C_1}{1!} + \frac{^2C_0 + {}^2C_1 + {}^2C_2}{2!} + \frac{^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3}{3!} + \ldots$

Then $\frac{2b}{a^2}$ is equal to ______

Ans. (8)

Sol. $f(x) = 1 + \frac{(1+x)}{1!} + \frac{(1+x)^2}{2!} + \frac{(1+x)^3}{3!} + \ldots..$

$\frac{e^{(1+x)}}{1+x} = \frac{1}{1+x} + 1 + \frac{(1+x)}{2!} + \frac{(1+x)^2}{3!} + \frac{(1+x)^2}{4!}$

coef x^2 in RHS : $1 + \frac{^2C_2}{3} + \frac{^3C_2}{4} + \ldots. = a$

coeff. x^2 in L.H.S.

$e\left(1 + x + \frac{x^2}{2!}\right)\ldots.\left(1 - x + \frac{x^2}{2!}\ldots..\right)$

is $e - e + \frac{e}{2!} = a$

$b = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \ldots.. = e^2$

$\frac{2b}{a^2} = 8$

27. Let A be a 3 × 3 matrix of non-negative real elements such that $A\begin{bmatrix}1\\1\\1\end{bmatrix} = 3\begin{bmatrix}1\\1\\1\end{bmatrix}$. Then the maximum value of det(A) is _____

Ans. (27)

Sol. Let A = $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$A\begin{bmatrix}1\\1\\1\end{bmatrix} = 3\begin{bmatrix}1\\1\\1\end{bmatrix}$

$\Rightarrow a_1 + a_2 + a_3 = 3$ …..(1)

$\Rightarrow b_1 + b_2 + b_3 = 3$ …..(2)

$\Rightarrow c_1 + ca_2 + c_3 = 3$ …..(3)

Now,

$|A| = (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2)$

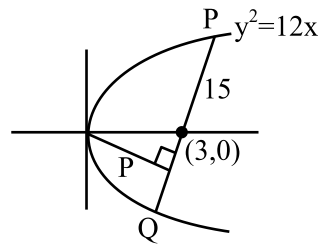
$- (a_3b_2c_1 + a_2b_1c_3 + a_1b_3c_2)$

$\therefore$ From above in formation, clearly $|A|_{max} = 27$, when $a_1 = 3, b_2 = 3, c_3 = 3$

28. Let the length of the focal chord PQ of the parabola $y^2 = 12x$ be 15 units. If the distance of PQ from the origin is p, then $10p^2$ is equal to ____

Ans. (72)

Sol.



length of focal chord = 4a cosec$^2\theta$ = 15

$12\text{cosec}^2\theta = 15$

$\sin^2\theta = \frac{4}{5}$

$\tan^2\theta = 4$

$\tan\theta = 2$

equation $\frac{y-0}{x-3} = 2$

$y = 2x - 6$

$2x - y - 6 = 0$

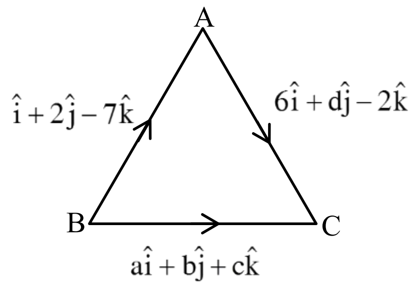
$P = \frac{6}{\sqrt{5}}$

$10p^2 = 10.\frac{36}{5} = 72$

29. Let ABC be a triangle of area $15\sqrt{2}$ and the vectors $\overrightarrow{AB} = \hat{i} + 2\hat{j} - 7\hat{k}$, $\overrightarrow{BC} = a\hat{i} + b\hat{j} + c\hat{k}$ and $\overrightarrow{AC} = 6\hat{i} + d\hat{j} - 2\hat{k}$, d > 0. Then the square of the length of the largest side of the triangle ABC is

Ans. (54)

Sol.



$$\text{Area} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -7 \\ 6 & d & -2 \end{vmatrix} = 15\sqrt{2}$$

$(-4 + 7d)\hat{i} - \hat{j}(-2 + 42) + \hat{k}(d - 12)$

$(7d - 4)^2 + (40)^2 + (d - 12)^2 = 1800$

$50d^2 - 80d - 40 = 0$

$5d^2 - 8d - 4 = 0$

$5d^2 - 10d - 2d - 4$

$5d(d - 2) + 2(d - 2) = 0$

$d = 2 \text{ or } d = -\frac{2}{5}$

$\because d > 0, d = 2$

$(a + 1)\hat{i} + (b + 2)\hat{j} + (c - 7)\hat{k} = 6\hat{i} + 2\hat{j} - 2\hat{k}$

$a + 1 = 6 \ \& \ b + 2 = 2, c - 7 = -2$

$a = 5 \quad b = 0 \quad c = 5$

$|AB| = \sqrt{1+4+49} = \sqrt{54}$

$|BC| = \sqrt{25+25} = \sqrt{50}$

$|AC| = \sqrt{86+4+4} = \sqrt{44}$

Ans. 54

30. If $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{1+\sin x \cos x} dx = \frac{1}{a}\log_e\left(\frac{a}{3}\right) + \frac{\pi}{b\sqrt{3}}$, where a, b $\in$ N, then a + b is equal to ______

Ans. (8)

Sol. $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\frac{1}{2}\sin 2x} dx = \int_0^{\frac{\pi}{4}} \frac{1-\cos 2x}{2+\sin 2x} dx$

$\int \frac{1}{2+\sin 2x} - \int \frac{\cos 2x}{2+\sin 2x}$

$(I_1) \quad - \quad (I_2)$

$(I_1) = \int \frac{dx}{2+\frac{2\tan x}{1+\tan^2 x}}$

$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x \, dx}{2\tan^2 x + 2\tan x + 2}$

$\tan x = t$

$\frac{1}{2}\int_0^1 \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \frac{3}{4}} = \frac{\pi}{6\sqrt{3}}$

$I_2 = \int_0^{\pi/4} \frac{\cos 2x}{2+\sin 2x} dx = \frac{1}{2}\left(\ell n \frac{3}{2}\right)$

$I_1 - I_2 = \frac{1}{\sqrt{3}}\frac{\pi}{6} + \frac{1}{2}\ell n \frac{2}{3}$

$\Rightarrow a = 2, b = 6$

Ans. 8

SECTION-A

31. An electron is projected with uniform velocity along the axis inside a current carrying long solenoid. Then :

(1) the electron will be accelerated along the axis.

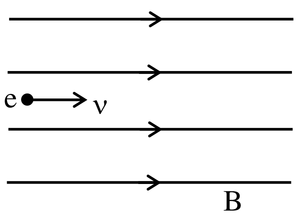
(2) the electron will continue to move with uniform velocity along the axis of the solenoid.

(3) the electron path will be circular about the axis.

(4) the electron will experience a force at 45° to the axis and execute a helical path.

Ans. **(2)**

Sol.



Since $\vec{v} \parallel \vec{B}$ so force on electron due to magnetic field is zero. So it will move along axis with uniform velocity.

