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Secondary School Certificate Examination

September 2021

Marking Scheme — Mathematics (Standard)/30/3/1

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2. "Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/ Website etc may invite action under IPC."
3. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-X, while evaluating two competency based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.
4. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
5. Evaluators will mark (✓) wherever answer is correct. For wrong answer 'X" be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.
6. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
7. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
8. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.
9. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
10. A full scale of marks __________(example 0-100 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.

11. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).
12. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrongquestion wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
13. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
14. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
15. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.
16. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
17. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 30/3/1
EXPECTED ANSWER/VALUE POINTS
SECTION I

1. Write the quadratic equation in x whose roots are 2 and –5.

Ans. $(x-2)(x+5) = x^2 + 3x - 10 = 0$ $\frac{1}{2}+\frac{1}{2}$

2. Find the exponent of 2 in the prime factorisation of 288.

Ans. $288 = 2^5 \times 3^2$ $\therefore$ exponent of 2 = 5 $\frac{1}{2}+\frac{1}{2}$

3. (a) If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta$.

Ans. $\alpha + \beta = 1, \quad \alpha\beta = -4$ $\frac{1}{2}$

$$\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta = \frac{\alpha+\beta}{\alpha\beta}-\alpha\beta = \frac{1}{-4}-(-4) = \frac{15}{4}$$ $\frac{1}{2}$

OR

(b) If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then find the value of k.

Ans. $2^2 + 3 \times 2 + k = 0 \Rightarrow k = -10$ $\frac{1}{2}+\frac{1}{2}$

4. (a) If $\frac{3}{5}$, a, 4 are three consecutive terms of an A.P., then find the value of a.

Ans. $2a = 4 + \frac{3}{5} \quad \therefore a = \frac{23}{10}$ $\frac{1}{2}+\frac{1}{2}$

OR

(b) In an A.P., if the common difference d = –3 and the eleventh term $a_{11} = 15$, then find the first term.

1

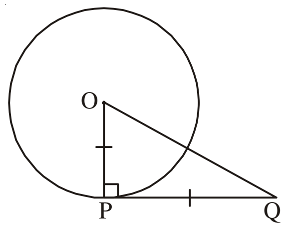
Ans. $a + 10 \times (-3) = 15 \quad \Rightarrow a = 45$ $\frac{1}{2}+\frac{1}{2}$

5. A man goes 5 metres due West and then 12 metres due North. How far is he from the starting point?

Ans. Required distance = $\sqrt{5^2+12^2} = 13$ m 1

6. PQ is a tangent to a circle with centre O at the point P on the circle. If ΔOPQ is an isosceles triangle, then find ∠OQP.

Ans.

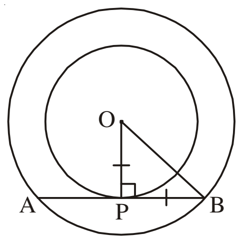


∠OPQ = 90° and OP = OQ $\frac{1}{2}$

∴ ∠OQP = 45° $\frac{1}{2}$

7. Two concentric circles have radii 10 cm and 6 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Ans.



PB = $\sqrt{10^2-6^2} = 8$ cm $\frac{1}{2}$

∴ AB = 2 × 8 cm = 16 cm $\frac{1}{2}$

8. (a) If 3 sin A = 1, then find the value of sec A.

Ans. $\sin A = \frac{1}{3} \Rightarrow \cos A = \frac{2\sqrt{2}}{3}$ $\frac{1}{2}$

$\therefore \sec A = \frac{3}{2\sqrt{2}}$ or $\frac{3\sqrt{2}}{4}$ $\frac{1}{2}$

OR

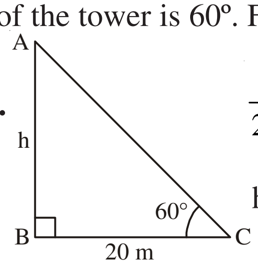
(b) Show that :

$$\frac{1+\cot^2\theta}{1+\tan^2\theta} - \cot^2\theta$$

Ans. L.H.S = $\frac{1+\cot^2\theta}{1+\tan^2\theta} = \frac{\operatorname{cosec}^2\theta}{\sec^2\theta} = \cot^2\theta$ = R.H.S. $\frac{1}{2}+\frac{1}{2}$

9. From a point on the ground, 20 m away from the foot of a vertical tower, the angle of elevation of the top of the tower is 60°. Find the height of the tower.

