QUESTION PAPER CODE 65/1/C

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $(x + 3)2x - (-2)(-3x) = 8$ $\frac{1}{2}$

$x = 2$ $\frac{1}{2}$

2. $\begin{pmatrix} 2 & 5 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}$ $\frac{1}{2} + \frac{1}{2}$

3. No. of possible matrices $= 3^4$ or 81 } 1

4. $\dfrac{2(2\vec{a} + 3\vec{b}) + 1(3\vec{a} - 2\vec{b})}{2+1}$ $\frac{1}{2}$

$= \frac{7}{3}\vec{a} + \frac{4}{3}\vec{b}$ (or enternal division may also be considered) $\frac{1}{2}$

5. 2 1

6. $\frac{x}{3} + \frac{y}{-4} + \frac{z}{2} = 1$ $\frac{1}{2}$

$\Rightarrow \vec{r}\cdot(4\hat{i} - 3\hat{j} + 6\hat{k}) = 12$ or $\vec{r}\cdot\left(\frac{\hat{i}}{3} - \frac{\hat{j}}{4} + \frac{\hat{k}}{2}\right) = 1$ $\frac{1}{2}$

SECTION B

7. Equation of line through A(3, 4, 1) and B(5, 1, 6)

$\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5} = k\text{(say)}$ 1

General point on the line:

$x = 2k + 3,\ y = -3k + 4,\ z = 5k + 1$ $\frac{1}{2}$

line crosses xz plane i.e. $y = 0$ if $-3k + 4 = 0$

$\therefore \quad k = \frac{4}{3}$ 1

Co-ordinate of required point $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$ $\frac{1}{2}$

Angle, which line makes with xz plane:

$\sin\theta = \left|\dfrac{2(0) + (-3)(1) + 5(0)}{\sqrt{4+9+25}\sqrt{1}}\right| = \dfrac{3}{\sqrt{38}} \Rightarrow \theta = \sin^{-1}\left(\dfrac{3}{\sqrt{38}}\right)$ 1

8. let $\vec{d}_1$ & $\vec{d}_2$ be the two diagonal vectors:

$\therefore \quad \vec{d}_1 = 4\hat{i} - 2\hat{j} - 2\hat{k}, \; \vec{d}_2 = -6\hat{j} - 8\hat{k}$ $\frac{1}{2}+\frac{1}{2}$

or $\quad \vec{d}_2 = 6\hat{j} + 8\hat{k}$

Unit vectors parallel to the diagonals are:

$\hat{d}_1 = \frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$ $\frac{1}{2}$

$\hat{d}_2 = -\frac{3}{5}\hat{j} - \frac{4}{5}\hat{k} \quad \left(\text{or } \hat{d}_2 = \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}\right)$ $\frac{1}{2}$

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8 \end{vmatrix} = 4\hat{i} + 32\hat{j} - 24\hat{k}$$ 1

Area of parallelogram $= \frac{1}{2}\left|\vec{d}_1 \times \vec{d}_2\right| = \sqrt{404}$ or $2\sqrt{101}$ sq. units 1

9. let X = Amount he wins then x = ₹ 5, 4, 3, – 3 1

P = Probability of getting a no. >4 = $\frac{1}{3}$, q = $1 - p = \frac{2}{3}$ $\frac{1}{2}$

X:	5	4	3	–3
P(x)	$\frac{1}{3}$	$\frac{2}{3}\cdot\frac{1}{3} = \frac{2}{9}$	$\left(\frac{2}{3}\right)^2\cdot\frac{1}{3} = \frac{4}{27}$	$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

2

Expected amount he wins $= \Sigma\, XP(X) = \frac{5}{3} + \frac{8}{9} + \frac{12}{27} - \frac{24}{27}$

$= ₹\,\frac{19}{9}$ or $₹\,2\frac{1}{9}$ $\frac{1}{2}$

OR

E_1 = Event that all balls are white,
E_2 = Event that 3 balls are white and 1 ball is non white
E_3 = Event that 2 balls are white and 2 balls are non-white
A = Event that 2 balls drawn without replacement are white
1

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ $\frac{1}{2}$

$P(A/E_1) = 1$, $P(A/E_2) = \frac{3}{4}\cdot\frac{2}{3} = \frac{1}{2}$, $P(A/E_3) = \frac{2}{4}\cdot\frac{1}{3} = \frac{1}{6}$ $1\frac{1}{2}$

$$P(E_1/A) = \frac{1.\frac{1}{3}}{1\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{6}} = \frac{3}{5}$$ 1