32. The electric field in an electromagnetic wave is given by $\vec{E} = \hat{i}40\cos\omega\left(t - \frac{z}{c}\right)NC^{-1}$. The magnetic field induction of this wave is (in SI unit):

(1) $\vec{B} = \hat{i}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$

(2) $\vec{B} = \hat{j}40\cos\omega\left(t - \frac{z}{c}\right)$

(3) $\vec{B} = \hat{k}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$

(4) $\vec{B} = \hat{j}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$

Ans. **(4)**

Sol. $\vec{E} = \hat{i}40\cos\omega\left(t - \frac{z}{c}\right)$

$\vec{E}$ is along +x direction

$\vec{v}$ is along +z direction

So direction of $\vec{B}$ will be along +y and magnitude of B will be $\frac{E}{c}$

So answer is $\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)\hat{j}$

33. Which of the following nuclear fragments corresponding to nuclear fission between neutron $\left({}_0^1n\right)$ and uranium isotope $\left({}_{92}^{235}U\right)$ is correct:

(1) ${}_{56}^{144}Ba + {}_{36}^{89}Kr + 4{}_0^1n$ (2) ${}_{56}^{140}Xe + {}_{38}^{94}Sr + 3{}_0^1n$

(3) ${}_{51}^{153}Sb + {}_{41}^{99}Nb + 3{}_0^1n$ (4) ${}_{56}^{144}Ba + {}_{36}^{89}Kr + 3{}_0^1n$

Ans. **(4)**

Sol. Balancing mass number and atomic number

${}_{92}^{235}U + {}_0^1n \rightarrow {}_{56}^{144}Ba + {}_{36}^{89}Kr + 3{}_0^1n$

34. In an experiment to measure focal length (f) of convex lens, the least counts of the measuring scales for the position of object (u) and for the position of image (v) are Δu and Δv, respectively. The error in the measurement of the focal length of the convex lens will be :

(1) $\frac{\Delta u}{u} + \frac{\Delta v}{v}$ (2) $f^2\left[\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}\right]$

(3) $2f\left[\frac{\Delta u}{u} + \frac{\Delta v}{v}\right]$ (4) $f\left[\frac{\Delta u}{u} + \frac{\Delta v}{v}\right]$

Ans. **(2)**

Sol. $f^{-1} = v^{-1} - u^{-1}$

$-f^{-2}\,df = -v^{-2}dv - u^{-2}du$

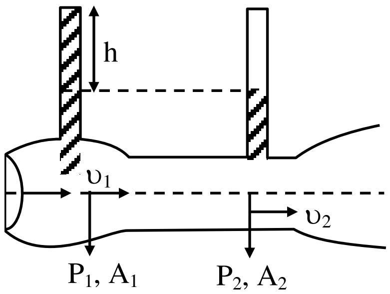
$\frac{df}{f^2} = \frac{dv}{v^2} + \frac{du}{u^2}$

$df = f^2\left[\frac{dv}{v^2} + \frac{du}{u^2}\right]$

35. Given below are two statements :

Statement I : When speed of liquid is zero everywhere, pressure difference at any two points depends on equation $P_1 - P_2 = \rho g (h_2 - h_1)$

Statement II : In ventury tube shown $2gh = \upsilon_1^2 - \upsilon_2^2$



In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and Statement II are correct.

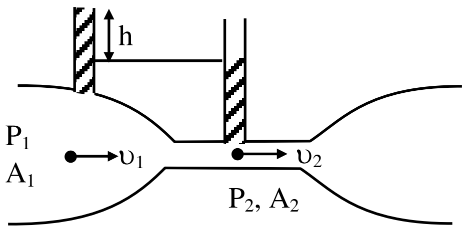
(2) Statement I is incorrect but Statement II is correct.

(3) Both Statement I and Statement II are incorrect.

(4) Statement I is correct but Statement II is incorrect.

Ans. **(4)**

Sol.



Applying Bernoulli's equation

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

[h_1 & h_2 are height of point from any reference level]

Given $V_1 = V_2 = 0$ (for statement-1)

$\therefore P_1 - P_2 = \rho g(h_2 - h_2)$

For statement-2

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$P_1 - P_2 = \rho g h$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$\rho g h = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$2gh = v_2^2 - v_1^2$$

Hence answer (4)

36. The resistances of the platinum wire of a platinum resistance thermometer at the ice point and steam point are 8 Ω and 10 Ω respectively. After inserting in a hot bath of temperature 400°C, the resistance of platinum wire is :

(1) 2Ω (2) 16 Ω (3) 8 Ω (4) 10 Ω

Ans. **(2)**

Sol. Given $R_0 = 8\Omega$, $R_{100} = 10\Omega$

$\therefore R_{100} = R_0 (1 + \alpha\Delta T)$

Also, $R_{400} = R_0 (1 + \alpha\Delta T^1)$

$\therefore 10 = 8 (1 + \alpha \times 100) \Rightarrow 100\alpha = \frac{1}{4}$

$\therefore R_{400} = 8 (1 + 400\alpha) = 8 (1 + 1) = 16\Omega$

Hence option (2)

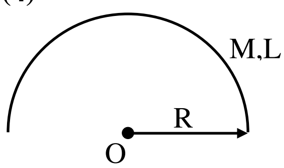
37. A metal wire of uniform mass density having length L and mass M is bent to form a semicircular arc and a particle of mass m is placed at the centre of the arc. The gravitational force on the particle by the wire is:

(1) $\frac{GMm\pi}{2L^2}$ (2) 0

(3) $\frac{GmM\pi^2}{L^2}$ (4) $\frac{2GmM\pi}{L^2}$

Ans. **(4)**

Sol.



We have $R = \frac{L}{\pi}$

$$g_0 = \frac{2G\frac{M}{L}}{R} = \frac{2GM\pi}{L^2}$$

$$\therefore F_m = mg_0 = \frac{2GM\pi m}{L^2}$$

Hence option (4)

38. On celcius scale the temperature of body increases by 40°C. The increase in temperature on Fahrenheit scale is:

(1) 70°F

(2) 68°F

(3) 72°F

(4) 75°F

Ans. **(3)**

Sol. We know that per °C change is equivalent to 1.8° change in °F.

∴ 40° change on celcius scale will corresponds to 72° change on Fahrenheit scale.

Hence option (3)

39. An effective power of a combination of 5 identical convex lenses which are kept in contact along the principal axis is 25 D. Focal length of each of the convex lens is :

(1) 20 cm

(2) 50 cm

(3) 500 cm

(4) 25 cm

Ans. **(1)**

Sol. We know that $P_{eq} = \Sigma P_i$

∴ given all lenses are identical

∴ 5P = 25D

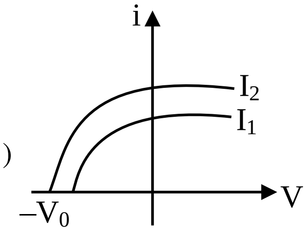
∴ P = 5D

$$\therefore \frac{1}{f} = 5 \Rightarrow f = \frac{1}{5}m = 20cm$$

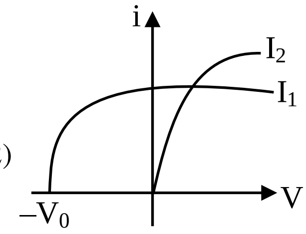
Hence option (1)

40. Which figure shows the correct variation of applied potential difference (V) with photoelectric current (I) at two different intensities of light ($I_1 < I_2$) of same wavelengths :

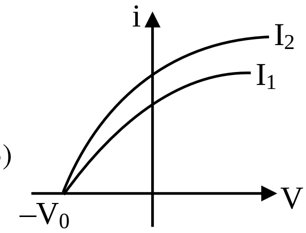
(1)



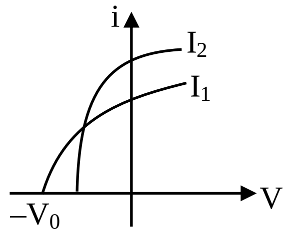
(2)



(3)



(4)



Ans. **(3)**

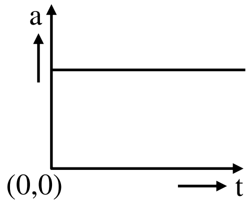
Sol. Given lights are of same wavelength.