Ans.



$\frac{h}{20} = \tan 60° \quad \therefore h = 20\sqrt{3}$ $\frac{1}{2}+\frac{1}{2}$

height of tower = $20\sqrt{3}$ m

10. (a) Find the area of a circle whose circumference is 66 cm.

Ans. $2\pi r = 66 \Rightarrow r = \frac{66 \times 7}{2 \times 22} = \frac{21}{2}$ cm $\frac{1}{2}$

Area of circle $= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$ cm^2 $= \frac{693}{2}$ cm^2 $\frac{1}{2}$

OR

(b) The perimeter of a semi-circular protractor is 108 cm. Find its diameter.

Ans. $\pi r + 2r = 108 \Rightarrow r = \frac{108}{\pi + 2} = \frac{108 \times 7}{36} = 21$ cm $\frac{1}{2}$

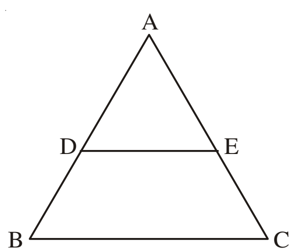
$\therefore$ diameter = 42 cm. $\frac{1}{2}$

11. Write the relationship between three measures of central tendency – Mean, Median and Mode.

Ans. 3 Median = mode + 2 Mean 1

12. In a Δ ABC, if DE is parallel to BC, $\frac{AD}{DB} = \frac{4}{5}$ and AC = 15 cm, then find the length of AE.

Ans.



Let AE = x cm $\Rightarrow$ EC = (15 – x) cm

$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{4}{5} = \frac{x}{15 - x}$ $\frac{1}{2}$

$\Rightarrow x = \frac{20}{3}$ cm $\frac{1}{2}$

$\therefore$ AE $= \frac{20}{3}$ cm

13. Simplify :

cosec^2 60º sin^2 30º – sec^2 60º

Ans. cosec^2 60º sin^2 30º – sec^2 60º

$\frac{\not{4}}{3} \times \frac{1}{\not{4}} - 4 = -\frac{11}{3}$ $\frac{1}{2} + \frac{1}{2}$

14. If tan θ + cot θ $= \frac{4\sqrt{3}}{3}$, then find the value of tan^2 θ + cot^2 θ.

Ans. $\tan\theta + \cot\theta = \dfrac{4\sqrt{3}}{3}$

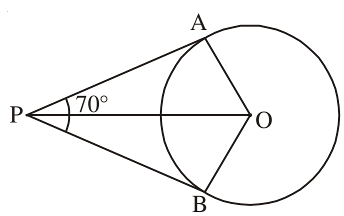
$\Rightarrow (\tan\theta + \cot\theta)^2 = \dfrac{16}{3}$ $\frac{1}{2}$

$\Rightarrow \tan^2\theta + \cot^2\theta + 2 = \dfrac{16}{3}$

$\therefore \tan^2\theta + \cot^2\theta = \dfrac{16}{3} - 2 = \dfrac{10}{3}$ $\frac{1}{2}$

15. If tangents PA and PB from an external point P to a circle with centre O are inclined to each other at an angle of 70º, then find ∠POA.

Ans.



$\angle AOB = 180^\circ - 70^\circ = 110^\circ$ $\frac{1}{2}$

$\angle POA = \dfrac{110^\circ}{2} = 55^\circ$ $\frac{1}{2}$

16. (a) How many outcomes are possible when three dice are thrown together?

Ans. Total number of outcomes = 6^3 $\frac{1}{2}$

= 216 $\frac{1}{2}$

OR

(b) If P(E) = 0.015, then find P(not E).

Ans. P(not E) = 1 – 0.015 = 0.985 1

SECTION II

Case study based questions (Q. No. 17 – 20) are compulsory. Attempt any 4 sub-parts from each question. Each sub-part carries 1 mark.