10. let $y = u + v,\ u = x^{\sin x},\ v = (\sin x)^{\cos x}$

$\log u = \sin x.\log x \Rightarrow \dfrac{du}{dx} = x^{\sin x}\cdot\left\{\cos x.\log x + \dfrac{\sin x}{x}\right\}$ $\quad \frac{1}{2}+1$

$\log v = \cos x.\log(\sin x) \Rightarrow \dfrac{dv}{dx} = (\sin x)^{\cos x}\cdot\{\cos x\cdot\cot x - \sin x\cdot\log(\sin x)\}$ $\quad \frac{1}{2}+1$

$\dfrac{dy}{dx} = \dfrac{du}{dx}+\dfrac{dv}{dx} = x^{\sin x}\cdot\left\{\cos x\cdot\log x + \dfrac{\sin x}{x}\right\} + (\sin x)^{\cos x}\{\cos x\cdot\cot x - \sin x\cdot\log(\sin x)\}$ $\quad \frac{1}{2}+\frac{1}{2}$

OR

$\dfrac{dy}{dx} = \dfrac{-2\sin(\log x)}{x} + \dfrac{3\cos(\log x)}{x}$ $\quad 1$

$\Rightarrow\ x\dfrac{dy}{dx} = -2\sin(\log x) + 3\cos(\log x)$, differentiate w.r.t 'x' $\quad \frac{1}{2}$

$\Rightarrow\ x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = \dfrac{-2\cos(\log x)}{x} - \dfrac{3\sin(\log x)}{x}$ $\quad 2$

$\Rightarrow\ x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = -y \ \Rightarrow\ x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = 0$ $\quad \frac{1}{2}$

11. $\dfrac{dx}{dt} = 2a\cos 2t\,(1+\cos 2t) - 2a\sin 2t\cdot\sin 2t$ $\quad 1\frac{1}{2}$

$\dfrac{dy}{dt} = -2b\sin 2t\,(1-\cos 2t) + 2b\cos 2t\cdot\sin 2t$ $\quad 1$

$\left.\dfrac{dy}{dx}\right]_{t=\frac{\pi}{4}} = \left.\dfrac{2b\cos 2t\cdot\sin 2t - 2b\sin 2t\,(1-\cos 2t)}{2a\cos 2t\,(1+\cos 2t) - 2a\sin 2t\cdot\sin 2t}\right]_{t=\frac{\pi}{4}} = \dfrac{b}{a}$ $\quad \frac{1}{2}+1$

12. $y^2 = ax^3 + b \Rightarrow 2y\dfrac{dy}{dx} = 3ax^2\ \therefore\ \dfrac{dy}{dx} = \dfrac{3a}{2}\dfrac{x^2}{y}$ $\quad 1$

Slope of tangent at (2, 3) $= \left.\dfrac{dy}{dx}\right]_{(2,3)} = \dfrac{3a}{2}\cdot\dfrac{4}{3} = 2a$ $\quad 1$

Comparing with slope of tangent $y = 4x - 5$, we get, $2a = 4\ \therefore\ \boxed{a = 2}$ $\quad 1$

Also (2, 3) lies on the curve $\therefore\ 9 = 8a + b$, put $a = 2$, we get $b = -7$ $\quad 1$

13. Let $x^2 = t\ \therefore\ \dfrac{x^2}{x^4+x^2-2} = \dfrac{x^2}{(x^2-1)(x^2+2)} = \dfrac{t}{(t-1)(t+2)} = \dfrac{A}{t-1} + \dfrac{B}{t+2}$ $\quad 1$

Solving for A and B to get, $A = \dfrac{1}{3},\ B = \dfrac{2}{3}$ $\quad 1$

$\displaystyle\int\frac{x^2}{x^4+x^2-2}dx = \frac{1}{3}\int\frac{1}{x^2-1}dx + \frac{2}{3}\int\frac{1}{x^2+2}dx = \frac{1}{6}\log\left|\frac{x-1}{x+1}\right| + \frac{\sqrt{2}}{3}\tan^{-1}\frac{x}{\sqrt{2}} + C$ $\quad 1 + 1$

14. Let $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$, Also $I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x} dx$ 1

