BSEH Practice Paper (March 2024)

(2023-24)

Marking Scheme

MATHEMATICS **SET-D**

CODE: 835

⇨ Important Instructions: ● All answers provided in the Marking scheme are SUGGESTIVE
● Examiners are requested to accept all possible alternative correct answer(s).

	SECTION – A (1Mark × 20Q)	
Q. No.	**EXPECTED ANSWERS**	**Marks**
Question 1.	Let R be the relation in the set **R** given by R = {(a, b) : $a \leq b^2$ }. Choose the correct answer.	
Solution:	**(D) (9, 2) ∈ R**	1
Question 2	$\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is equal to	
Solution:	**(D)** $\frac{\pi}{6}$	1
Question 3	If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then A′A is:	
	(A) I	1
Question 4.	If A is an invertible matrix of order 2, then det (A^{-1}) is equal to	
Solution:	(B) $\frac{1}{det(A)}$	1
Question 5.	If the vertices of a triangle are (3, 8), (-4, 2) and (5, 1), then by using determinants its area is	
Solution:	**(B)** $\frac{61}{2}$	1
Question 6.	If $y = x^2 \log x$, then $\frac{d^2y}{dx^2}$ is equal to :	
Solution:	(A) 3 + 2logx	1
Question 7.	The antiderivative of $\frac{x^2+3x+4}{\sqrt{x}}$ equals:	
Solution:	(B) $\frac{2}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}} + C$	1

Question 8.	$\int e^x\left(\tan^{-1}x + \frac{1}{1+x^2}\right)dx$ equals:	
Solution:	(A) $e^x \tan^{-1}x + C$	1
Question 9.	The value of $\int_{-\pi/2}^{\pi/2} \sin^3x\, dx$ is	
Solution:	(C) 0	1
Question10.	The order of the differential equation $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ is :	
Solution:	(C) 3	1
Question11.	The number of arbitrary constants in the particular solution of a differential equation of third order are:	
Solution:	Since order of differential equation is 3 therefore number of arbitrary constants in the particular solution is 3.	1
Question12.	The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at x = 0, then find the value of k.	
Solution:	$\lim_{X\to 0} f(x) = \lim_{X\to 0} \frac{\sin 3x}{x}$ $=3\lim_{X\to 0} \frac{\sin 3x}{3x}$ $= 3(1) = 3$ Since f(x) is continuous at x = 0 $\therefore \lim_{X\to 0} f(x) = f(0)$ **k = 3**	1
Question13.	Find the direction cosines of y-axis.	
Solution:	$< 0, 1, 0 >$	1
Question14.	Compute P(A∩B), if P(B) = 0.8 , P(A|B) = 0.4 .	
Solution:	P(A∩B) = ? , P(A|B) = 0.4, P(B) = 0.8 $P(A/B) = \frac{P(A\cap B)}{P(B)}$ P(A∩B) = P(A/B).P(B)	1

	P(A∩B) = (0.4).(0.8) = 0.32	
Question15.	Two vectors having same magnitude are collinear. (True / False)	
Solution:	False	1
Question16.	Let A and B are independent events. Then P(A and **B**) = P(A) + P(B) (True / False)	
Solution:	False	1
Question17.	Let A and B be two events. If P(A|B) = P(A) , then A is _____ of B.	
Solution:	Independent	1
Question18.	The projection vector of $\vec{a} = \hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ on $\vec{b} = 7\hat{\imath} - \hat{\jmath} + 8\hat{k}$ is ________.	
Solution:	Projection of vector of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{\lvert\vec{b}\rvert} = \frac{7-3+56}{\sqrt{49+1+64}} = \frac{60}{\sqrt{114}}$	
Question19.	**Assertion (A):** Let L be the collection of all lines in a plane and R_1 be the relation on L as $R_1 = \{(L_1, L_2): L_1 \perp L_2)$ is a symmetric relation. **Reason (R) :** A relation R is said to be symmetric if (a, b) ∈ R => (b, a) ∈ R .	
Solution:	**(A)**	1
Question20.	**Assertion (A):** Vector form of the equation of a line $\frac{(x-2)}{3} = \frac{(y-1)}{2} = \frac{(3-z)}{-1}$ is $\vec{r} = (2\hat{\imath} + \hat{\jmath} + 3\hat{k}) + \lambda(3\hat{\imath} + 2\hat{\jmath} + \hat{k})$ **Reason (R):** Cartesian equation of a line passing through the point (2, 1,3) and parallel to the line $\frac{(x-3)}{1} = \frac{(y-2)}{2} = \frac{(z-4)}{-2}$ is 2x – 4 = y –1 = 3 –z	
Solution:	(B)	1
	SECTION – B (2Marks × 5Q)	
Question21.	Check the injectivity and surjectivity of the function f: R−{0} → R−{0} given by $f(x) = \frac{1}{x}$	
Solution:	Here given function is $f(x) = \frac{1}{x}$ Let a, b ∈ R – {0} and f(a) = f(b) ⇨ $\frac{1}{a} = \frac{1}{b}$	

