BSEH Practice Paper (March 2024)

(2023-24)

Marking Scheme

MATHEMATICS **SET-B**

CODE: 835

⇨ Important Instructions: ● All answers provided in the Marking scheme are SUGGESTIVE
● Examiners are requested to accept all possible alternative correct answer(s).

	SECTION – A (1Mark × 20Q)	
Q. No.	**EXPECTED ANSWERS**	**Marks**
Question 1.	Let R be the relation in the set **N** given by R = {(a, b) : b = a + 1, b > 5}. Choose the correct answer.	
Solution:	(B) (7, 8) ∈ R	1
Question 2.	$\cos^{-1}(\cos \frac{7\pi}{6})$ is equal to	
Solution:	(B) $\frac{5\pi}{6}$	1
Question 3.	If A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then A′A is:	
Solution:	(A) I	1
Question 4.	If A and B are invertible matrices, then which of the following is not correct	
Solution:	(D) $(A + B)^{-1} = B^{-1} + A^{-1}$	1
Question 5.	If the vertices of a triangle are (−2, −3), (3, 2) and (−1, −8), then by using determinants its area is	
Solution:	(A) 15	1
Question 6.	If $y = \log x^2$, then $\frac{d^2y}{dx^2}$ is equal to :	
Solution:	(A) $\frac{-2}{x^2}$	1
Question 7.	The antiderivative of $(1 - x)\sqrt{x}$ equals:	
Solution:	(B) $\frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + C$	1
Question 8.	$\int e^x \sec x\,(1 + \tan x)\,dx$ equals	
Solution:	(C) $e^x \sec x + C$	1
Question 9.	The value of $\int_{-\pi/2}^{\pi/2} \tan^5 x\,dx$ is	
Solution:	(C) 0	1
Question 10.	The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is :	

Solution:	(D) not defined	1
Question 11.	How many number of arbitrary constants are there in the general solution of a differential equation of fourth order?	
Solution:	**4**	1
Question 12.	The function $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at x = 0, then find the value of k	
Solution:	$\lim\limits_{X\to 0} f(x) = \lim\limits_{X\to 0}\left(\frac{\sin x}{x} + \cos x\right)$ $= 1 + 1$ $= 2$ Since f(x) is continuous at x = 0 $\therefore \lim\limits_{X\to 0} f(x) = f(0)$ ⇨ $\mathbf{2 = k}$	1
Question 13.	If a line makes angles 90°, 135°, 45° with the x, y and z-axes respectively, find its direction cosines.	
Solution:	Line makes angles 90°, 135°, 45° with the x,y and z-axes respectively ∴ Direction Cosines are $l = \cos 90°$, $m = \cos 135°$, $n = \cos 45°$ $l = 0$, $m = \frac{-1}{\sqrt{2}}$, $n = \frac{1}{\sqrt{2}}$ ⇨ D.C.'s are $< 0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$	1
Question 14.	If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.	
Solution:	Since A and B are independent therefore $P(A \cap B) = P(A).P(B)$ $\therefore P(A \cap B) = \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$	1
Question 15.	$\vec{a}$ and $-\vec{a}$ aer collinear. (True / False)	
Solution:	True	1
Question 16.	The probability of obtaining an even prime number on each die, when a pair of dice is rolled is $\frac{6}{36}$. (True / False)	
Solution:	**False**	1
Question 17.	If A and B are any two events such that $P(A) + P(B) - P(A \cap B) = P(A)$, then $P(A\|B)$ ____.	
Solution:	**1**	1
Question 18.	The projection vector of $\vec{a} = \hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ on $\vec{b} = 7\hat{\imath} - \hat{\jmath} + 8\hat{k}$ is ____ **.**	