Adding to get, $2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \frac{1}{\cos(x - \pi/4)} dx$ $\frac{1}{2} + 1$

$\Rightarrow 2I = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \sec(x - \pi/4)\, dx = \frac{1}{\sqrt{2}} \log\left|\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right|_0^{\pi/2}$ 1

$\Rightarrow 2I = \frac{1}{\sqrt{2}}\left\{\log\left|\sqrt{2}+1\right| - \log\left|\sqrt{2}-1\right|\right\}$

$\Rightarrow I = \frac{1}{2\sqrt{2}}\left\{\log\left|\sqrt{2}+1\right| - \log\left|\sqrt{2}-1\right|\right.$ or $\frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$ $\frac{1}{2}$

OR

$\int_0^{3/2} |x \cos \pi x|\, dx = \int_0^{1/2} x \cos \pi x\, dx - \int_{1/2}^{3/2} x \cos \pi x\, dx$ $1\frac{1}{2}$

$= \left\{\frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right\}_0^{1/2} - \left\{\frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right\}_{1/2}^{3/2}$ $1\frac{1}{2}$

$= \frac{1}{2\pi} - \frac{1}{\pi^2} - \left(-\frac{3}{2\pi} - \frac{1}{2\pi}\right) = \frac{5}{2\pi} - \frac{1}{\pi^2}$ 1

15. $\int (3x+1)\sqrt{4 - 3x - 2x^2}\, dx = -\frac{3}{4}\int (-4x-3)\sqrt{4-3x-2x^2}\, dx - \frac{5}{4}\int \sqrt{4-3x-2x^2}\, dx$ 1

$= -\frac{1}{2}(4 - 3x - 2x^2)^{3/2} - \frac{5}{4}\sqrt{2}\int \sqrt{\left(\frac{\sqrt{41}}{4}\right)^2 - \left(x + \frac{3}{4}\right)^2}\, dx$ 1 + 1

$= -\frac{1}{2}(4 - 3 - 2x^2)^{3/2} - \frac{5}{4}\sqrt{2}\left\{\frac{4x+3}{8}\sqrt{\frac{41}{16} - \left(x+\frac{3}{4}\right)^2} + \frac{41}{32}\cdot\sin^{-1}\left(\frac{4x+3}{\sqrt{41}}\right)\right\} + C$

$= -\frac{1}{2}(4 - 3x - 2x^2)^{3/2} - \frac{5}{4}\left\{\frac{4x+3}{8}\sqrt{4-3x-2x^2} + \frac{41\sqrt{2}}{32}\cdot\sin^{-1}\left(\frac{4x+3}{\sqrt{41}}\right)\right\} + C$ 1

16. The differential equation can be re-written as:

$\frac{dy}{dx} = \frac{x-y}{x+y}$, put $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2} + \frac{1}{2}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{1-v}{1+v} \Rightarrow \frac{1+v}{1-2v-v^2} dv = \frac{1}{x} dx$ 1

integrating we get

$\Rightarrow \frac{1}{2}\int \frac{2V+2}{V^2+2V-1} dv = -\int \frac{1}{x} dx = \frac{1}{2}\log|V^2 + 2V - 1| = -\log x + \log C$ $1\frac{1}{2}$

$\therefore$ Solution of the differential equation is:

$\frac{1}{2}\log\left|\frac{y^2}{x^2} + \frac{2y}{x} - 1\right| = \log C - \log x$ or, $y^2 + 2xy - x^2 = C^2$ $\frac{1}{2}$

17. Let radius of any of the circle touching co-ordinate axes in the second quadrant be "a" then centre is (–a, a)

$\therefore$ Equation of the family of circles is:

$(x + a)^2 + (y - a)^2 = a^2,\ a \in R$ $\quad 1\frac{1}{2}$

$\Rightarrow \quad x^2 + y^2 + 2ax - 2ay + a^2 = 0$

Differentiate w.r.t. "x", $2x + 2yy' + 2a - 2ay' = 0 \Rightarrow a = \frac{x + yy'}{y' - 1}$ $\quad 1\frac{1}{2}$

$\therefore$ The differential equation is:

$$\left.\begin{array}{l}\left(x + \frac{x + yy'}{y' - 1}\right)^2 + \left(y - \frac{x + yy'}{y' - 1}\right)^2 = \left(\frac{x + yy'}{y' - 1}\right)^2 \\ \Rightarrow \left(\frac{xy' + yy'}{y' - 1}\right)^2 + \left(\frac{x + y}{y' - 1}\right)^2 = \left(\frac{x + yy'}{y' - 1}\right)^2\end{array}\right\} \quad 1$$