	$\Rightarrow$ a = b $\therefore$ f is one - one. Let b $\in$ R – {0} , then b $\neq$ 0 And $f\left(\frac{1}{b}\right) = \frac{1}{\frac{1}{b}} = b$ Thus, f is both one - one and onto	**1** 1
OR Question21.	Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$	
Solution:	$\tan^{-1}\left[2\cos\left(2.\frac{\pi}{6}\right)\right] = \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$ $= \tan^{-1}\left[2.\frac{1}{2}\right]$ $= \tan^{-1}1$ $= \frac{\pi}{4}$	1 1
Question22.	Construct a 3 × 2 matrix whose elements are given by $a_{ij} = \frac{1}{2}(i + 2j)^2$.	
Solution:	Since it is 3 x 2 Matrix It has 3 rows and 2 columns Let the matrix be A Where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ Now it is given that $a_{ij} = \frac{1}{2}(i + 2j)^2$ Hence the required matrix is $a_{11} = \frac{1}{2}(1 + 2)^2 = 9/2$ $a_{12} = \frac{1}{2}(1 + 2(2))^2 = 25/2$	 1/2

	$a_{21} = \frac{1}{2}(2+2(1))^2 = 8$ $\quad a_{22} = \frac{1}{2}(2+2(2))^2 = 18$ $a_{31} = \frac{1}{2}(3+2(1))^2 = 25/2$ $\quad a_{32} = \frac{1}{2}(3+2(2))^2 = 49/2$ ⇨ $A = \begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \\ 25/2 & 49/2 \end{bmatrix}$	$1\frac{1}{2}$
Question23.	Find the value of k so that the function is continuous is at x = 3. $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x = 3 \\ k & x \neq 3 \end{cases}$	
Solution:	Given function is $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ k & x = 3 \end{cases}$ Now $\lim_{x\to 3} f(x) \Rightarrow \lim_{x\to 3} \frac{x^2-9}{x-3}$ $\lim_{x\to 3} \frac{(x-3)(x+3)}{x-3} = \lim_{x\to 3}(x+3) = 6$ Since function is continuous, therefore $\lim_{x\to 3} f(x) = f(3)$ $k = 6$	1 1
Question24.	Verify that the function $y = e^{-3x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$	
Solution:	The given function is $y = e^{-3x}$ $\frac{dy}{dx} = -3e^{-3x}$ $\frac{d^2y}{dx^2} = 9e^{-3x}$ L.H.S. $= \frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y$ $= 9e^{-3x} + (-3e^{-3x}) - 6e^{-3x}$	$\frac{1}{2}$ $\frac{1}{2}$ 1

	$= 9e^{-3x} - 9e^{-3x} = 0$	
OR Question24.	Find the general solution of the differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$	
Solution:	The given equation is $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ ⇨ $\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ ⇨ $\frac{dy}{\sqrt{1-y^2}} = -\frac{dx}{\sqrt{1-x^2}}$ Integrating both sides , we have $\sin^{-1} y = -\sin^{-1} x + C$ $\sin^{-1} y + \sin^{-1} x = C$ which is the required solution.	**$\frac{1}{2}$** **$1\frac{1}{2}$**
Question25.	Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that first ball is black and second is red.	
Solution:	Total number of balls = 10 black balls + 8 red balls = 18 balls Probability of getting a black ball in the first draw = $\frac{10}{18} = \frac{5}{9}$ As the ball is replaced after the first throw, ∴ Probability of getting a red ball in the second draw = $\frac{8}{18} = \frac{4}{9}$ Since the two balls are drawn with replacement, the two draws are independent. P(both balls are red) = P(first ball is red) x P(second ball is red) Now, the probability of getting both balls red = $\frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$	**1** **1**
	SECTION – C (3Marks × 8Q)	
Question26.	Show that the relation R defined in the set A of all polygons as R = {(P_1, P_2): P_1, and P_2 have same number of sides}, is an equivalence relation.	

Solution:	Set A is the set of all the polygons . R ={(P_1, P_2): P_1, and P_2 have same number of sides } Now R is **reflexive** since (P, P) ∈ R as P and P has the same number of sides. Let (P_1, P_2) ∈ R ⇒ P_1 and P_2 have same number of sides ⇒ P_2 and P_1 have same number of sides ⇒ (P_2, P_1) ∈ R Therefore R is **symmetric** Now let (P_1, P_2) ∈ R and (P_2, P_3) ∈ R ⇒ P_1 and P_2 have same number of sides and P_2 and P_3 have same number of sides ⇒ P_1 and P_3 have same number of sides ⇒ (P_1, P_3) ∈ R Therefore R is **transitive.** Hence **R** is an **equivalence relation**.	$\frac{1}{2}$ 1 1 $\frac{1}{2}$
OR Question26.	Solve for x: $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$, $x > 0$	
Solution:	We have $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x$ ...(1) We know $\tan^{-1}\left(\frac{A-B}{1+AB}\right) = \tan^{-1}A - \tan^{-1}B$ Therefore, From equation (1) ⇨ $\tan^{-1}1 - \tan^{-1}x = \frac{1}{2}\tan^{-1}x$ ⇨ $2\tan^{-1}1 - 2\tan^{-1}x = \tan^{-1}x$ ⇨ $2(\frac{\pi}{4}) = 3\tan^{-1}x$ ⇨ $\frac{\pi}{2} = 3\tan^{-1}x$ ⇨ $\tan^{-1}x = \frac{\pi}{6}$	$\frac{1}{2}$ 1 $1\frac{1}{2}$