Solution:	$\frac{\mathbf{60}}{\sqrt{\mathbf{114}}}$	
Question 19.	**Assertion (A):** Let A = {1,2} and B = {3,4}. Then, number of relations from A to B is 16. **Reason (R):** If n(A) = p and n(B) = q, then number of relations is 2^{pq}.	
Solution:	**(A).** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)	1
Question 20.	**Assertion (A):** The direction cosines of line $\frac{x-5}{3} = \frac{y+4}{2} = \frac{z+8}{1}$ is $\frac{3}{\sqrt{14}}$, $\frac{2}{\sqrt{14}}$, $\frac{1}{\sqrt{14}}$. **Reason (R):** The distance between two parallel lines $\vec{r} = \overrightarrow{a_1} + \lambda \vec{b}$ and $\vec{r} = \overrightarrow{a_2} + \mu\vec{b}$ is given by d = $\frac{\lvert(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}\rvert}{\lvert\vec{b}\rvert}$.	
Solution:	**(B)** Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A)	1
	SECTION – B (2Marks × 5Q)	
Question 21.	Show that the function f: R→ R, defined as $f(x) = x^2$, is neither one-one nor onto.	
Solution:	$f(x) = x^2$ Checking for ONE-ONE let x_1 and x_2 are be any two real numbers. $f(x_1) = x_1^2$ and $f(x_2) = x_2^2$ Now $f(x_1) = f(x_2)$ ⇨ $x_1^2 = x_2^2$ ⇨ $x_1 = x_2$, $x_1 = -x_2$ ⇨ Since x_1 does not have a unique image so f(x) is not one-one. Checking for ONTO Let f(x) = y such that y ∈ **R** ⇨ $x^2 = y$ ⇨ $x = \pm\sqrt{y}$ Note that y is a real number, so it can be negative also ⇨ f(x) is not an onto function. (*Note: Students can also use some illustrations to show f(x) neither one-one nor onto.*)	1 1
OR Question 21.	Find the value of: $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$	
Solution:	Let $\tan^{-1}\sqrt{3} = x$. Then $\tan x = \sqrt{3} = \tan(\pi/3)$	

	We know that the range of the principal value branch of $\tan^{-1}$ is $(\frac{-\pi}{2}, \frac{\pi}{2})$ $\therefore \tan^{-1}\sqrt{3} = \pi/3$ Let $\sec^{-1}(-2) = y$. Then, $\sec y = -2 = -\sec(\pi/3)$ $\sec y = -2 = \sec(\pi - \frac{\pi}{3})$ We know that the range of the principal value branch of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ $\therefore \sec^{-1}(-2) = 2\pi/3$ Now $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \pi/3 - 2\pi/3$ $= -\pi/3$	$\frac{1}{2}$ $\frac{1}{2}$ 1
Question 22.	Construct a 3 × 2 matrix whose elements are given by $a_{ij} = \frac{1}{2} \| i - 3j \|$.	
Solution:	Since it is 3 x 2 Matrix It has 3 rows and 2 columns Let the matrix be A Where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ Now it is given that $a_{ij} = \frac{1}{2} \| i - 3j \|$ Hence the required matrix is $a_{11} = 1 \quad a_{12} = 5/2$ $a_{21} = ½ \quad a_{22} = 2 \quad \Rightarrow \quad A = \begin{bmatrix} 1 & 5/2 \\ 1/2 & 2 \\ 0 & 3/2 \end{bmatrix}$ $a_{31} = 0 \quad a_{32} = 3/2$	$\frac{1}{2}$ $\frac{1}{2}$ 1
Question 23.	Find the value of k so that the function is continuous is at x = 1. $f(x) = \begin{cases} \frac{x^2 - 1}{x-1}, & x \neq 1 \\ k & x = 1 \end{cases}$	
Solution:		

	Given function is $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ k & x = 1 \end{cases}$ Now $\lim_{x\to 1} f(x) \Rightarrow \lim_{x\to 1} \frac{x^2-1}{x-1}$ $\lim_{x\to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x\to 1}(x+1) = 2$ ….(1) Since function is continuous, therefore $\lim_{x\to 1} f(x) = f(1)$ k = 2	1 1
Question 24.	Verify that the function y = a cosx + b sinx , where a,b ∈ **R** is a solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$	
Solution:	Given: y = a cosx + b sinx …(1) Diff. w.r.t. 'x' , and we get $\frac{dy}{dx} = -a\sin x + b\cos x$ Again differentiate (1) w.r.t. 'x', we get $\frac{d^2y}{dx^2} = -a\cos x - b\sin x$ …..(2) Now, substitute (1) and (2) in the given differential equation, and we get the following: L.H.S = $\frac{d^2y}{dx^2} + yx$ $= (-a\cos x - b\sin x) + (a\cos x + b\sin x)$ $= -a\cos x - b\sin x + a\cos x + b\sin x$ = 0 = R.H.S As L.H.S = R.H.S, the given function is the solution of the corresponding differential equation.	$\frac{1}{2}$ $\frac{1}{2}$ **1**
OR Question 24.	Find the general solution of the differential equation y log y *dx* – x *dy* = 0	
Solution:	Since y log y *dx* – x *dy* = 0, therefore separating the variables, the given differential equation can be written as	