17. The residents of a housing society, on the occasion of environment day, decided to build two straight paths in the central park of the society and also plant trees along the boundary lines of each path.

Taking one corner of the park as origin and the two mutually perpendicular lines as the x-axis and y-axis, the paths were represented by the two linear equations $2x - 3y = 5$ and $6x + 9y = 7$.

Based on the above, answer the following questions:

(i) Two paths represented by the two equations here are

(A) intersecting

(B) overlapping

(C) parallel

(D) mutually perpendicular

Ans. (C) parallel 1

(ii) Which one of the following points lie on the line $2x - 3y = 5$?

(A) $(-4, 1)$

(B) $(4, -1)$

(C) $(4, 1)$

(D) $(-4, -1)$

Ans. (C) $(4, 1)$ 1

(iii) If the line $6x + 9y = 7$ intersects the y-axis at a point, then its coordinates are :

(A) $\left(0, \frac{7}{9}\right)$

(B) $\left(\frac{7}{9}, 0\right)$

(C) $\left(-\frac{7}{6}, 0\right)$

(D) $\left(0, -\frac{7}{6}\right)$

Ans. (A) $\left(0, \frac{7}{9}\right)$ 1

(iv) If a pair of equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has a unique solution, then

(A) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(B) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(C) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(D) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Ans. (B) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ 1

(v) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

(A) parallel

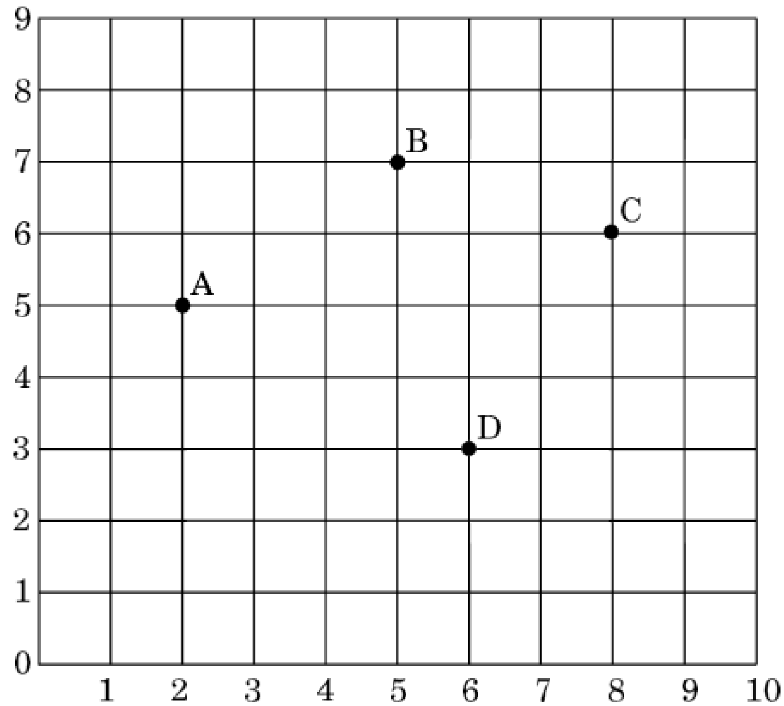
(B) coincident

(C) intersecting

(D) perpendicular to each other

Ans. (B) coincident 1

18. Students of a school are standing in rows and columns in their school playground to celebrate their annual sports day. A, B, C and D are the positions of four students as shown in the figure.



Based on the above, answer the following questions :

(i) The figure formed by the four points A, B, C and D is a

(A) square

(B) parallelogram

(C) rhombus

(D) quadrilateral

Ans. (D) quadrilateral 1

(ii) If the sports teacher is sitting at the origin, then which of the four students is closest to him?