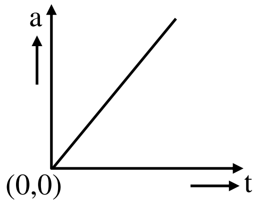
Hence stopping potential will remain same.

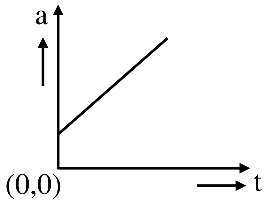
Since $I_2 > I_1$, hence saturation current corresponding to I_2 will be greater than that corresponding to I_1.

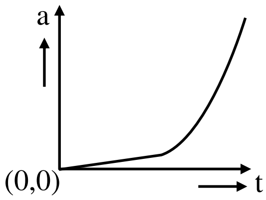
Hence option (3)

41. A wooden block, initially at rest on the ground, is pushed by a force which increases linearly with time t. Which of the following curve best describes acceleration of the block with time :

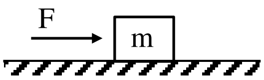
(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol. 

$$F = ma \Rightarrow a = \frac{F}{m} = \frac{kt}{m}$$

a vs t will be straight line passing through origin.

Since option (2).

42. If a rubber ball falls from a height h and rebounds upto the height of h/2. The percentage loss of total energy of the initial system as well as velocity ball before it strikes the ground, respectively, are :

(1) 50%, $\sqrt{\frac{gh}{2}}$ (2) 50%, $\sqrt{gh}$

(3) 40%, $\sqrt{2gh}$ (4) 50%, $\sqrt{2gh}$

Ans. (4)

Sol. Velocity just before collision = $\sqrt{2gh}$

Velocity just after collision = $\sqrt{2g\left(\frac{h}{2}\right)}$

$$\therefore \Delta KE = \frac{1}{2}m(2gh) - \frac{1}{2}mgh$$

$$= \frac{1}{2}mgh$$

$\therefore$ % loss in energy

$$= \frac{\Delta KE}{KE_i} \times 100 = \frac{\frac{1}{2}mgh}{\frac{1}{2}mg2h} \times 100 = 50\%$$

Hence option (4)

43. The equation of stationary wave is :

$$y = 2a\sin\left(\frac{2\pi nt}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$$

Which of the following is NOT correct

(1) The dimensions of nt is [L]

(2) The dimensions of n is [LT^{-1}]

(3) The dimensions of n/λ is [T]

(4) The dimensions of x is [L]

Ans. (3)

Sol. Comparing the given equation with standard equation of standing $\frac{2\pi n}{\lambda} = \omega$ & $\frac{2\pi}{\lambda} = k$

$$\left[\frac{n}{\lambda}\right] = [\omega] = T^{-1}$$

[nt] = [λ] = L

[n] = [λω] = LT^{-1}

[x] = [λ] = L

Hence option (3)

44. A body travels 102.5 m in n^{th} second and 115.0 m in $(n + 2)^{th}$ second. The acceleration is :

(1) 9 m/s^2 (2) 6.25 m/s^2

(3) 12.5 m/s^2 (4) 5 m/s^2

Ans. **(2)**

Sol. Given, $102.5 = u + \frac{a}{2}(2n - 1)$ &

$115 = u + \frac{a}{2}(2n + 3)$

$\Rightarrow 102.5 = u + an - \frac{a}{2}$ &

$115 = u + an + \frac{3a}{2}$

$12.5 = 2a \Rightarrow a = 6.25 \text{ m/s}^2$

Hence option (2)

45. To measure the internal resistance of a battery, potentiometer is used. For R = 10 Ω, the balance point is observed at ℓ = 500 cm and for R = 1 Ω the balance point is observed at ℓ = 400 cm. The internal resistance of the battery is approximately :

(1) 0.2 Ω (2) 0.4 Ω

(3) 0.1 Ω (4) 0.3 Ω

Ans. **(4)**

Sol. Let potential gradient be λ.

$\therefore\ i \times 10 = \lambda \times 500 = \varepsilon - ir_s$

$\Rightarrow 500\lambda = \varepsilon - 50\lambda r_s$

Also,

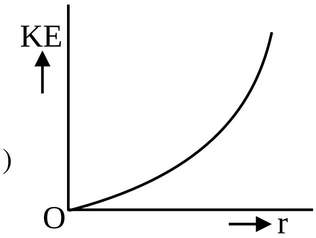
$i' \times 1 = \lambda \times 400 = \varepsilon - i'r_s$

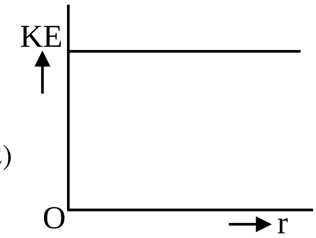
$\Rightarrow 400\lambda = \varepsilon - 400\ \lambda r_s$

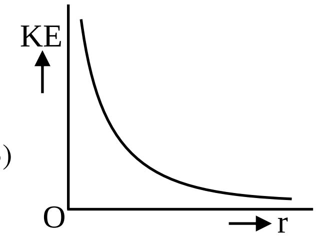
$\therefore\ 100\lambda = 350\lambda\ r_s \Rightarrow r_s = \frac{10}{35} \approx 0.3\Omega$

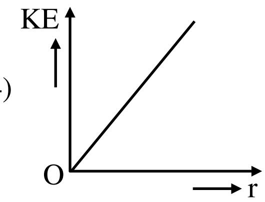
Hence option (4)

46. An infinitely long positively charged straight thread has a linear charge density λ Cm^{-1}. An electron revolves along a circular path having axis along the length of the wire. The graph that correctly represents the variation of the kinetic energy of electron as a function of radius of circular path from the wire is :

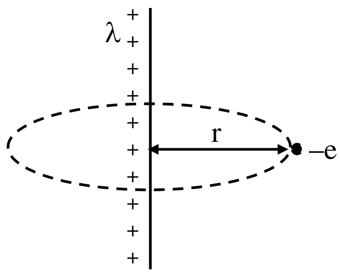
(1)


(2)


(3)


(4)


Ans. **(2)**

Sol.


Electric field E at a distance r due to infinite long wire is $E = \frac{2k\lambda}{r}$

Force of electron $\Rightarrow F = eE$

$$F = e\left(\frac{2k\lambda}{r}\right)$$

$$F = \frac{2k\lambda e}{r}$$

This force will provide required centripetal force

$$F = \frac{mv^2}{r} = \frac{2k\lambda e}{r}$$

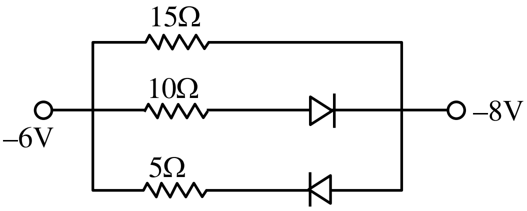
$$v = \sqrt{\frac{2k\lambda e}{m}}$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{2k\lambda e}{m}\right)$$

$$= k\lambda e$$

This is constant so option (2) is correct.

47. The value of net resistance of the network as shown in the given figure is :

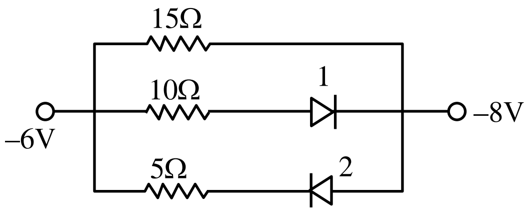


(1) $\left(\frac{5}{2}\right)\Omega$ (2) $\left(\frac{15}{4}\right)\Omega$

(3) 6Ω (4) $\left(\frac{30}{11}\right)\Omega$

Ans. **(3)**

Sol.