18. $\sin^{-1}x + \sin^{-1}(1 - x) = \cos^{-1}x \;\Rightarrow\; \sin^{-1}(1 - x) = \frac{\pi}{2} - 2\sin^{-1}x$ $\quad 1$

$\Rightarrow 1 - x = \sin\left(\frac{\pi}{2} - 2\sin^{-1}x\right) \Rightarrow 1 - x = \cos(2\sin^{-1}x) \Rightarrow 1 - x = 1 - 2\sin^2(\sin^{-1}x)$ $\quad 1$

$\Rightarrow 1 - x = 1 - 2x^2$ $\quad 1$

Solving we get, $x = 0$ or $x = \frac{1}{2}$ $\quad 1$

OR

From the equation: $\cos^{-1}\frac{x}{a} = \alpha - \cos^{-1}\frac{y}{b}$

$\frac{x}{a} = \cos\left(\alpha - \cos^{-1}\frac{y}{b}\right) \;\Rightarrow\; \frac{x}{a} = \cos\alpha \cdot \cos\left(\cos^{-1}\frac{y}{b}\right) + \sin\alpha \cdot \sin\left(\cos^{-1}\frac{y}{b}\right)$ $\quad 1 + 1$

$\Rightarrow \frac{x}{a} = \frac{y \cdot \cos\alpha}{b} + \sin\alpha\sqrt{1 - \frac{y^2}{b^2}} \;\Rightarrow\; \frac{x}{a} - \frac{y}{b}\cos\alpha = \sin\alpha\sqrt{1 - \frac{y^2}{b^2}}$ $\quad 1$

Squaring both sides,

$\Rightarrow \left(\frac{x}{a} - y\frac{\cos\alpha}{b}\right)^2 = \left(\sin\alpha\sqrt{1 - \frac{y^2}{b^2}}\right)^2$ $\quad \frac{1}{2}$

$\Rightarrow \frac{x^2}{a^2} - \frac{2xy}{ab} \cdot \cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha.$ $\quad \frac{1}{2}$

19. let ₹ x be invested in first bond

and ₹ y be invested in second bond

then the system of equations is:

$$\left.\begin{array}{l}\frac{10x}{100} + \frac{12y}{100} = 2800 \\ \frac{12x}{100} + \frac{10y}{100} = 2700\end{array}\right\} \Rightarrow \left.\begin{array}{l}5x + 6y = 140000 \\ 6x + 5y = 135000\end{array}\right\} \quad 1$$

let $A = \begin{bmatrix} 5 & 6 \\ 6 & 5 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 140000 \\ 135000 \end{bmatrix}$

$\therefore \; A \cdot X = B$

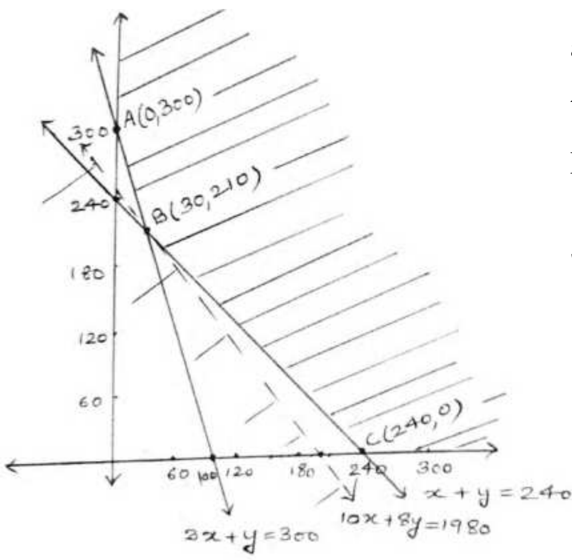
$|A| = -11; \; A^{-1} = \frac{1}{-11}\begin{bmatrix} 5 & -6 \\ -6 & 5 \end{bmatrix}$ 1

$\left.\begin{array}{l} \therefore \text{ Solution is } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} 5 & -6 \\ -6 & 5 \end{bmatrix}\begin{bmatrix} 140000 \\ 135000 \end{bmatrix} = \begin{bmatrix} 10000 \\ 15000 \end{bmatrix} \\ \therefore x = 10000, \; y = 15000, \therefore \text{ Amount invested} = ₹\, 25000 \end{array}\right\}$ $\frac{1}{2}+\frac{1}{2}$

Value: caring elders 1

SECTION C

20.