(A) A

(B) B

(C) C

(D) D

Ans. (A) A 1

(iii) The distance between A and C is

(A) $\sqrt{37}$ units

(B) $\sqrt{35}$ units

(C) 6 units

(D) 5 units

Ans. (A) $\sqrt{37}$ units 1

(iv) The coordinates of the mid-point of line segment AC are

(A) $\left(\frac{5}{2}, 11\right)$

(B) $\left(\frac{5}{2}, \frac{11}{2}\right)$

(C) $\left(5, \frac{11}{2}\right)$

(D) (5, 11)

Ans. (C) $\left(5, \frac{11}{2}\right)$ 1

(v) If a point P divides the line segment AD in the ratio 1 : 2, then coordinates of P are

(A) $\left(\frac{8}{3}, \frac{8}{3}\right)$

(B) $\left(\frac{10}{3}, \frac{13}{3}\right)$

(C) $\left(\frac{13}{3}, \frac{10}{3}\right)$

(D) $\left(\frac{16}{3}, \frac{11}{3}\right)$

Ans. (B) $\left(\frac{10}{3}, \frac{13}{3}\right)$ 1

19. During the annual sports meet in a school, all the athletes were very enthusiastic. They all wanted to be the winner so that their house could stand first. The instructor noted down the time taken by a group of students to complete a certain race. The data recorded is given below :

Time (in sec.) :	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
Number of students :	1	4	3	7	5

Based on the above, answer the following questions :

(i) What is the class mark of the modal class?

(A) 60

(B) 70

(C) 80

(D) 140

Ans. (B) 70 1

(ii) The mode of the given data is

(A) 70.33

(B) 71.33

(C) 72.33

(D) 73.33

Ans. (D) 73.33 1

(iii) The median class of the given data is

(A) 20 – 40

(B) 40 – 60

(C) 80 – 100

(D) 60 – 80

Ans. (D) 60 – 80 1

(iv) The sum of the lower limits of median class and modal class is

(A) 80

(B) 140

(C) 120

(D) 100

Ans. (C) 120 1

(v) The median time (in seconds) of the given data is

(A) 65.7

(B) 85.7

(C) 45.7

(D) 25.7

Ans. (A) 65.7 1

20. During summer break, Harish wanted to play with his friends but it was too hot outside, so he decided to play some indoor game with his friends. He collects 20 identical cards and writes the numbers 1 to 20 on them (one number on one card). He puts them in a box. He and his friends make a bet for the chances of drawing various cards out of the box. Each was given a chance to tell the probability of picking one card out of the box.

Based on the above, answer the following questions :

(i) The probability that the number on the card drawn is an odd prime number, is

(A) $\frac{3}{5}$

(B) $\frac{2}{5}$

(C) $\frac{9}{20}$

(D) $\frac{7}{20}$

Ans. (D) $\frac{7}{20}$ 1

(ii) The probability that the number on the card drawn is a composite number is

(A) $\frac{11}{20}$

(B) $\frac{3}{5}$

(C) $\frac{4}{5}$

(D) $\frac{1}{2}$

Ans. (A) $\frac{11}{20}$ 1

(iii) The probability that the number on the card drawn is a multiple of 3, 6 and 9 is

(A) $\frac{1}{20}$

(B) $\frac{1}{10}$

(C) $\frac{3}{20}$

(D) 0

Ans. (A) $\frac{1}{20}$ 1

(iv) The probability that the number on the card drawn is a multiple of 3 and 7 is

(A) $\frac{3}{10}$

(B) $\frac{1}{10}$

(C) 0

(D) $\frac{2}{5}$

Ans. (C) 0 1

(v) If all cards having odd numbers written on them are removed from the box and then one card is drawn from the remaining cards, the probability of getting a card having a prime number is

(A) $\frac{1}{20}$

(B) $\frac{1}{10}$

(C) 0

(D) $\frac{1}{5}$

Ans. (B) $\frac{1}{10}$ 1

PART B

SECTION III

All questions are compulsory. In case of internal choices, attempt any one.

21. (a) Check whether the points P(5, – 2), Q(6, 4) and R(7, –2) are the vertices of an isosceles triangle PQR.

Ans. $PQ = \sqrt{37}$ units, $QR = \sqrt{37}$ units, PR = 2 units $1\frac{1}{2}$

$PQ = QR \neq PR$ $\frac{1}{2}$

$\therefore$ P, Q, R are the vertices of an isosceles triangle

OR

(b) Find the ratio in which P(4, 5) divides the join of A(2, 3) and B(7, 8).