	⇨ $x = \tan\frac{\pi}{6}$ => $x = \frac{1}{\sqrt{3}}$	
Question27.	Find X and Y, if $2X + Y = \begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix}$ and $X - 2Y = \begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$	
Solution:	Given equations are $2X + Y = \begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix}$ …(1) $X - 2Y = \begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ …(2) Multiplying equation (1) by 2 and then adding to equation (2), we have $2(2X + Y) + (X - 2Y) = 2\begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix} + \begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ $5X = \begin{bmatrix} 5 & 10 & 15 \\ 15 & 5 & 10 \end{bmatrix}$ $X = \frac{1}{5}\begin{bmatrix} 5 & 10 & 15 \\ 15 & 5 & 10 \end{bmatrix}$ $X = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ Using the value of matrix X in (1) equation , we have $2\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} + Y = \begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix}$ ⇨ $\begin{bmatrix} 2 & 4 & 6 \\ 6 & 2 & 4 \end{bmatrix} + Y = \begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix}$ ⇨ $Y = \begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ ⇨ $Y = \begin{bmatrix} 3 & 2 & 4 \\ 4 & 2 & 2 \end{bmatrix}$	**$\frac{1}{2}$** **1** **$\frac{1}{2}$** **1**
Question28.	Find $\frac{dy}{dx}$ of the function $xy = e^{(x-y)}$.	
Solution:	Given: $xy = e^{(x-y)}$ Taking log on both sides, we have ⇨ $\log x + \log y = (x - y)\log e$	**1**

	⇨ $\log x + \log y = x - y$ [∵ log e = 1] Diff. w.r.t. 'x' ⇨ $\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$ ⇨ $\left(\frac{1}{y} + 1\right)\frac{dy}{dx} = 1 - \frac{1}{x}$ ⇨ $\left(\frac{1+y}{y}\right)\frac{dy}{dx} = \frac{x-1}{x}$ ⇨ $\frac{dy}{dx} = \frac{y(x-1)}{x(1+y)}$	**1** **1**
Question29.	Find the intervals in which the function f is given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is strictly increasing or strictly decreasing.	
Solution:	Given function: $f(x) = 4x^3 - 6x^2 - 72x + 30$ Diff. w.r.t. 'x' $f'(x) = 12x^2 - 12x - 72 = 12(x^2 - x - 6)$ $f'(x) = 12(x-3)(x+2)$,.......(1) Now for increasing or decreasing, $f'(x) = 0$ $12(x-3)(x+2) = 0$ $x - 3 = 0$ or $x + 2 = 0$ $x = 3$ or $x = -2$ Therefore, we have sub-intervals are $(-\infty, -2)$, $(-2, 3)$ and $(3, \infty)$ For interval $(-\infty, -2)$, picking $x = -3$, from equation (1), $f'(x) = (+ve)(-ve)(-ve) = (+ve) > 0$ Therefore, f is strictly increasing in $(-\infty, -2)$ For interval $(-2, 3)$, picking $x = 0$, from equation (1), $f'(x) = (+ve)(-ve)(+ve) = (-ve) < 0$	**1** $\frac{1}{2}$ $\frac{1}{2}$

	Therefore, f is strictly decreasing in (–2, 3). For interval (3, ∞), picking x = 4 , from equation (1), f' (x) = (+ve)(+ve)(+ve) = (+ve) > 0 Therefore, is strictly increasing in (3, ∞). So, f is strictly increasing in (–∞, –2) and (3, ∞). f is strictly decreasing in (–2, 3).	$\frac{1}{2}$ $\frac{1}{2}$
Question 30	Integrate: $\int x \tan^{-1} x \ \ dx$	
Solution:	$I = \int x \tan^{-1} x \ \ dx$ Using $\int U.V \ dx = U\int V \ dx - \int(\frac{dU}{dx}.\int V\,dx) \ dx$ $\int x \tan^{-1} x \ dx = \tan^{-1} x \int x \ dx - \int(\frac{d(\tan^{-1} x)}{dx}.\int x.dx) \ dx$ ⇨ $\tan^{-1} x\,(\frac{x^2}{2}) - \int \frac{1}{1+x^2}.(\frac{x^2}{2}) \ \ dx$ ⇨ $\frac{x^2.\tan^{-1} x}{2} - \frac{1}{2}\int \frac{x^2}{1+x^2} \ dx$ ⇨ $\frac{x^2.\tan^{-1} x}{2} - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2} \ dx$ ⇨ $\frac{x^2.\tan^{-1} x}{2} - \frac{1}{2}\int(\frac{1+x^2}{1+x^2} - \frac{1}{1+x^2}) \ dx$ ⇨ $\frac{x^2.\tan^{-1} x}{2} - \frac{1}{2}\int(1 - \frac{1}{1+x^2}) \ dx$ ⇨ $\frac{x^2.\tan^{-1} x}{2} - \frac{1}{2}(x - \tan^{-1} x) + C$	$\frac{1}{2}$ $\frac{1}{2}$ 1 1
OR Question30.	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{cos^5 x}{sin^5 x + cos^5 x} \ dx$	
Solution:	$I = \int_0^{\frac{\pi}{2}} \frac{cos^5 x}{sin^5 x + cos^5 x} \ dx$ …(1)	