	$\frac{dy}{y \log y} = \frac{dy}{x}$ .....(1) Integrating both sides of equation (1), we get $\int \frac{dy}{y \log y} = \int \frac{dy}{x}$ $\log \log y = \log x + C$ which is the general solution of equation (1)	$\frac{1}{2}$ $1\frac{1}{2}$
Question 25.	An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?	
Solution:	Total number of balls = 10 black balls + 5 red balls = 15 balls Let A be the event of drawing a black ball in first draw and B be the events of drawing a black ball in second draw. P(A) = Probability of getting a black ball in the first draw = $\frac{10}{15} = \frac{2}{3}$ As the ball is not replaced after the first throw, ∴ P(B/A) = Probability of getting another black ball in the second draw = $\frac{8}{14} = \frac{4}{7}$ Since the two balls are drawn without replacement, the two draws are not independent. P(both balls are black) = P(A) × P(B/A) Now, the probability of getting both balls red = $\frac{2}{3} \times \frac{4}{7} = \frac{8}{21}$	$\frac{1}{2}$ $\frac{1}{2}$ 1
	SECTION – C (3Marks × 8Q)	
Question 26.	Let R be a relation on the set A of ordered pairs of positive integers defined by (x, y) R (u, v) if and only if xv = yu. Show that R is an equivalence relation.	
Solution:	Clearly, (x, y) R (x, y), ∀ (x, y) ∈ A	

	Since xy = yx This shows that R is reflexive. Further, (x, y) R (u, v) => xv = yu => uy = vx => (u, v) R (x, y) $\forall$ (x, y), (u, v) $\in$ A This shows that R is symmetric. Similarly, (x, y) R (u, v) and (u, v) R (a, b) ⇨ xv = yu and ub = va ⇨ $\frac{x}{y} = \frac{u}{v}$ and $\frac{u}{v} = \frac{a}{b}$ ⇨ $\frac{x}{y} = \frac{a}{b}$ ⇨ xb = ya Hence (x, y) R (a, b) $\forall$ (x, y), (u, v) (a, b) $\in$ A Thus, R is transitive. Thus, R is an equivalence relation.	**1** **1** **1**
OR Question 26.	Prove that: $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$	
Solution:	$\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$ We know $\cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$ where x <1 ⇨ $\cos^{-1}\frac{12}{13} = \sin^{-1}\sqrt{1-\left(\frac{12}{13}\right)^2}$ ⇨ $= \sin^{-1}\sqrt{\frac{25}{169}}$ ⇨ $\cos^{-1}\frac{12}{13} = \sin^{-1}\frac{5}{13}$ Now taking L.H.S. $= \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{3}{5}$ We know that, $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$ if xy < 1 $\therefore$ $= \sin^{-1}\left[\frac{5}{13}\sqrt{1-\left(\frac{3}{5}\right)^2} + \frac{3}{5}\sqrt{1-\left(\frac{5}{13}\right)^2}\right]$ $= \sin^{-1}\left[\frac{5}{13}\sqrt{\frac{16}{25}} + \frac{3}{5}\sqrt{\frac{144}{169}}\right]$ $= \sin^{-1}\left[\frac{5}{13}\left(\frac{4}{5}\right) + \frac{3}{5}\left(\frac{12}{13}\right)\right] = \sin^{-1}\frac{56}{65}$ L.H.S. = R.H.S.	**1** $\frac{1}{2}$ $1\frac{1}{2}$

Question 27.	If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that F(x).F(y) = F(x + y).	
Solution:	$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ F(x).F(y) = F(x + y) $F(x).F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ⇨ F(x).F(y) = F(x + y)	$\frac{1}{2}$ $1\frac{1}{2}$ 1
Question 28.	Find $\frac{dy}{dx}$ of the function $(\cos x)^y = (\cos y)^x$.	
Solution:	Given: $(\cos x)^y = (\cos y)^x$ $(\cos x)^y = (\cos y)^x$ Taking log on both sides $\log((\cos x)^y) = \log((\cos y)^x)$ $y.\log(\cos x) = x.\log(\cos y)$ Diff. on both sides w.r.t. ‘x’ $\frac{d}{dx}(y.\log(\cos x)) = \frac{d}{dx}(x.\log(\cos y))$	**1**