Ans. Let P divides AB in the ratio k : 1

Coordinates of P are $\left(\frac{7k+2}{k+1}, \frac{8k+3}{k+1}\right) = (4,5)$ 1

$\Rightarrow \frac{7k+2}{k+1} = 4, \frac{8k+3}{k+1} = 5$

$\therefore$ k = $\frac{2}{3}$ 1

So, P divides AB in the ratio 2 : 3.

22. (a) The sum of the numerator and the denominator of a fraction is 18. If the denominator is increased by 2, the fraction reduces to $\frac{1}{3}$. Find the fraction.

Ans. Let the numberator be x and denominator be y

x + y = 18 ...(1) $\frac{1}{2}$

and $\frac{x}{y+2} = \frac{1}{3} \Rightarrow 3x - y = 2$ ...(2) $\frac{1}{2}$

Solving equation (1) and (2), we get

x = 5 $\frac{1}{2}$

y = 13 $\frac{1}{2}$

$\therefore$ Fraction = $\frac{5}{13}$

OR

(b) Find the value of k for which the system of equations x + 2y = 5 and 3x + ky + 15 = 0 has no solution.

Ans. For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ $\frac{1}{2}$

$\Rightarrow \frac{1}{3} = \frac{2}{k} \neq \frac{-5}{15} \Rightarrow k = 6$ $1 + \frac{1}{2}$

23. Explain why 2 × 3 × 5 + 5 and 5 × 7 × 11 + 7 × 5 are composite numbers.

Ans. 2 × 3 × 5 + 5

= 5 × (2 × 3 + 1) = 5 × 7

which is the product of two factors other than 1. 1

∴ Given number is composite

5 × 7 × 11 + 7 × 5 = 5 × 7 × (11 + 1)

= 5 × 7 × 12

which is the product of two number other than 1. 1

∴ Given number is composite.

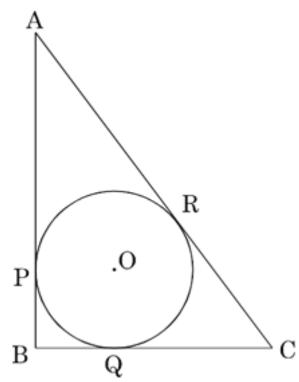
24. Find the mean of first 10 composite numbers.

Ans. First 10 composite numbers are

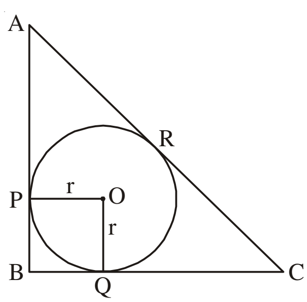
4, 6, 8, 9, 10, 12, 14, 15, 16, 18 1

Mean = $\frac{112}{10} = 11.2$ 1

25. ABC is right triangle, right-angled at B, with BC = 6 cm and AB = 8 cm. A circle with centre O and radius r cm has been inscribed in ABC as shown in the figure. Find the value of r.



Ans.



∠PBQ = ∠OQB = ∠OPQ = 90º

⇒ OPBQ is a square as OP = OQ

⇒ PB = BQ = r

Now AP = AR = 8 – r

and CQ = CR = 6 – r 1

$AC = \sqrt{AB^2 + BC^2} = \sqrt{8^2 + 6^2} = 10 \text{ cm}$

$AC = AR + CR \Rightarrow 8 - r + 6 - r = 10$

$\therefore\ r = 2 \text{ cm}$ 1

26. Draw a circle of radius 5 cm. From a point 8 cm away from its centre, construct a pair of tangents to the circle.

Ans. Correct construction of tangents 2

SECTION IV

27. Divide the polynomial $f(x) = 5x^3 + 10x^2 - 30x - 15$ by the polynomial $g(x) = x^2 + 1 + x$ and hence, find the quotient and the remainder.