	Using property of definite integral $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ $I = \int_0^{\frac{\pi}{2}} \frac{\cos^5(\frac{\pi}{2}-x)}{\sin^5(\frac{\pi}{2}-x)+\cos^5(\frac{\pi}{2}-x)}\ dx$ $I = \int_0^{\frac{\pi}{2}} \frac{\sin^5 x}{\cos^5 x+\sin^5 x}\ dx$ …(2) Adding (1) and (2) $2I = \int_0^{\frac{\pi}{2}} \frac{\cos^5 x+\sin^5 x}{\sin^5 x+\cos^5 x}\ dx$ $2I = \int_0^{\frac{\pi}{2}} 1\ dx$ $2I = \lvert x\rvert_0^{\frac{\pi}{2}}$ $2I = \frac{\pi}{2}$ $I = \frac{\pi}{4}$	1 1 1
Question31.	Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$.	
Solution:	$\vec{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 2\hat{\imath} - 7\hat{\jmath} + \hat{k}$ Area of a parallelogram = $\lvert \vec{a} \times \vec{b} \rvert$ $\lvert \vec{a} \times \vec{b} \rvert = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$ $= \lvert \hat{\imath}(-1 + 21) - \hat{\jmath}(1-6) + \hat{k}(-7+2) \rvert$ $= \lvert 20\hat{\imath} + 5\hat{\jmath} - 5\hat{k} \rvert$ $= \sqrt{(20)^2 + (5)^2 + (-5)^2}$ $= \sqrt{450} = 15\sqrt{2}$ sq. unit	$\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$

	SECTION – C (5Marks × 4Q)	
Question32.	Solve the system of linear equations, using matrix method. $x - y + 2z = 7$ $3x + 4y - 5z = -5$ $2x - y + 3z = 12$	
Solution:	$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$ $\lvert A\rvert = 1(12 - 5) + 1(9 + 10) + 2(-3 - 8) = 1(7) + 1(19) + 2(-11)$ $= 7 + 19 - 22$ $= 4 \neq 0$;	**1**
	Inverse of matrix A, exists. To find the inverse of matrix: Cofactors of matrix: $A_{11} = 7$, $A_{12} = -19$, $A_{13} = -11$ $A_{21} = 1$, $A_{22} = -1$, $A_{23} = -1$ $A_{31} = -3$, $A_{32} = 11$, $A_{33} = 7$ ⇨ $\text{adj.}A = \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}' = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$	**1**
	So, $A^{-1} = \frac{\text{adj.}A}{\lvert A\rvert}$ $A^{-1} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$	**1**
	Now, matrix of equations can be written as: AX=B	

	$\begin{bmatrix}1 & -1 & 2\\3 & 4 & -5\\2 & -1 & 3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}7\\-5\\12\end{bmatrix}$ And, X = A^{-1} B $\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{4}\begin{bmatrix}7 & 1 & -3\\-19 & -1 & 11\\-11 & -1 & 7\end{bmatrix}\begin{bmatrix}7\\-5\\12\end{bmatrix}$ $\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{4}\begin{bmatrix}49-5-36\\-133+5+132\\-77+5+84\end{bmatrix}$ $\begin{bmatrix}x\\y\\z\end{bmatrix} = \frac{1}{4}\begin{bmatrix}8\\4\\12\end{bmatrix} => \begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}2\\1\\3\end{bmatrix}$ Therefore, x = 2, y = 1 and z =3.	**1** **1**
Question33.	Find the area of the region bounded by the ellipse $\frac{x^2}{36} + \frac{y^2}{4} = 1$	
Solution:	Here $\frac{x^2}{36} + \frac{y^2}{4} = 1$ …(1) It is a horizontal ellipse having center at origin and is symmetrical about both axes (if we change y to -y or x to -x, equation remain same). Standard equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ By comparing, a = 6 and b = 2 From equation (1) $\Rightarrow y^2 = \frac{4}{36}(36 - x^2) \Rightarrow y^2 = \frac{1}{9}(36 - x^2)$ $\Rightarrow y = \pm\frac{1}{3}\sqrt{36 - x^2}$ …..(2) Points of Intersections of ellipse (1) with x-axis (y = 0) Put y = 0 in equation (1), we have	$\frac{1}{2}$

$x^2/36 = 1$

$\Rightarrow x^2 = 36$

$\Rightarrow x = \pm 6$ **½**

Therefore, Intersections of ellipse(1) with x-axis are (6, 0) and (-6, 0).