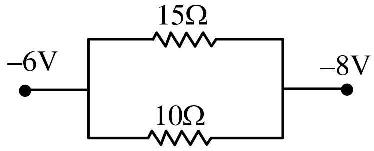


Diode 2 is in reverse bias

So current will not flow in branch of 2^{nd} diode, So we can assume it to be broken wire.

Diode 1 is in forward bias

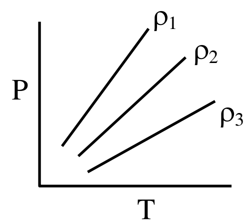
So it will behave like conducting wire. So new circuit will be



$$R_{eq} = \frac{15\times10}{15+10} = \frac{15\times10}{25} = 6\Omega$$

Correct answer (3)

48. P-T diagram of an ideal gas having three different densities ρ_1, ρ_2, ρ_3 (in three different cases) is shown in the figure. Which of the following is correct :



(1) $\rho_2 < \rho_3$ (2) $\rho_1 > \rho_2$

(3) $\rho_1 < \rho_2$ (4) $\rho_1 = \rho_2 = \rho_3$

Ans. **(2)**

Sol. For ideal gas

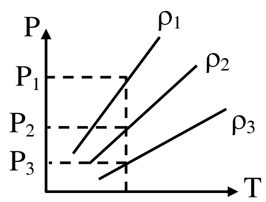
$PV = nRT$

$$PV = \frac{m}{M}RT$$

$$P = \left(\frac{M}{V}\right)\frac{RT}{M}$$

$$P = \frac{\rho RT}{M}$$

(Where m is mass of gas and M is molecular mass of gas)



for same temperature $P_1 > P_2 > P_3$

So $\rho_1 > \rho_2 > \rho_3$

So correct answer is (2)

49. The co-ordinates of a particle moving in x-y plane are given by :

$x = 2 + 4t$, $y= 3t + 8t^2$.

The motion of the particle is :

(1) non-uniformly accelerated.

(2) uniformly accelerated having motion along a straight line.

(3) uniform motion along a straight line.

(4) uniformly accelerated having motion along a parabolic path.

Ans. (4)

Sol. $x = 2 + 4t$

$$\frac{dx}{dt} = v_x = 4$$

$$\frac{dv_x}{dt} = a_x = 0$$

$y = 3t + 8t^2$

$$\frac{dy}{dt} = v_y = 3 + 16t$$

$$\frac{dv_y}{dt} = a_y = 16$$

the motion will be uniformly accelerated motion.

For path

$x = 2 + 4t$

$$\frac{(x-2)}{4} = t$$

Put this value of t is equation of y

$$y = 3\left(\frac{x-2}{4}\right) + 8\left(\frac{x-2}{4}\right)^2$$

this is a quadratic equation so path will be parabola.

Correct answer (4)

50. In an ac circuit, the instantaneous current is zero, when the instantaneous voltage is maximum. In this case, the source may be connected to :

A. pure inductor.

B. pure capacitor.

C. pure resistor.

D. combination of an inductor and capacitor.

Choose the correct answer from the options given below :

(1) A, B and C only (2) B, C and D only

(3) A and B only (4) A, B and D only

Ans. (4)

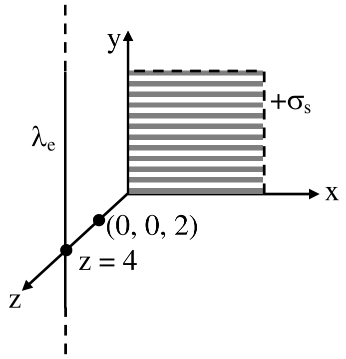
Sol. This is possible when phase difference is $\frac{\pi}{2}$ between current and voltage so correct answer will be (4)

SECTION-B

51. An infinite plane sheet of charge having uniform surface charge density $+\sigma_s$ C/m^2 is placed on x-y plane. Another infinitely long line charge having uniform linear charge density $+\lambda_e$ C/m is placed at z = 4m plane and parallel to y-axis. If the magnitude values $|\sigma_s| = 2|\lambda_e|$ then at point (0, 0, 2), the ratio of magnitudes of electric field values due to sheet charge to that of line charge is $\pi\sqrt{n} : 1$. The value of n is________.

Ans. (16)

Sol.



$$\frac{E_S}{E_\ell} = \frac{\sigma}{2\epsilon_0} \times \frac{2\pi\epsilon_0 r}{\lambda}$$

$$= \frac{\pi \times \sigma r}{\lambda}$$

$$= \frac{\pi \times 2\lambda \times 2}{\lambda} = \frac{4\pi}{1}$$

$\therefore$ n = 16

52. A hydrogen atom changes its state from n = 3 to n = 2. Due to recoil, the percentage change in the wave length of emitted light is approximately 1×10^{-n}. The value of n is__________.

[Given Rhc = 13.6 eV, hc = 1242 eV nm, h = 6.6×10^{-34} J s, mass of the hydrogen atom = 1.6×10^{-27} kg]

Ans. **(7)**

Sol. $\Delta E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$ = 1.9 eV

$$\Delta E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\Delta E}$$

$P_i = P_f$

$$0 = -mv + \frac{h}{\lambda'}$$

$$\Rightarrow v = \frac{h}{m\lambda'}$$

$$\Delta E = \frac{1}{2}mv^2 + \frac{hc}{\lambda'}$$

$$= \frac{1}{2}m\left(\frac{h}{m\lambda'}\right)^2 + \frac{hc}{\lambda'}$$

Now

$$\Delta E = \frac{h^2}{2m\lambda'^2} + \frac{hc}{\lambda'}$$

$$\lambda'^2 \Delta E - hc\lambda' - \frac{h^2}{2m} = 0$$

$$\lambda' = \frac{hc \pm \sqrt{h^2c^2 + \frac{4\Delta E h^2}{2m}}}{2\Delta E}$$

$$\lambda' = \frac{hc \pm hc\sqrt{1 + \frac{2\Delta E}{mc^2}}}{2\Delta E}$$

$$\frac{\lambda'}{\lambda} = \frac{1 + \left(1 + \frac{2\Delta E}{mc^2}\right)^{\frac{1}{2}}}{2} = \frac{1 + 1 + \frac{\Delta E}{mc^2}}{2}$$

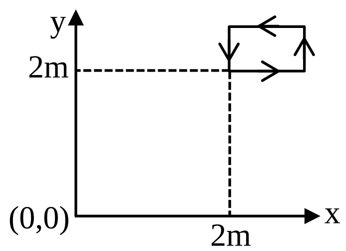
$$\frac{\lambda'}{\lambda} = 1 + \frac{\Delta E}{2mc^2}$$

$$\frac{\lambda' - \lambda}{\lambda} = \frac{\Delta E}{2mc^2} = \frac{1.9 \times 1.6 \times 10^{-19}}{2 \times 1.67 \times 10^{-27} \times 9 \times 10^{16}} = 10^{-9}$$

$\therefore$ % change $\approx 10^{-7}$

Correct answer 7

53. The magnetic field existing in a region is given by $\vec{B} = 0.2(1 + 2x)\hat{k}T$. A square loop of edge 50 cm carrying 0.5 A current is placed in x-y plane with its edges parallel to the x-y axes, as shown in figure. The magnitude of the net magnetic force experienced by the loop is______ mN.



Ans. **(50)**

Sol. Force on segment parallel to x-axis will cancel each other. Hence F_{net} will be due to portion parallel to y-axis.