Ans.

$$
\begin{array}{r|l}
x^2 + x + 1 & 5x^3 + 10x^2 - 30x - 15 \quad (5x + 5 \\
 & \underline{\underset{-}{5x^3} \underset{-}{+} 5x^2 \underset{-}{+} 5x} \\
 & 5x^2 - 35x - 15 \\
 & \underline{\underset{-}{5x^2} \underset{-}{+} 5x \underset{-}{+} 5} \\
 & -40x - 20
\end{array}
$$

2

Quotient = $5x + 5$, Remainder = $-40x - 20$ $\frac{1}{2} + \frac{1}{2}$

28. Prove that $3 + \sqrt{2}$ is an irrational number, given that $\sqrt{2}$ is an irrational number.

Ans. Let us assume, to the contrary that $3\sqrt{2}$ is a rational number

$\therefore\ 3 + \sqrt{2} = \frac{p}{q}$, where p and q are integers and $q \neq 1$ 1

$\Rightarrow\ \sqrt{2} = \frac{p - 3q}{q}$ 1

Since, 3, p and q are integers

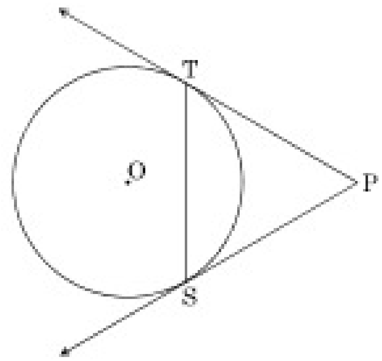
$\Rightarrow\ \frac{p - 3q}{q}$ is rational

$\Rightarrow\ \sqrt{2}$ is also rational which is a contradiction.

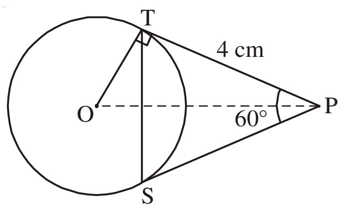
So, our assumption that $3 + \sqrt{2}$ is rational is wrong.

Hence, $3 + \sqrt{2}$ is an irrational number. 1

29. In the given figure, PT and PS are tangents to a circle with centre O, from a point P, such that PT = 4 cm and $\angle TPS = 60^\circ$. Find the length of the chord TS. Also, find the radius of the circle.



Ans.



In ΔTPS, $\angle P = 60^\circ$

$\angle PTS = \angle PST$ $\quad (\because TP = TS)$

$\therefore$ ΔPTS is an equilateral triangle.

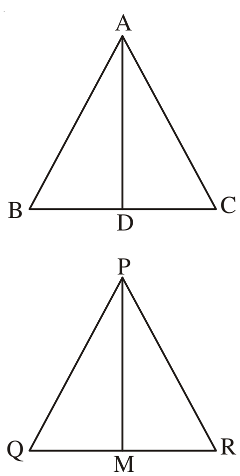
$\Rightarrow$ TS = 4 cm. $\quad 1\frac{1}{2}$

In ΔOTP, $\frac{OT}{PT} = \tan 30^\circ$

$$\Rightarrow \ OT = \frac{4}{\sqrt{3}} \text{cm} = \frac{4\sqrt{3}}{3} \text{cm}$$ $\quad 1\frac{1}{2}$

30. The areas of two similar triangles are 121 cm^2 and 64 cm^2 respectively. If one median of the first triangle is 12.1 cm long, then find the length of the corresponding median of the other triangle.

Ans.



$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AD}{PM}\right)^2$$

$$\Rightarrow \frac{121}{64} = \left(\frac{12.1}{PM}\right)^2$$ $\quad 1$

$$\Rightarrow \frac{12.1}{PM} = \frac{11}{8}$$ $\quad 1$

Solving, we get PM = 8.8 cm. $\quad 1$

31. (a) Prove :

$$\frac{1}{(\cot A)(\sec A) - \cot A} - \operatorname{cosec} A = \operatorname{cosec} A - \frac{1}{(\cot A)(\sec A) + \cot A}$$

Ans. L.H.S. $= \dfrac{1}{\cot A \sec A - \cot A} - \operatorname{cosec} A$

$= \dfrac{1}{\dfrac{\cos A}{\sin A} \times \dfrac{1}{\cos A} - \dfrac{\cos A}{\sin A}} - \dfrac{1}{\sin A}$