Now again,

Points of Intersections of ellipse (1) with y-axis (x = 0)

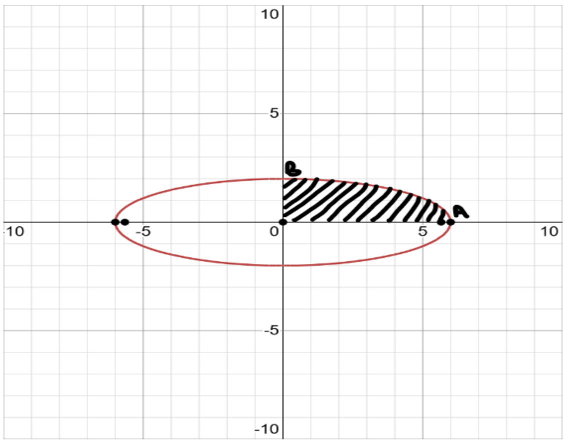
Putting x = 0 in equation (1), $y^2/4 = 1$

$\Rightarrow y^2 = 4$

$\Rightarrow y = \pm 2$ **1**

Therefore, Intersections of ellipse (1) with y-axis are (0, 2) and (0, –2)

for arc of ellipse in first quadrant.



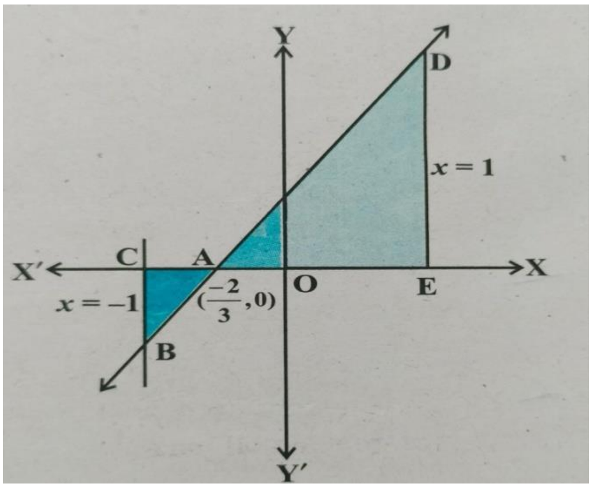
Now, Area of region bounded by ellipse (1)

Total shaded area = 4 x Area OAB of ellipse in first quadrant

$= 4\left|\int_0^6 y.\,dx\right|$ [∵ at end B of arc AB of ellipse: x = 0 and at end A of arc AB; x = 2] **½**

$= 4\left|\int_0^6 \frac{1}{3}\sqrt{36 - x^2}.\,dx\right| = \frac{4}{3}\left|\int_0^6 \sqrt{6^2 - x^2}.\,dx\right|$

$= \frac{4}{3}\left|\frac{x}{2}\sqrt{6^2 - x^2} + \frac{6^2}{2}\sin^{-1}\frac{x}{6}\right|_0^6$ $\left[\because \int\sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$ **1**

	$\frac{4}{3}$ [((6/2)$\sqrt{36-36}$ + 18 sin^{-1}1) – (0 + 18 sin^{-1}0)] = $\frac{4}{3}$ [18($\frac{\pi}{2}$)] = 12(π) = 12π sq. units	**1**
OR Question33.	Find the area of the region bounded by the line y = 3x +2, the x-axis and the ordinates x = −1 and x = 1.	
Solution:	The line y = 3x +2 …(1) It is a straight line passing through the points (−1, −1) and (1, 5). x = −1 and x + 1 are two straight lines parallel to y-axis. Put x = −1 in equation(1) y = −1 Point is (−1, −1) Put x = 1 in equation (1) y = 5 Point is (1, 5) Making a rough hand sketch for the given lines. We have ,  Now, line (1) is meets x-axis at x = $\frac{-2}{3}$ (i.e. where y =0) Therefore required region is lying below the x – axis for x ∈ (−1, $\frac{-2}{3}$) And lying above the x-axis for x ∈ ($\frac{-2}{3}$, 1). Required area = Area of the egion ACBA + Area of the region ADEA ⇨ = $\int_{-1}^{\frac{-2}{3}}$(−y of line) .dx + $\int_{\frac{-2}{3}}^{1}$(y of line) .dx ⇨ = − $\int_{-1}^{\frac{-2}{3}}$ 3x + 2.dx + $\int_{\frac{-2}{3}}^{1}$ 3x + 2 .dx	**$\frac{1}{2}$** **1** **1** **1**