F = 0.5 × 0.5 × 6 × 0.2 – 0.5 × 0.5 × 0.2 × 5

= 0.5 × 0.5 × 0.2

= 0.25 × 0.2

= 50×10^{-3} N

= 50 mN

54. A alternating current at any instant is given by $i=\left[6+\sqrt{56}\sin\left(100\pi t+\frac{\pi}{3}\right)\right]A$. The rms value of the current is____________ A.

Ans. **(8)**

Sol. $I_{rms}=\sqrt{\frac{\int i^2dt}{\int dt}}$

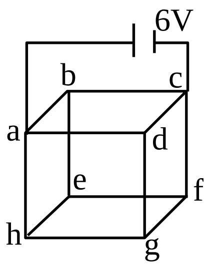
$I_{rms}=\sqrt{(6)^2+\frac{(\sqrt{56})^2}{2}}$

$=\sqrt{36+28}$

$=\sqrt{64}$

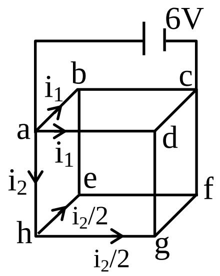
= 8A

55. Twelve wires each having resistance 2Ω are joined to form a cube. A battery of 6 V emf is joined across point a and c. The voltage difference between e and f is_____ V.



Ans. **(1)**

Sol.



From symmetry, current through e-b & g-d = 0

$\therefore R_{eq}=\frac{3}{4}\times R=\frac{3}{2}\Omega$

$\therefore$ Current through battery $=\frac{6\times 2}{3}=4A$

$i_2=\frac{4}{8}\times 2=1A$

$\therefore$ ΔV across e-f = $\frac{i_2}{2}\times R=\frac{1}{2}\times 2=1V$

56. A soap bubble is blown to a diameter of 7 cm. 36960 erg of work is done in blowing it further. If surface tension of soap solution is 40 dyne/cm then the new radius is ______ cm. Take : $\left(\pi=\frac{22}{7}\right)$.

Ans. **(7)**

Sol. $\omega=\Delta U=S\Delta A$

$36960\text{ erg}=\frac{40\text{dyne}}{\text{cm}}8\pi\left[(r)^2-\left(\frac{7}{2}\right)^2\right]\text{cm}^2$

r = 7 cm

57. Two wavelengths λ_1 and λ_2 are used in Young's double slit experiment λ_1 = 450 nm and λ_2 = 650 nm. The minimum order of fringe produced by λ_2 which overlaps with the fringe produced by λ_1 is n. The value of n is ____.

Ans. **(9)**

Sol. $n_2\lambda_2=n_1\lambda_1$

$\frac{n_2}{n_1}=\frac{\lambda_1}{\lambda_2}=\frac{450}{650}=\frac{9}{13}$

$n_2=9$

58. An elastic spring under tension of 3 N has a length a. Its length is b under tension 2 N. For its length (3a – 2b), the value of tension will be_____ N.

Ans. **(5)**

Sol. $3=K(a-\ell)$

$2=K(b-\ell)$

$T=K(3a-2b-\ell)$

$T=K(3(a-\ell)-2(b-\ell)$

$=K\left[3\left(\frac{3}{K}\right)-2\left(\frac{2}{K}\right)\right]$

= 9 – 4

= 5 N

59. Two forces $\vec{F}_1$ and $\vec{F}_2$ are acting on a body. One force has magnitude thrice that of the other force and the resultant of the two forces is equal to the force of larger magnitude. The angle between $\vec{F}_1$ and $\vec{F}_2$ is $\cos^{-1}\left(\frac{1}{n}\right)$. The value of $|n|$ is ____.

Ans. (6)

Sol. $|\vec{F}_1| = F$

$|\vec{F}_R| = |\vec{F}_2| = 3F$

$F_R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$9F^2 = F^2 + 9F^2 + 6F^2\cos\theta$

$\cos\theta = -\frac{1}{6}$

$\theta = \cos^{-1}\left(\frac{1}{-6}\right)$

$n = -6$

$|n| = 6$

60. A solid sphere and a hollow cylinder roll up without slipping on same inclined plane with same initial speed v. The sphere and the cylinder reaches upto maximum heights h_1 and h_2, respectively, above the initial level. The ratio $h_1 : h_2$ is $\frac{n}{10}$. The value of n is_____.

Ans. (7)

Sol Gain in P.E. = Loss in K.E.

$$mgh = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$$

$$h \propto 1+\frac{K^2}{R^2}$$

$$\frac{h_1}{h_2} = \frac{1+\frac{2}{5}}{1+1} = \frac{7}{5\times 2} = \frac{7}{10}$$

$n = 7$

SECTION-A

61. What pressure (bar) of H_2 would be required to make emf of hydrogen electrode zero in pure water at 25ºC ?

(1) 10^{-14} (2) 10^{-7} (3) 1 (4) 0.5

NTA Ans. (3)

Sol. $2e^{-} + 2H^{+}(aq) \rightarrow H_2(g)$

$$E = E^{o} - \frac{0.059}{n}\log\frac{P_{H_2}}{[H^{+}]^{2}}$$

$$0 = 0 - \frac{0.059}{2}\log\frac{P_{H_2}}{(10^{-7})^{2}}$$

$$\log\frac{P_{H_2}}{(10^{-7})^{2}} = 0$$

$$\frac{P_{H_2}}{10^{-14}} = 1$$

$$P_{H_2} = 10^{-14}\,\text{bar}$$

62. The correct sequence of ligands in the order of decreasing field strength is :

(1) $CO > H_2O > F^{-} > S^{2-}$

(2) $^{-}OH > F^{-} > NH_3 > CN^{-}$

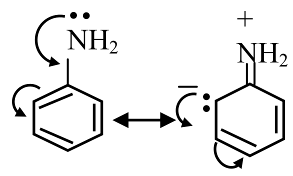
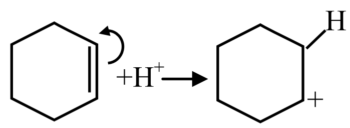
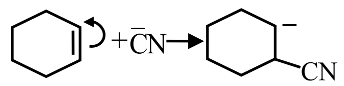
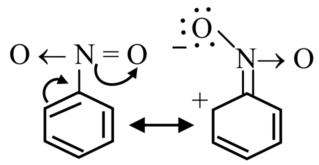
(3) $NCS^{-} > EDTA^{4-} > CN^{-} > CO$

(4) $S^{2-} > ^{-}OH > EDTA^{4-} > CO$

Ans. (1)

Sol. According to spectrochemical series ligand field strength is $CO > H_2O > F^{-} > S^{2-}$

63. Match List -I with List II:

List - I Mechanism steps		List - II Effect	
(A)		(I)	– E effect
(B)		(II)	– R effect
(C)		(III)	+ E effect
(D)		(IV)	+ R effect

Choose the **correct** answer from the options given below :

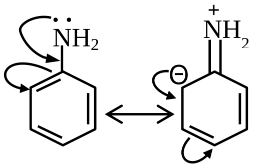
(1) (A) – (IV), (B) – (III), (C) – (I), (D) – (II)

(2) (A) – (III), (B) – (I), (C) – (II), (D) – (IV)

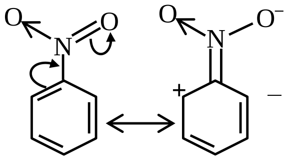
(3) (A) – (II), (B) – (IV), (C) – (III), (D) – (I)

(4) (A) – (I), (B) – (II), (C) – (IV), (D) – (III)

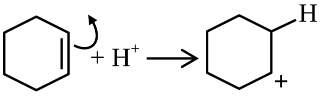
Ans. (1)

Sol. 