	$y.\frac{1}{\cos x}(-\sin x) + \log(\cos x).\frac{dy}{dx} = x.\frac{1}{\cos y}(-\sin y).\frac{dy}{dx} + \log(\cos y).1$ $-y.(\tan x) + \log(\cos x).\frac{dy}{dx} = -x.(\tan y).\frac{dy}{dx} + \log(\cos y)$ $(\log(\cos x) + x(\tan y)).\frac{dy}{dx} = \log(\cos y) - y.(\tan x)$ $\frac{dy}{dx} = \frac{\log(\cos y) - y.(\tan x)}{\log(\cos x) + x.(\tan y)}$	$1\frac{1}{2}$ $\frac{1}{2}$
Question 29.	Find the intervals in which the function f is given by $(x) = -2x^3 - 9x^2 - 12x + 1$ is strictly increasing or strictly decreasing.	
Solution:	Given function: $f(x) = -2x^3 - 9x^2 - 12x + 1$ $f'(x) = -6x^2 - 18x - 12 = -6(x^2 + 3x + 2)$ $f'(x) = -6(x + 2)(x + 1)$,.......(1) Now for increasing or decreasing, $f'(x) = 0$ $-6(x + 2)(x + 1) = 0$ $x + 2 = 0$ or $x + 1 = 0$ $x = -2$ or $x = -1$ Therefore, we have sub-intervals are $(-\infty,-2)$, $(-2, -1)$ and $(-1, \infty)$ For interval $(-\infty,-2)$, picking $x = -3$, from equation (1), $f'(x) = (-ve)(-ve)(-ve) = (-ve) < 0$ Therefore, f is strictly decreasing in $(-\infty,-2)$ For interval $(-2, -1)$, picking $x = -1.5$, from equation (1), $f'(x) = (-ve)(+ve)(-ve) = (+ve) > 0$ Therefore, f is strictly increasing in $(-2, -1)$. For interval $(-1, \infty)$, picking $x = 4$, from equation (1), $f'(x) = (-ve)(+ve)(+ve) = (-ve) < 0$ Therefore, is strictly decreasing in $(-1, \infty)$. So, f is strictly decreasing in $(-\infty, -2)$ and $(-1, \infty)$. f is strictly increasing in $(-2, -1)$.	$\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
Question 30.	Integrate: $\int \frac{4x+1}{\sqrt{2x^2+x-3}}dx$	
Solution:	It is given that $I = \int \frac{4x+1}{\sqrt{2x^2+x-3}}dx$	

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Here form of integral is $\int \frac{px+q}{\sqrt{ax^2+bx+c}}\,dx$
$\therefore\ 4x + 1 = A\frac{d}{dx}(2x^2 + x - 3) + B$
$4x + 1 = A(2x + 1) + B \qquad \ldots(1)$
On comparing the like terms , we have
$2A = 4$ and $A + B = 1$
⇨ $A = 2$ and $B = -1$
⇨ $4x + 1 = 2(4x + 1) - 1 \qquad \ldots$from (1)