	$\Rightarrow \; = -\left\|\frac{3x^2}{2} + 2x\right\|_{-1}^{\frac{-2}{3}} + \left\|\frac{3x^2}{2} + 2x\right\|_{\frac{-2}{3}}^{1}$ $\Rightarrow \; = -\left[\left\{\frac{3}{2}\left(\frac{-2}{3}\right)^2 + 2\left(\frac{-2}{3}\right)\right\} - \left\{\frac{3}{2}(-1)^2 + 2(-1)\right\}\right] + \left[\left\{\frac{3}{2}(1)^2 + 2(1)\right\} - \left\{\frac{3}{2}\left(\frac{-2}{3}\right)^2 + 2\left(\frac{-2}{3}\right)\right\}\right]$ $\Rightarrow \; = -\left[\left\{\frac{2}{3} - \frac{4}{3}\right\} - \left\{\frac{3}{2} - 2\right\}\right] + \left[\left\{\frac{3}{2} + 2\right\} - \left\{\frac{2}{3} - \frac{4}{3}\right\}\right]$ $\Rightarrow \; = -\left[\left\{-\frac{2}{3}\right\} - \left\{\frac{-1}{2}\right\}\right] + \left[\left\{\frac{7}{2}\right\} - \left\{\frac{-2}{3}\right\}\right]$ $\Rightarrow \; = \frac{2}{3} - \frac{1}{2} + \left[\frac{7}{2} + \frac{2}{3}\right]$ $\Rightarrow \; = \frac{1}{6} + \frac{25}{6} = \frac{13}{3}$ sq. units	$1\frac{1}{2}$
Question34.	Find the shortest distance between the line $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$.	
Solution:	Given lines are $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ $\therefore$ Corresponding vector equations of given lines are $\vec{r} = 3\hat{\imath} + 8\hat{\jmath} + 3\hat{k} + \lambda(3\hat{\imath} - \hat{\jmath} + \hat{k})$ ...(1) and $\vec{r} = -3\hat{\imath} - 7\hat{\jmath} + 6\hat{k} + \mu(-3\hat{\imath} + 2\hat{\jmath} + 4\hat{k})$ ...(2) Comparing (1) and (2) with $\vec{r} = \overrightarrow{a_1} + \lambda\,\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\,\overrightarrow{b_2}$ respectively, we get $\overrightarrow{a_1} = 3\hat{\imath} + 8\hat{\jmath} + 3\hat{k}$, and $\overrightarrow{b_1} = 3\hat{\imath} - \hat{\jmath} + \hat{k}$ $\overrightarrow{a_2} = -3\hat{\imath} - 7\hat{\jmath} + 6\hat{k}$ and $\overrightarrow{b_2} = -3\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$ Therefore $\overrightarrow{a_2} - \overrightarrow{a_1} = -6\hat{\imath} - 15\hat{\jmath} + 3\hat{k}$ And $\overrightarrow{b_1} \times \overrightarrow{b_2} = (3\hat{\imath} - \hat{\jmath} + \hat{k}) \times (-3\hat{\imath} + 2\hat{\jmath} + 4\hat{k})$	$\frac{1}{2}$ $\frac{1}{2}$

	$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} = -6\hat{\imath} - 15\hat{\jmath} + 3\hat{k}$ $\|\overrightarrow{b_1} \times \overrightarrow{b_2}\| = \sqrt{36 + 225 + 9} = \sqrt{270}$ Hence, the shortest distance between the given lines is given by $D = \frac{\|(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2})\|}{\|\overrightarrow{b_1} \times \overrightarrow{b_2}\|} = \frac{\|(-6\hat{\imath}-15\hat{\jmath}+3\hat{k}).(-6\hat{\imath}-15\hat{\jmath}+3\hat{k})\|}{\sqrt{270}}$ $\frac{\|36+225+9\|}{\sqrt{270}} = \frac{270}{\sqrt{270}} = \sqrt{270} = 3\sqrt{30}$	**1** $\frac{1}{2}$ $1\frac{1}{2}$ **1**
OR Question34.	Find the vector equation of the line passing through the point (−1, 3,−2) and perpendicular to the two lines : $\frac{x-5}{1} = \frac{y-3}{2} = \frac{z+1}{6}$ and $\frac{2-x}{3} = \frac{y-1}{2} = \frac{z+4}{5}$.	
Solution:	The vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\,\vec{b}$ .It is given that, the line passes through (−1, 3,−2). So, $\vec{a} = -1\hat{\imath} + 3\,\hat{\jmath} - 2\hat{k}$ Given lines are $\frac{x-5}{1} = \frac{y-3}{2} = \frac{z+1}{6}$ and $\frac{x-2}{-3} = \frac{y-1}{2} = \frac{z+4}{5}$ It is also given that, line is perpendicular to both given lines. So we can say that the required line is perpendicular to both parallel vectors of two given lines. We know that, $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ & $\vec{b}$, so let $\vec{b}$ is cross product of parallel vectors of both lines i.e. $\vec{b} = \overrightarrow{b_1} \times \overrightarrow{b_2}$	1 2