$= \dfrac{\sin A}{1 - \cos A} - \dfrac{1}{\sin A}$ $1\frac{1}{2}$

$= \dfrac{\sin^2 A - (1 - \cos A)}{(1 - \cos A)(\sin A)}$

$= \dfrac{(1 - \cos^2 A) - (1 - \cos A)}{(1 - \cos A) \times \sin A}$

$= \dfrac{(1 - \cos A) \cos A}{(1 - \cos A) \sin A} = \cot A$

R.H.S $= \operatorname{cosec} A - \dfrac{1}{\cot A \sec A + \cot A}$

$= \dfrac{1}{\sin A} - \dfrac{1}{\dfrac{\cos A}{\sin A} \times \dfrac{1}{\cos A} + \dfrac{\cos A}{\sin A}}$

$= \dfrac{1}{\sin A} - \dfrac{\sin A}{1 + \cos A}$ $1\frac{1}{2}$

$= \dfrac{(1 + \cos A) - \sin^2 A}{\sin A (1 + \cos A)}$

$= \dfrac{(1 + \cos A) \times \cos A}{\sin A (1 + \cos A)} = \cot A$

$\therefore$ L.H.S = R.H.S

OR

(b) Prove :

$\sin^6 A + 3 \sin^2 A \cos^2 A = 1 - \cos^6 A$

Ans. L.H.S. $= \sin^6 A + 3 \sin^2 A \cos^2 A$

$= (1 - \cos^2 A)^3 + 3\sin^2 A \cos^2 A$ 1

$= 1 - \cos^6 A - 3 \cos^2 A (1 - \cos^2 A) + 3\sin^2 A \cos^2 A$ 1

$= 1 - \cos^6 A - 3 \cos^2 A \sin^2 A + 3\sin^2 A \cos^2 A$

$= 1 - \cos^6 A$ 1

$=$ R.H.S.

32. (a) One root of the quadratic equation $2x^2 - 8x - k = 0$ is $\frac{5}{2}$. Find the value of k. Also, find the other root.

Ans. $\frac{5}{2}$ is a root of $2x^2 - 8x - k = 0$

$\Rightarrow 2 \times \frac{25}{4} - 8 \times \frac{5}{2} - k = 0$

$\Rightarrow k = -\frac{15}{2}$ $1\frac{1}{2}$

$2x^2 - 8k + \frac{15}{2} = 0 \Rightarrow 4x^2 - 16x + 15 = 0$

$\Rightarrow (2x - 5)(2x - 3) = 0$ $1\frac{1}{2}$

$\therefore$ other root $= \frac{3}{2}$

OR

(b) Using quadratic formula, solve the following equation for x :

$abx^2 + (b^2 - ac)x - bc = 0$

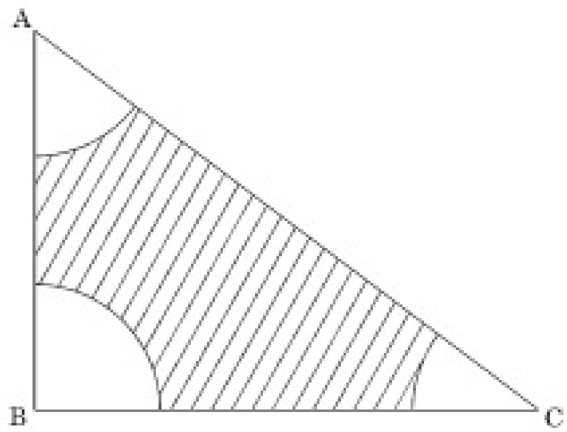
Ans. $abx^2 + b^2x - acx - bc = 0$

Getting, $D = b^2 + ac$ 1

$x = \frac{-(b^2 - ac) \pm (b^2 + ac)}{2ab}$ 1

$= \frac{c}{a}, -\frac{b}{a}$ 1

33. With vertices A, B and C of a triangle ABC as centres, arcs are drawn with radii 2 cm each as shown in the figure. If AB = 6 cm, BC = 8 cm and AC = 10 cm, then find the area of the shaded region.