Let x kg of fertilizer A be used

and y kg of fertilizer B be used

then the linear programming problem is:

Minimise cost: $z = 10x + 8y$ 1

Subject to $\left.\begin{array}{l} \frac{12x}{100} + \frac{4y}{100} \geq 12 \Rightarrow 3x + y \geq 300 \\ \frac{5x}{100} + \frac{5y}{100} \geq 12 \Rightarrow x + y \geq 240 \\ x, y \geq 0 \end{array}\right\}$ 2

Correct Graph $1\frac{1}{2}$

Value of Z at corners of the unbounded region ABC:

Corner	Value of Z
A (0, 300)	₹ 2400
B(30, 210)	₹ 1980 (Minimum)
C(240, 0)	₹ 2400

1

The region of $10x + 8y < 1980$ or $5x + 4y < 990$ has no point in common to the feasible region. Hence, minimum cost = ₹ 1980 at $x = 30$ and $y = 210$ $\frac{1}{2}$

21. Let X = Number of bad oranges out of 4 drawn = 0, 1, 2, 3, 4 1

P = Probability of a bad orange = $\frac{1}{5}, q = 1 - p = \frac{4}{5}$ $\frac{1}{2}$

$\therefore$ Probability distribution is:

X:	0	1	2	3	4
P(X):	${}^4C_0\left(\frac{4}{5}\right)^4 = \frac{256}{625}$	${}^4C_1\,\frac{1}{5}\left(\frac{4}{5}\right)^3$	${}^4C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2$	${}^4C_3\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)$	${}^4C_4\left(\frac{1}{5}\right)^4$
		$= \frac{256}{625}$	$= \frac{96}{625}$	$= \frac{16}{625}$	$= \frac{1}{625}$

$2\frac{1}{2}$

Mean (μ) = $\Sigma X.P(X) = 0\times\frac{256}{625}+1\times\frac{256}{625}+2\times\frac{96}{625}+3\times\frac{16}{625}+4\times\frac{1}{625}=\frac{4}{5}$ 1

Variance (σ^2) $= \Sigma x^2.P(x) - [\Sigma x.P(x)]^2$

$$= 0\times\frac{256}{625}+\frac{1\times256}{625}+\frac{4\times96}{625}+\frac{9\times16}{625}+\frac{16}{625}-\left(\frac{4}{5}\right)^2=\frac{16}{25}$$ 1

22. Line through 'P' and perpendicular to plane is:

$\vec{r}=(2\hat{i}+3\hat{j}+4\hat{k})+\lambda(2\hat{i}+\hat{j}+3\hat{k})$ 1

General point on line is: $\vec{r}=(2+2\lambda)\hat{i}+(3+\lambda)\hat{j}+(4+3\lambda)\hat{k}$

For some $\lambda \in R$, $\vec{r}$ is the foot of perpendicular, say Q, from P to the plane, since it lies on plane

$\therefore$ $[(2+2\lambda)\hat{i}+(3+\lambda)\hat{j}+(4+3\lambda)\hat{k}]\cdot(2\hat{i}+\hat{j}+3\hat{k})-26=0$

$\Rightarrow$ $4+4\lambda+3+\lambda+12+9\lambda-26=0 \Rightarrow \lambda=\frac{1}{2}$ $1\frac{1}{2}$

$\therefore$ Foot of perpendicular is $Q\left(3\hat{i}+\frac{7}{2}\hat{j}+\frac{11}{2}\hat{k}\right)$ 1

let $P'(a\hat{i}+b\hat{j}+c\hat{k})$ be the image of P in the plane then Q is mid point of PP′

$\therefore$ $Q\left(\frac{a+2}{2}\hat{i}+\frac{b+3}{2}\hat{j}+\frac{c+4}{2}\hat{k}\right)=Q\left(3\hat{i}+\frac{7}{2}\hat{j}+\frac{11}{2}\hat{k}\right)$ 1