	where $\overrightarrow{b_1} = \hat{\imath} + 2\hat{\jmath} + 6\hat{k}$ and $\overrightarrow{b_2} = -3\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$ and Required Normal $\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 6 \\ -3 & 2 & 5 \end{vmatrix}$ $= \hat{\imath}(10 - 12) - \hat{\jmath}(5 + 18) + \hat{k}(2 + 6)$ $\vec{b} = -2\hat{\imath} - 23\hat{\jmath} + 8\hat{k}$ Now, by substituting the value of $\vec{a}$ & $\vec{b}$ in the formula $\vec{r} = \vec{a} + \lambda \vec{b}$, we get $\vec{r} = (-1\hat{\imath} + 3\hat{\jmath} - 2\hat{k}) + \lambda(-2\hat{\imath} - 23\hat{\jmath} + 8\hat{k})$	1 1
Question35.	Solve the following problem graphically: Minimise and Maximise Z = 30x + 60y Subject to the constraints: $2x + y \leq 70$ $x + y \leq 40$ $x + 3y \leq 90$ $x \geq 0, y \geq 0$	
Solution:	Z = 30x + 60y ...(1) $2x + y \leq 70$ ...(2) $x + y \leq 40$ ...(3) $x + 3y \leq 90$ ...(4) $x \geq 0, y \geq 0$ ...(5) First of all, let us graph the feasible region of the system of linear inequalities (2) to (5). Let Z = 30x + 60y ....(1) Converting inequalities to equalities 2x + y = 70 X | 0 | 35 Y | 70 | 0 Points are (0, 70), (35, 0) Now Put (0, 0) in (2) inequation we have $0 \leq 70$ which is true. ∴ Req. region lies towards the origin. x + y = 40	$\frac{1}{2}$

X	0	40
Y	40	0

Points are (0, 40), (40, 0) $\frac{1}{2}$

Now Put (0, 0) in (3) inequation we have, $0 \leq 40$ which is true.

$\therefore$ Req. region lies towards the origin.

$x + 3y = 90$

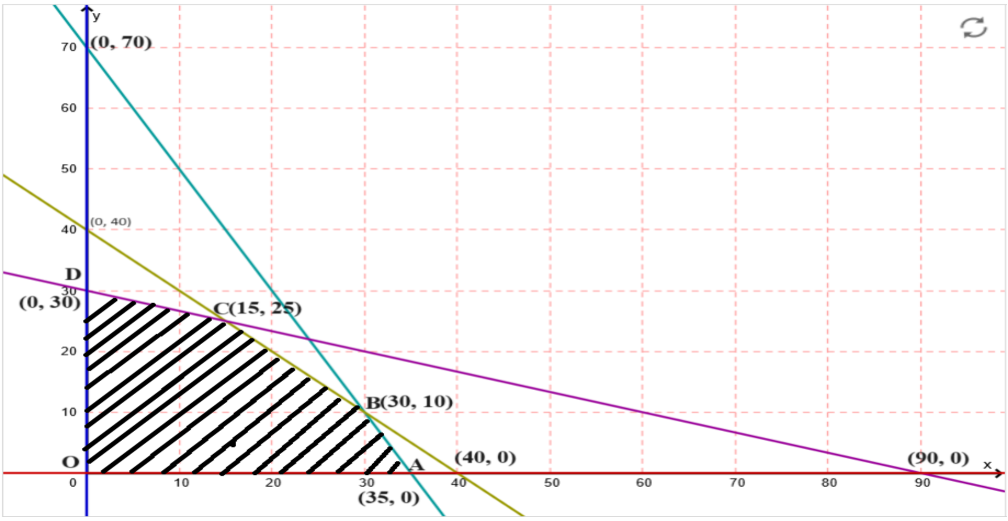
X	0	30
Y	90	0

Points are (0, 90), (30,0)

Now Put (0, 0) in (4) inequation we have, $0 \leq 90$ which is true. $\frac{1}{2}$

$\therefore$ Req. region lies towards the origin.

Plot the graph for the set of points



1

To find minimum and maximum value of Z.

The feasible region ABCD is shown in the figure. Note that the region is bounded. The coordinates of the corner points A, B, C and D are (35, 0), (30, 10), (15, 25) and (0, 30) respectively. 1

Corner Point	Corresponding Value of $Z = 30x + 60y$

A (35, 0)	**1050←Minimum**
B (30, 10)	1500
C (15, 25)	**1950←Maximum**
D (0, 30)	1800

From the table, we find that,

∴The maximum value of Z is 1950 at the point B (15, 25). $1\frac{1}{2}$

The minimum value of Z is 1050 at the point C (35, 0).

SECTION – E (4Marks × 3Q)

Question36. An architect designs an auditorium for a school for its cultural activities. The floor of the auditorium is rectangular in shape and has a fixed perimeter P.

Based on the above information, answer the following questions.

(i) If x and y represents the length and breadth of the rectangular region, then find the relation between the variable.

(ii) Find the area A of the rectangular region, as a function of x.

(iii) Find the value of y, for which the area of the floor is maximum.