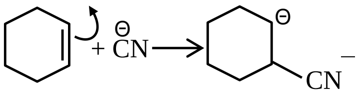
+ R effect ($\ddot{N}H_2$ electron donating)



– R effect (NO_2 electron withdrawing)



+E Effect H^+ electron deficient species)



– E Effect ($\bar{C}N$ electron deficient species)

64. What will be the decreasing order of basic strength of the following conjugate bases ?

$^-OH, R\bar{O}, CH_3CO\bar{O}, C\bar{l}$

(1) $C\bar{l} > OH > R\bar{O} > CH_3CO\bar{O}$
(2) $R\bar{O} > {}^-OH > CH_3CO\bar{O} > C\bar{l}$
(3) $^-OH > R\bar{O} > CH_3CO\bar{O} > C\bar{l}$
(4) $C\bar{l} > R\bar{O} > {}^-OH > CH_3CO\bar{O}$

Ans. (2)

Sol. Strong acid have weak conjugate base
Acidic strength :
H–Cl > $CH_3COOH > H_2O >$ R–OH
Conjugate base strength :
$Cl^- < CH_3COO^- < \bar{O}H < RO^-$

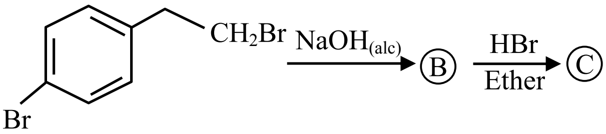
65. In the precipitation of the iron group (III) in qualitative analysis, ammonium chloride is added before adding ammonium hydroxide to :
(1) prevent interference by phosphate ions
(2) decrease concentration of ^-OH ions
(3) increase concentration of Cl^- ions
(4) increase concentration of NH_4^+ ions

Ans. (2)

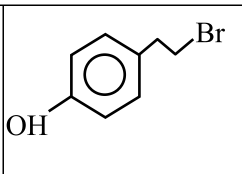
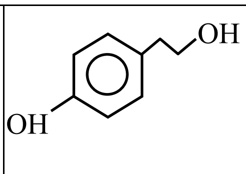
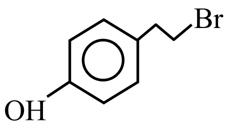
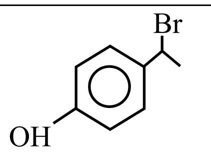
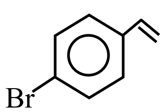
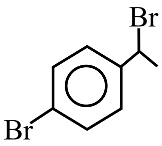
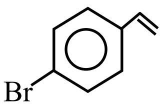
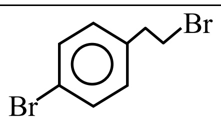
Sol. $NH_4OH \rightleftharpoons NH_4^+ + OH^-$

$NH_4Cl \rightarrow NH_4^+ + Cl$

Due to common ion effect of NH_4^+,
$[OH^-]$ decreases in such extent that only group-III cation can be precipitated , due to their very low K_{sp} in the range of 10^{-38}.

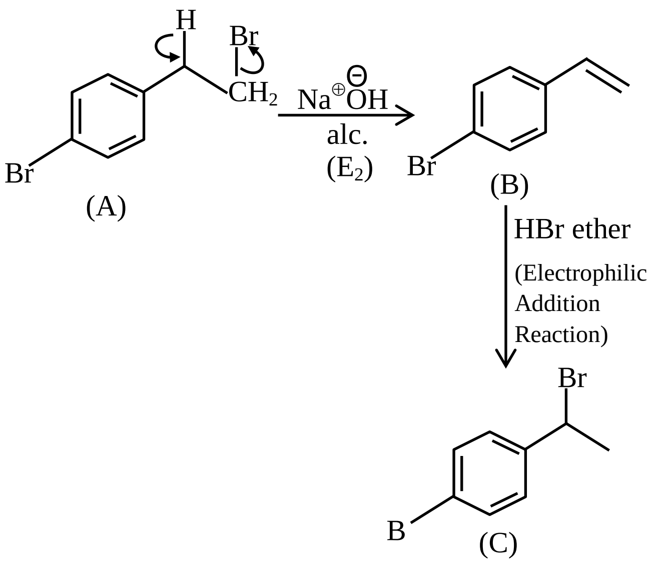
66. 

Identify Ⓑ and Ⓒ and how are Ⓐ and Ⓒ related ?

	(B)	(C)	
(1)			functional group isomers
(2)			Derivative
(3)			position isomers
(4)			chain isomers

Ans. (3)

Sol.



A and C are position isomer.

67. One of the commonly used electrode is calomel electrode. Under which of the following categories calomel electrode comes ?

(1) Metal – Insoluble Salt – Anion electrodes

(2) Oxidation – Reduction electrodes

(3) Gas – Ion electrodes

(4) Metal ion – Metal electrodes

Ans. **(1)**

Sol. Theory based

68. Number of complexes from the following with even number of unpaired "d" electrons is____.

$[V(H_2O)_6]^{3+}$, $[Cr(H_2O)_6]^{2+}$, $[Fe(H_2O)_6]^{3+}$, $[Ni(H_2O)_6]^{3+}$, $[Cu(H_2O)_6]^{2+}$

[Given atomic numbers : V = 23, Cr = 24, Fe = 26, Ni = 28, Cu = 29]

(1) 2 (2) 4

(3) 5 (4) 1

Ans. **(1)**

Sol. $[V(H_2O)_6]^{3+} \rightarrow d^2sp^3$

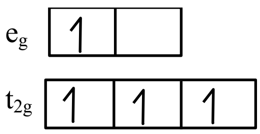
$_{23}V$:- $[Ar]3d^3 4s^2$

V^{+3} :- $[Ar]3d^2$, n = 2 (even number of unpaired e^-)

$[Cr(H_2O)_6]^{2+} \rightarrow sp^3d^2$

$_{24}Cr$:- $[Ar]3d^5 4s^1$

Cr^{+2} :- $[Ar]3d^4$, n = 4 (even number of unpaired e^-)



$[Fe(H_2O)_6]^{3+} \rightarrow sp^3d^2$

Fe^{3+} :– $[Ar]3d^5 4s^0$

n = 5 (odd number of unpaired e^-)

$[Ni(H_2O)_6]^{3+} \rightarrow sp^3d^2$

Ni :- $[Ar]3d^8 4s^2$

Ni^{+3} :- $[Ar]3d^7$, n = 3 (odd number of unpaired e^-)

$[Cu(H_2O)_6]^{2+} \rightarrow sp^3d^2$

Cu :- $[Ar]3d^9 4s^0$

n = 1 (odd number of unpaired e^-)

69. Which one of the following molecules has maximum dipole moment ?

(1) NF_3 (2) CH_4

(3) NH_3 (4) PF_5

Ans. **(3)**

Sol. CH_4 & PF_5 , $\mu_{net} = 0$ (non polar)

μ_{NH_3} > μ_{NF_3}

Vector addition of bond moment & lone pair moment (for NH_3); Vector subtraction of bond moment & lone pair moment (for NF_3)

70. Number of molecules/ions from the following in which the central atom is involved in sp^3 hybridization is ________.

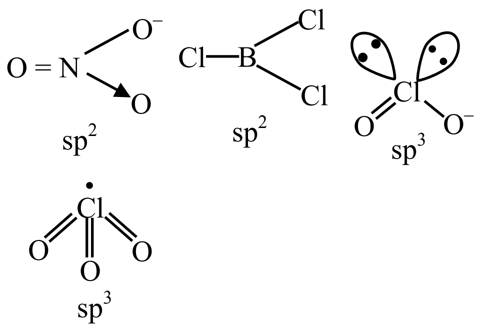
NO_3^-, BCl_3, ClO_2^-, ClO_3

(1) 2 (2) 4

(3) 3 (4) 1

Ans. **(1)**

Sol.