$\Rightarrow$ $\frac{a+2}{2}=3, \frac{b+3}{2}=\frac{7}{2}, \frac{c+4}{2}=\frac{11}{2} \Rightarrow a=4, b=4, c=7 \quad \therefore P'(4\hat{i}+4\hat{j}+7\hat{k})$ 1

Perpendicular distance of P from plane = $PQ=\sqrt{(2-3)^2+\left(3-\frac{7}{2}\right)^2+\left(4-\frac{11}{2}\right)^2}=\sqrt{\frac{7}{2}}$ $\frac{1}{2}$

23. Commutative: For any elements a, b $\in$ A

a * b = a + b + ab = b + a + ba = b * a. Hence * is commutative $1\frac{1}{2}$

Associative: For any three elements a, b, c, $\in$ A

a * (b * c) = a * (b + c + bc) = a + b + c + bc + ab + ac + abc
(a * b) * c = (a + b + ab) * c = a + b + ab + c + ac + bc + abc $1\frac{1}{2}$

$\therefore$ a * (b * c) = (a * b) * c, Hence * is Associative.

Identity element: let e $\in$ A be the identity element them a * e = e * a = a

$\Rightarrow$ a + e + ae = e + a + ea = a $\Rightarrow$ e (1 + a) = 0, as a $\neq$ – 1

e = 0 is the identity element $1\frac{1}{2}$

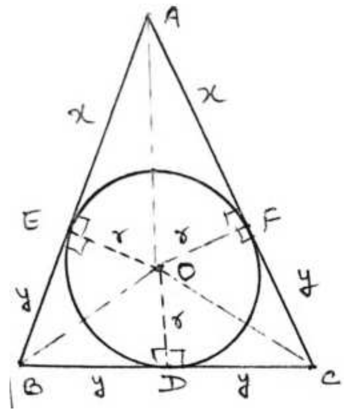
Invertible: let a, b $\in$ A so that 'b' is inverse of a

$\therefore$ a * b = b * a = e

$\Rightarrow$ a + b + ab = b + a + ba = 0

As $a \neq -1$, $b = \frac{-a}{1+a} \in A$. Hence every element of A is invertible $1\frac{1}{2}$

24. Correct Figure 1



Let ΔABC be isosceles with inscribed circle of radius 'r' touching sides AB, AC and BC at E, F and D respectively.

let AE = AF = x, BE = BD = y, CF = CD = y then

area (ΔABC) = ar(ΔAOB) + ar(ΔAOC) + ar(ΔBOC)

$$\Rightarrow \frac{1}{2}.2y\left(r+\sqrt{r^2+x^2}\right) = \frac{1}{2}\{2yr+2(x+y)r\} \Rightarrow x = \frac{2r^2y}{y^2-r^2}$$ 1

Then,

$$P(\text{Perimeter of } \Delta ABC) = 2x + 4y = \frac{4r^2y}{y^2-r^2}+4y$$ 1

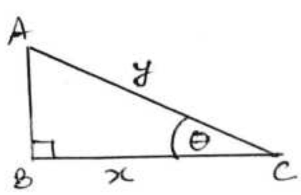
$$\frac{dP}{dy} = \frac{-4r^2(r^2+y^2)}{(y^2-r^2)^2}+4 \text{ and } \frac{dP}{dy}=0 \Rightarrow y=\sqrt{3}r$$ $1+\frac{1}{2}$

$$\left.\frac{d^2P}{dy^2}\right]_{y=\sqrt{3}r} = \frac{4r^2y(2y^2+6r^2)}{(y^2-r^2)^3} = \frac{6\sqrt{3}}{r} > 0$$ $\frac{1}{2}$

$\therefore$ Perimeter is least iff $y = \sqrt{3}r$ and least perimeter is

$$P = 4y+\frac{4r^2y}{y^2-r^2} = 4\sqrt{3}r+\frac{4r^2\sqrt{3}r}{2r^2} = 6\sqrt{3}r$$ 1

OR



let ABC be the right triangle with $\angle B = 90°$

$\angle ACB = \theta$, AC = y, BC = x, x + y = k (constant)

$$A \text{ (Area of triange)} = \frac{1}{2}.BC.AB = \frac{1}{2}.x\sqrt{y^2-x^2}$$ $1\frac{1}{2}$

$$\text{let } z = A^2 = \frac{1}{4}x^2(y^2-x^2) = \frac{1}{4}x^2\{(k-x)^2-x^2\} = \frac{1}{4}(x^2k^2-2kx^3)$$ 1