Solution: Given length of the rectangular auditorium = x 1

Also breadth of the rectangular auditorium = y

Given perimeter of the rectangle = P

∴ relation between the variables $2x + 2y = P$

∴ Area of the floor (A) = length × breadth

$$A = x \times y$$ 1

$$A = \left(\frac{P - 2x}{2}\right)x \Rightarrow A = \frac{1}{2}(Px - 2x^2)$$

Area of the floor = A = xy

For the value of y for which area is maximum, expressing area in terms of y, we have

$$A = \left(\frac{P - 2y}{2}\right)y$$

	$A = \frac{1}{2}(Py - 2y^2)$ …(1) Diff. w.r.t. 'y' $\frac{dA}{dx} = \frac{1}{2}(P - 4y)$ …(2) For miaximum or minimum value of A, $\frac{dA}{dx} = 0$ we have ⇨ $P - 4y = 0$ ⇨ $y = \frac{P}{4}$ Diff. equation (2) again w.r.t. 'y', we have $\frac{d^2A}{dx^2} = \frac{1}{2}(0 - 4) = -2$ At $y = \frac{P}{4}$ $\frac{d^2A}{dx^2} = -2 = -ve$ ⇨ ∴ Area A is maximum at $y = \frac{P}{4}$ unit	2
Question37.	A linear differential equation is of the form $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then such equation is known as linear differential equation. Its solution is given by $y.(IF.) = \int Q(IF.)\,dx + c$, where I.F.(Integrating Factor) = $e^{\int Pdx}$ Now, suppose the given equation is $\cos^2 x \frac{dy}{dx} + y = \tan x$, $(0 \le x < \frac{\pi}{2})$ *Based on the above information, answer the following questions:* (i)What are the values of P and Q respectively ? (1) (ii) What is the value of I.F.? (1) (iii) Find the Solution of given equation. (2)	
Solution:	(i) Given differential equation is $\cos^2 x \frac{dy}{dx} + y = \tan x$ Dividing on both side by $\cos^2 x$, we have $\frac{dy}{dx} + \frac{1}{\cos^2 x} y = \frac{\tan x}{\cos^2 x}$ $\frac{dy}{dx} + \sec^2 x\, y = \tan x \,.\, \sec^2 x$	

	Comparing this differential equation with $\frac{dy}{dx}$ + Py = Q, we have $\Rightarrow$ P = sec^2x and Q = tan x . sec^2x	**1**
	(ii) I.F.(Integrating Factor) = $e^{\int Pdx}$ $= e^{\int sec^2x.dx}$ $= e^{\tan x}$ I.F. $= e^{\tan x}$	**1**
	(iii) Solution of given equation is y.(IF.) = $\int$ Q(IF.) dx +c $y\,(e^{\tan x}) = \int \tan x\ sec^2x.\, e^{\tan x}\ + c$ Put tan x = t $\Rightarrow$ sec^2x . dx = dt $y\ e^{\tan x} = \int e^t.t.dt$ Integrating by part, we have $y\ e^{\tan x} = t\int e^t\ dt - \int(\frac{dt}{dt}\int e^t\ dt\ )\ dt$ $y\ e^{\tan x} = t.e^t - \int e^t\ dt + C$ $y\ e^{\tan x} = t.e^t - e^t + C$ $y\ e^{\tan x} = (t-1)\ e^t + C$ $y\ e^{\tan x} = (\tan x - 1)\ e^{\tan x} + C$	**2**
Question 38.	In a school, teacher asks a question to three students Ravi, Mohit and Sonia. The probability of solving the question by Ravi, Mohit and Sonia are 30%, 25% and 45%, respectively. The probability of making error by Ravi, Mohit and Sonia are 1%, 1.2% and 2%, respectively. *Based on the above information, answer the following questions.* (i) Find the total probability of committing an error in solving the question. (2) (ii) If the solution of question is checked by teacher and has some error, then find the probability that the question is not solved by Ravi. (2)	

Solution:	Let E_1, E_2 and E_3 be the events that Ravi, Mohit and Sonia solve the question respectively. It is given that $P(E_1) = \frac{30}{100}$, $P(E_2) = \frac{25}{100}$ and $P(E_3) = \frac{45}{100}$ Let A be the event that students commit the error. It is given that $P(A/E_1) = \frac{1}{100}$, $P(A/E_2) = \frac{1.2}{100}$ and $P(A/E_3) = \frac{2}{100}$	
	(i) Required probability of committing an error in solving the question = P(A) Therefore, $P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)$ $P(A) = (\frac{30}{100})(\frac{1}{100}) + (\frac{25}{100})(\frac{1.2}{100}) + (\frac{45}{100})(\frac{2}{100})$ $P(A) = \frac{30}{10000} + \frac{30}{10000} + \frac{90}{10000}$ $P(A) = \frac{30+30+90}{10000}$ $P(A) = \frac{150}{10000}$ $P(A) = \frac{3}{200}$	**2**
	(ii) Probability that the question is not solved by Ravi when solution of question has some error $= P(\overline{E1/A})$ $\therefore\ 1 - P(E_1/A) = 1 - \frac{P(E1)P(A/E1)}{P(A)}$ $= 1 - \frac{(\frac{30}{100})(\frac{1}{100})}{\frac{3}{200}}$ $P(E_1/A) = 1 - \frac{30}{10000} \times \frac{200}{3}$	2

	$= 1 - \frac{1}{5} = \frac{4}{5}$	