Ans. Aea of shaded regions

= Area of ΔABC – Area of 3 sectors

$$= \frac{1}{2}\times 8\times 6 - \frac{22}{7}\times\frac{2^2}{360^\circ}(\angle ABC + \angle CAB + \angle ACB)$$ 1

$$= 24 - \frac{\overset{11}{\cancel{22}}}{7}\times\frac{4}{\underset{2}{\cancel{360^\circ}}}\times\cancel{180^\circ}$$ 1

$$= 24 - \frac{44}{7} = \frac{124}{7}\text{ cm}^2$$ 1

SECTION V

34. Water is being pumped out through a circular pipe whose internal diameter is 8 cm. If the rate of flow of water is 80 cm/s, then how many litres of water is being pumped out through this pipe in one hour?

Ans.

Volume of water flowing in 1 sec. = $\frac{22}{7}\times 4^2\times 80\text{ cm}^3$ $1\frac{1}{2}$

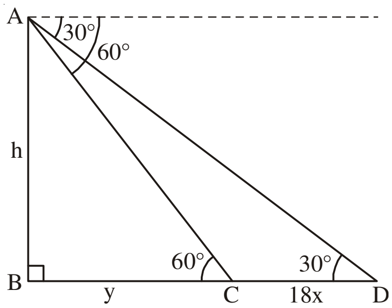
∴ Volume of water flowing in 1 hr. = $\frac{22}{7}\times 4^2\times 80\times 3600\text{ cm}^3$ $1\frac{1}{2}$

$$= \frac{22}{7}\times\frac{16\times 80\times 3600}{1000}\,l$$ 1

$$= \frac{101376}{7}\,l$$ 1

35. (a) A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 18 minutes for the angle of depression to change from 30 to 60 , how soon after this will the car reach the tower?

Ans.



Correct Figure 1

Let the speed of car be x m/min.

$\therefore$ CD = 18 x

Let AB = h and BC = y

In ΔABC, $\frac{h}{y} = \tan 60°$

$\Rightarrow h = y\sqrt{3}$ ...(1) $1\frac{1}{2}$

In ΔABD, $\frac{h}{y+18x} = \tan 30°$

$\Rightarrow h\sqrt{3} = y + 18x$

$\therefore$ 3y = y + 18 x (from (1)) $1\frac{1}{2}$

y = 9x

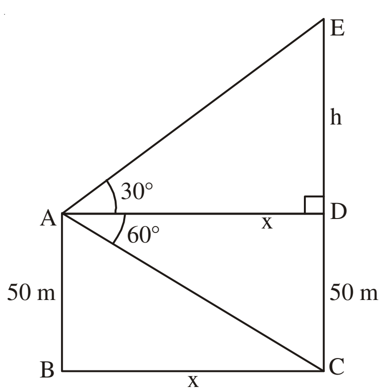
$\therefore$ Time taken by car to reach the tower

$= \frac{9x}{x} = 9$ minutes. 1

OR

(b) A girl on a ship standing on a wooden platform, which is 50 m above water level, observes the angle of elevation of the top of a hill as 30° and the angle of depression of the base of the hill as 60°. Calculate the distance of the hill from the platform and the height of the hill.

Ans.



Correct Figure 1

In ΔADE,

$\frac{h}{x} = \tan 30°$

$\Rightarrow h\sqrt{3} = x$ 1

In ΔADC,

$\frac{50}{x} = \tan 60°$

$\Rightarrow x = \frac{50}{\sqrt{3}}$ 1

$\therefore\ h = \frac{50}{3}$

$\therefore$ Distance of hill from the platform $= \frac{50\sqrt{3}}{3}$ m 1

height of hill $= \left(\frac{50}{3} + 50\right)$ m

$= \frac{200}{3}$ m 1

36. If S_n denotes the sum of first n terms of an A.P., prove that $S_{12} = 3\,(S_8 - S_4)$.

Ans. R.H.S. $= 3(S_8 - S_4)$

$= 3\left[\frac{8}{2}(2a+7d) - \frac{4}{2}(2a+3d)\right]$ 1

$= 3(8a + 28d - 4a - 6d)$ 1

$= 3(4a + 22d)$ 1

$= 3 \times \frac{4}{2}(2a + 11d)$ 1

$= \frac{12}{2}(2a + 11d)$

$= S_{12}$ = R.H.S. 1