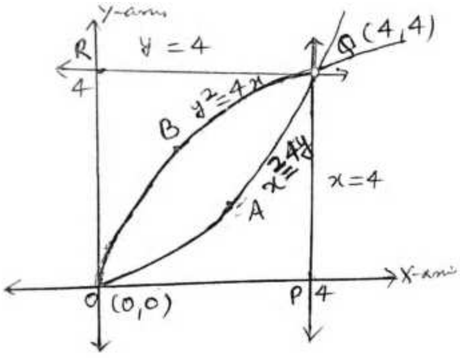
$$\frac{dz}{dx} = \frac{1}{4}(2xk^2-6kx^2) \text{ and } \frac{dz}{dx}=0 \Rightarrow x=\frac{k}{3},\ y=k-x=\frac{2k}{3}$$ 1+1

$$\left.\frac{d^2z}{dx^2}\right]_{x=\frac{k}{3}} = \left.\frac{1}{4}(2k^2-12kx)\right]_{x=\frac{k}{3}} = -\frac{k^2}{2} < 0$$ $\frac{1}{2}$

$\therefore$ z and area of ΔABC is max at $x = \frac{k}{3}$

$$\text{and, } \cos\theta = \frac{x}{y} = \frac{k}{3}.\frac{3}{2k} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$ 1

25. Point of intersection of $y^2 = 4x$ and $x^2 = 4y$ are (0, 0) and (4, 4);



Correct Graph $1\frac{1}{2}$

are (OAQBO) $= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx$ 1

$= \left\{\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right\}\Bigg]_0^4$

$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$ $\frac{1}{2}$

area (OPQAO) $= \int_0^4 \frac{x^2}{4} dx = \frac{1}{12}x^3\Bigg]_0^4 = \frac{16}{3}$ $1\frac{1}{2}$

area (OBQRO) $= \int_0^4 \frac{y^2}{4} dy = \frac{1}{12}y^3\Bigg]_0^4 = \frac{16}{3}$ $1\frac{1}{2}$

Hence the areas of the three regions are equal.

26. $\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$

Apply $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$\Leftrightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & \cos B - \cos A & \cos C - \cos A \\ \cos^2 A + \cos A & (\cos B - \cos A)(\cos B + \cos A + 1) & (\cos C - \cos A)(\cos C + \cos A + 1) \end{vmatrix} = 0$ 3

Taking (cos B – cos A), (cos C – cos A) common from C_2 & C_3

$\Leftrightarrow (\cos B - \cos A)(\cos C - \cos A)\begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & 1 & 1 \\ \cos^2 A + \cos A & \cos B + \cos A + 1 & \cos C + \cos A + 1 \end{vmatrix} = 0$ 1

Expand along R_1

$\Leftrightarrow (\cos B - \cos A)(\cos C - \cos A)(\cos C - \cos B) = 0$ 1

$\Leftrightarrow \cos A = \cos B$ $\Leftrightarrow A = B$ $\Leftrightarrow \Delta ABC$ is an isosceles triangle 1

or or

$\cos B = \cos C$ $B = C$

or or

$\cos C = \cos A$ $C = A$

OR

let the cost of one pen of veriety 'A', 'B' and 'C' be ₹ x. ₹ y and ₹ z respectively then the system of equations is:

$$\left.\begin{aligned} x + y + z &= 21 \\ 4x + 3y + 2z &= 60 \\ 6x + 2y + 3z &= 70 \end{aligned}\right\}$$ $1\frac{1}{2}$

Matrix form of the system is:

$$A{\cdot}X = B, \text{ where } A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$ $\frac{1}{2}$

$|A| = (5) - 1\ (0) + 1\ (-10) = -5$ 1

co-factors of the matrix A are:

$$\left.\begin{aligned} C_{11} &= 5; \quad C_{21} = -1; \quad C_{31} = -1 \\ C_{12} &= 0; \quad C_{22} = -3 \quad C_{32} = 2 \\ C_{13} &= -10; \quad C_{23} = 4; \quad C_{33} = -1 \end{aligned}\right\}$$ 2

$$\therefore\ A^{-1} = \frac{1}{|A|}\text{Adj } A = \frac{1}{-5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$ $\frac{1}{2}$

Solution of the matrix equation is $X = A^{-1} B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \therefore x = 5, y = 8, z = 8$$ $\frac{1}{2}$