71. Which among the following is **incorrect** statement?

(1) Electromeric effect dominates over inductive effect

(2) The electromeric effect is, temporary effect

(3) The organic compound shows electromeric effect in the presence of the reagent only

(4) Hydrogen ion (H^+) shows negative electromeric effect

Ans. **(4)**

Sol. Hydrogen ion (H^+) shows positive electromeric effect.

72. Given below are two statements :

Statement I : Acidity of α-hydrogens of aldehydes and ketones is responsible for Aldol reaction.

Statement II : Reaction between benzaldehyde and ethanal will NOT give Cross – Aldol product.

In the light of above statements, choose the **most appropriate** answer from the options given below.

(1) Both **Statement I** and **Statement II** are correct.

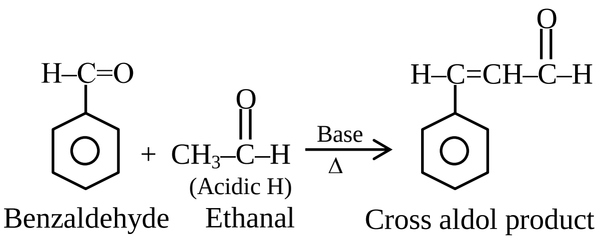
(2) Both **Statement I** and **Statement II** are incorrect.

(3) **Statement I** is incorrect but **Statement II** is correct.

(4) **Statement I** is correct but **Statement II** is incorrect.

Ans. (4)

Sol. Aldehyde and ketones having acidic α-hydrogen show aldol reaction



73. Which of the following nitrogen containing compound does not give Lassaigne's test ?

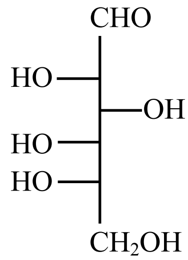
(1) Phenyl hydrazine (2) Glycene

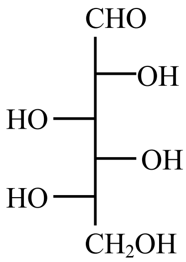
(3) Urea (4) Hydrazine

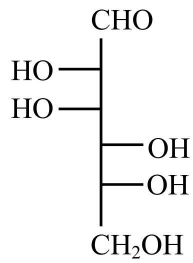
Ans. (4)

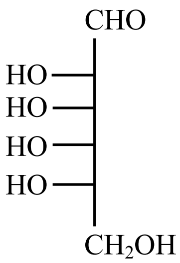
Sol. Hydrazine (NH_2–NH_2) have no carbon so does not show Lassaigne's test.

74. Which of the following is the correct structure of L-Glucose ?

(1) 

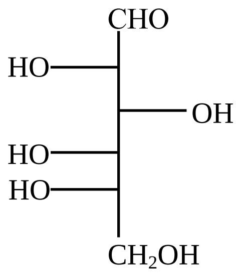
(2) 

(3) 

(4) 

Ans. (1)

Sol. Structure of L-Glucose is



75. The element which shows only one oxidation state other than its elemental form is :

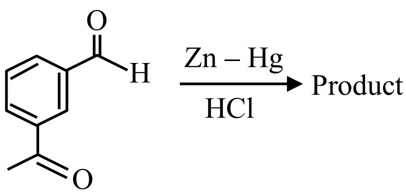
(1) Cobalt (2) Scandium

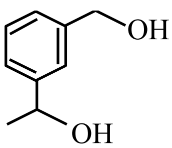
(3) Titanium (4) Nickel

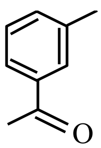
Ans. (2)

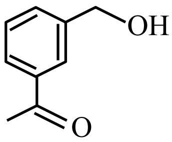
Sol. Co, Ti, Ni can show +2, +3 and +4 oxidation state, But 'Sc' only shows +3 stable oxidation state.

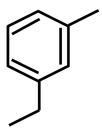
76. Identify the product in the following reaction :



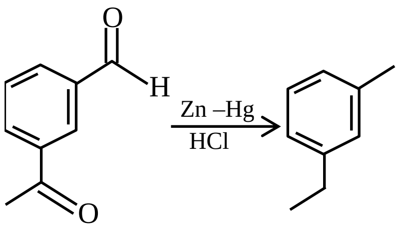
(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol. 

(Clemmensen reduction)

77. Number of elements from the following that CANNOT form compounds with valencies which match with their respective group valencies is ______.

B, C, N, S, O, F, P, Al, Si

(1) 7 (2) 5 (3) 6 (4) 3

Ans. (4)

Sol. N,O, F can't extend their valencies upto their group number due to the non-availability of vacant 2d like orbital.

78. The Molarity (M) of an aqueous solution containing 5.85 g of NaCl in 500 mL water is :

(Given : Molar Mass Na : 23 and Cl : 35.5 $gmol^{-1}$)

(1) 20 (2) 0.2

(3) 2 (4) 4

Ans. (2)

Sol. $M = \dfrac{n_{NaCl}}{V_{sol}\text{ (in L)}}$

$$M = \dfrac{\dfrac{5.85}{58.5}}{0.5} = 0.2\,M$$

79. Identify the correct set of reagents or reaction conditions 'X' and 'Y' in the following set of transformation.

$$CH_3-CH_2-CH_2-Br \xrightarrow{\text{'X'}} \text{Product} \xrightarrow{\text{'Y'}} CH_3-\underset{\substack{|\\Br}}{CH}-CH_3$$

(1) X = conc.alc. NaOH, 80°C, Y = $Br_2/CHCl_3$

(2) X = dil.aq. NaOH, 20°C, Y = HBr/acetic acid

(3) X = conc.alc. NaOH, 80°C, Y = HBr/acetic acid

(4) X = dil.aq. NaOH, 20°C, Y = $Br_2/CHCl_3$

Ans. (3)

Sol. $CH_3–CH_2–CH_2–Br \xrightarrow[80\,^{\circ}C]{X=\text{conc.alc.NaOH}}$

$$CH_3–CH=CH_2 \xrightarrow{Y\equiv HBr/\text{Acetic acid}} CH_3–CHBr – CH_3$$

80. The correct order of first ionization enthalpy values of the following elements is :

(A) O (B) N

(C) Be (D) F

(E) B

Choose the correct answer from the options given below :

(1) B < D < C < E < A (2) E < C < A < B < D

(3) C < E < A < B < D (4) A < B < D < C < E

Ans. (2)

Sol. Correct order of I[st] IE

Li < B < Be < C < O < N < F < Ne

↓ ↓ ↓ ↓ ↓

E < C < A < B < D

SECTION-B

81. The enthalpy of formation of ethane (C_2H_6) from ethylene by addition of hydrogen where the bond-energies of C – H, C – C, H – H are 414 kJ, 347 kJ, 615 kJ and 435 kJ respectively is - __________ kJ.

Ans. (125)

Sol. $C_2H_4(g) + H_2(g) \rightarrow C_2H_6(g)$

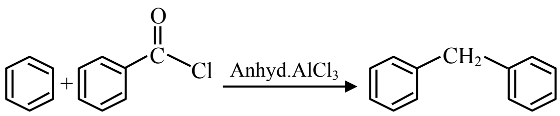
ΔH = BE(C = C) + 4BE (C – H) + BE (H – H) – BE(C – C) – 6BE (C – H)

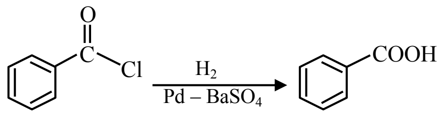
ΔH = BE(C = C) + BE(H – H) – BE(C – C) – 2BE(C – H)

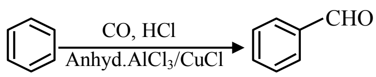
= 615 + 435 – 347 – 2 × 414

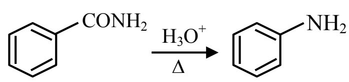
= – 125 kJ

82. The number of correct reaction(s) among the following is ________.

(A) 

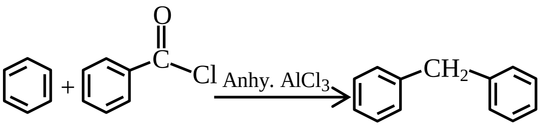
(B) 

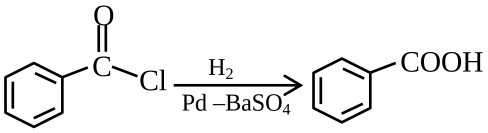
(C) 

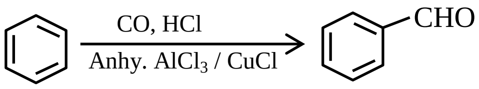
(D) 

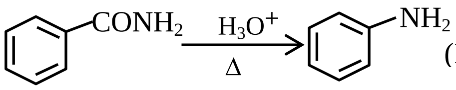
Ans. (1)

Sol.

(A)  (Incorrect)

(B)  (Incorrect)

(C)  (Correct)

(D)  (Incorrect)

83. X g of ethylamine is subjected to reaction with $NaNO_2/HCl$ followed by water; evolved dinitrogen gas which occupied 2.24 L volume at STP.

X is ______ $\times 10^{-1}$ g.

Ans. **(45)**

Sol.

$$\underset{\text{Mol.wt.45g}}{CH_3CH_2NH_2} \xrightarrow{NaNO_2+HCl} \xrightarrow{H_2O} CH_3CH_2\text{–}OH + \underset{14g}{N_2}$$

given : N_2 evolved is 2.24 L i.e. 0.1 mole.

i.e. $CH_3CH_2NH_2$ (ethyl amine) will be 4.5 g

(=0.1 mole)

Hence the answer = 45×10^{-1} g

84. The de-Broglie's wavelength of an electron in the 4^{th} orbit is _______ πa_0. (a_0 = Bohr's radius)

Ans. **(8)**

Sol. $2\pi r_n = n\lambda_d$

$$2\pi a_0 \frac{n^2}{Z} = n\lambda_d$$

$$2\pi a_0 \frac{4^2}{1} = 4\lambda_d$$

$$\lambda_d = 8\pi a_0$$

85. Only 2 mL of $KMnO_4$ solution of unknown molarity is required to reach the end point of a titration of 20 mL of oxalic acid (2 M) in acidic medium. The molarity of $KMnO_4$ solution should be ________ M.

NTA Ans. (50)

Sol. eq.($KMnO_4$) = eq.($H_2C_2O_4$)

$M \times 2 \times 5 = 2 \times 20 \times 2$

$M = 8M$

86. Consider the following reaction

$MnO_2 + KOH + O_2 \rightarrow A + H_2O$.

Product 'A' in neutral or acidic medium disproportionate to give products 'B' and 'C' along with water. The sum of spin-only magnetic moment values of B and C is __________ BM. (nearest integer)

(Given atomic number of Mn is 25)

Ans. **(4)**

Sol. $MnO_2 + KOH + O_2 \rightarrow \underset{(A)}{K_2MnO_4} + H_2O$

$$K_2MnO_4 \xrightarrow{\text{Neutral/acidic solution}} KMnO_4 + MnO_2$$

Mn^{+4} :- $[Ar]3d^3$

$n = 3,\ \mu = \sqrt{3(3+2)} = 3.87$ B.M.

Nearest integer is (4)

87. Consider the following transformation involving first order elementary reaction in each step at constant temperature as shown below.

$$A + B \underset{\text{Step 3}}{\overset{\text{Step 1}}{\rightleftharpoons}} C \xrightarrow{\text{Step 2}} P$$

Some details of the above reaction are listed below.

Step	Rate constant (sec^{-1})	Activation energy ($kJ\ mol^{-1}$)
1	k_1	300
2	k_2	200
3	k_3	Ea_3

If the overall rate constant of the above transformation (k) is given as $k = \frac{k_1 k_2}{k_3}$ and the overall activation energy (E_a) is 400 kJ mol^{-1}, then the value of Ea_3 is ______ kJ mol^{-1} (nearest integer)

Ans. **(100)**

Sol. $K = \frac{K_1 K_2}{K_3}$

$$Ae^{\frac{-E_a}{RT}} = \frac{A_1 e^{\frac{-E_{a_1}}{RT}} A_2 e^{\frac{-E_{a_2}}{RT}}}{A_3 e^{\frac{-E_{a_3}}{RT}}}$$

$$Ae^{\frac{-E_a}{RT}} = \frac{A_1 A_2}{A_3} e^{\frac{-(E_{a_1}+E_{a_2}-E_{a_3})}{RT}}$$

$$E_a = E_{a_1} + E_{a_2} - E_{a_3}$$

$$400 = 300 + 200 - E_{a_3}$$

$$E_{a_3} = 100 \text{ kJ/mole}$$

88. 2.5 g of a non-volatile, non-electrolyte is dissolved in 100 g of water at 25°C. The solution showed a boiling point elevation by 2°C. Assuming the solute concentration in negligible with respect to the solvent concentration, the vapour pressure of the resulting aqueous solution is _______ mm of Hg (nearest integer)

[Given : Molal boiling point elevation constant of water (K_b) = 0.52 K. kg mol^{-1},
1 atm pressure = 760 mm of Hg, molar mass of water = 18 g mol^{-1}]

Ans. **(707)**

Sol. $2 = 0.52 \times m$

$$m = \frac{2}{0.52}$$

According to question, solution is much diluted

so $\frac{\Delta P}{P^o} = \frac{n_{solute}}{n_{solvent}}$

$$\frac{\Delta P}{P^o} = \frac{m}{1000} \times M_{solvent}$$

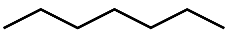
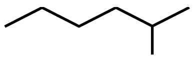
$$\Delta P = P^o \times \frac{m}{1000} \times M_{solvent}$$

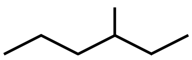
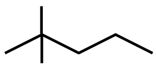
$$= 760 \times \frac{\frac{2}{0.52}}{1000} \times 18 = 52.615$$

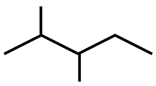
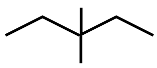
$P_5 = 760 - 52.615 = 707.385$ mm of Hg

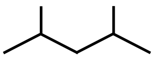
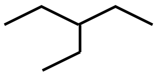
89. The number of different chain isomers for C_7H_{16} is __________.

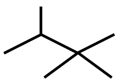
Ans. **(9)**

Sol. (i)  (ii) 

(iii)  (iv) 

(v)  (vi) 

(vii)  (viii) 

(ix) 

90. Number of molecules/species from the following having one unpaired electron is ________.

O_2, O_2^{-1}, NO, CN^{-1}, O_2^{2-}

Ans. **(2)**

Sol. According to M.O.T.

$O_2 \rightarrow$ no. of unpaired electrons = 2

$O_2^- \rightarrow$ no. of unpaired electron = 1

NO $\rightarrow$ no. of unpaired electron = 1

$CN^- \rightarrow$ no. of unpaired electron = 0

$O_2^{2-} \rightarrow$ no. of unpaired electron = 0