$I = \int \frac{2(4x+1)-1}{\sqrt{2x^2+x-3}}\,dx$
$I = 2\int \frac{4x+1}{\sqrt{2x^2+x-3}}\,dx - \int \frac{1}{\sqrt{2x^2+x-3}}\,dx$
Put $2x^2 + x - 3 = t \Rightarrow (4x + 1)\,dx = dt$
$I = 2\int \frac{1}{\sqrt{t}}\,dt - \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{x^2+\frac{1}{2}x-\frac{3}{2}}}\,dx$
$I = 4\sqrt{t} - \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{x^2+\frac{1}{2}x-\frac{3}{2}}}\,dx$
$I = 4\sqrt{2x^2 + x - 3} - \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{x^2+\frac{1}{2}x+\frac{1}{4}-\frac{1}{4}-\frac{3}{2}}}\,dx$ (completing the square)
$I = 4\sqrt{2x^2 + x - 3} - \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{7}}{4}\right)^2}}\,dx$
$I = 4\sqrt{2x^2 + x - 3} - \frac{1}{\sqrt{2}}\log\left\lvert\left(x+\frac{1}{2}\right) + \sqrt{\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt{7}}{4}\right)^2}\right\rvert + C$
$I = 4\sqrt{2x^2 + x - 3} - \frac{1}{\sqrt{2}}\log\left\lvert\frac{(2x+1)+\sqrt{2x^2+x-3}}{2}\right\rvert + C$
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<td>**OR**
Question 30.</td>
<td>Evaluate: $\int_2^8 \lvert x - 5 \rvert dx$</td>
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<td>**Solution:**</td>
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$I = \int_2^8 \lvert x-5 \rvert\,dx$
We know $\lvert x-5 \rvert = \begin{cases} -(x-5), & x \le 5 \\ (x-5), & x > 5 \end{cases}$
$I = \int_2^5 \lvert x-5 \rvert\,dx + \int_5^8 \lvert x-5 \rvert\,dx$
$I = \int_2^5 -(x-5)\,dx + \int_5^8 (x-5)\,dx$
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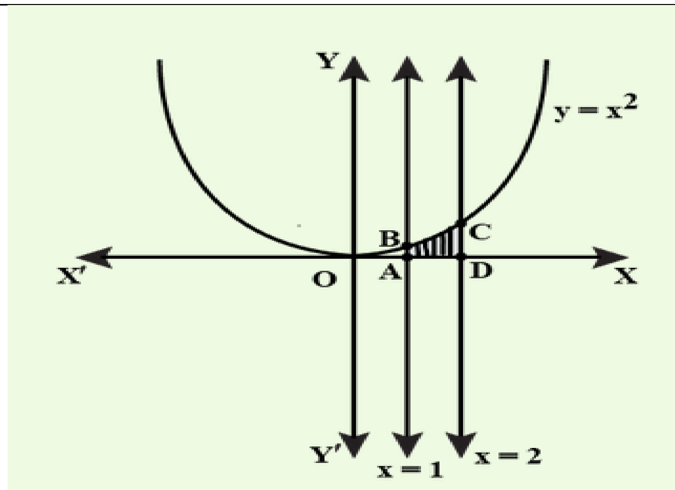
	$I = \left|\frac{-(x-5)^2}{2}\right|_2^5 + \left|\frac{(x-5)^2}{2}\right|_5^8$ $I = \left(\frac{-(0)^2}{2} - \frac{-(-3)^2}{2}\right) + \left(\frac{(3)^2}{2} - \frac{(0)^2}{2}\right)$ $I = \frac{9}{2} + \frac{9}{2}$ I = 9	**$1\frac{1}{2}$** 1
Question 31.	Find the area of a triangle having points A(1, 1, 1), B(1, 2, 3) and C(2, 3,1) as its vertices.	
Solution:	We have $\overrightarrow{AB}$ = (1-1) $\hat{\imath}$ + (2-1)$\hat{\jmath}$ + (3-1)$\hat{k}$ = $\hat{\jmath}$ + 2$\hat{k}$ and $\overrightarrow{AC}$ = (2-1)$\hat{\imath}$ – (3-1)$\hat{\jmath}$ – (1-1)$\hat{k}$ = $\hat{\imath}$ – 2$\hat{\jmath}$ Area of the given triangle is $\frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$. Now $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{vmatrix}$ $= -4\,\hat{\imath} \mp 2\hat{\jmath} - \hat{k}$ Therefore, $\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{16+4+1} = \sqrt{21}$ Thus the required area is $\frac{1}{2}\sqrt{21}$.	1/2 1/2 1 1
	SECTION – C (5Marks × 4Q)	
Question 32.	Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations x – y + 2z = 1; 2y – 3z = 1 ; 3x – 2y + 4z = 2	
Solution:	$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = =$ $\begin{bmatrix} -2-9+12 & -2+2 & 1+3-4 \\ 18-18 & 4-3 & -6+6 \\ -6-18+24 & -4+4 & 3+6-8 \end{bmatrix}$ $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	

	$x - y + 2z = 1$ $2y - 3z = 1$ $3x - 2y + 4z = 2$ $\therefore A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $\|A\| = 1(8 - 6) + 1(0 + 9) + 2(0 - 6) = 2 + 9 - 12$ $= -1 \neq 0$ $\therefore$ Inverse of matrix exists. Now by using the product the inverse of matrix A is $\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ $\Rightarrow A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ Now, matrix of equations can be written as: AX=B $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ And, $X = A^{-1} B$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$ Therefore, x = 0, y = 5 and z = 3.	$1\frac{1}{2}$ **1** **1** $1\frac{1}{2}$
Question 33.	Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$	
Solution:	Given lines are $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ $\therefore$ Corresponding vector equations of given lines are $\vec{r} = -\hat{\imath} - \hat{\jmath} - \hat{k} + \lambda(7\hat{\imath} - 6\hat{\jmath} + \hat{k})$ ...(1) and $\vec{r} = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k} + \mu(\hat{\imath} - 2\hat{\jmath} + \hat{k})$ ...(2) Comparing (1) and (2) with $\vec{r} = \overrightarrow{a_1} + \lambda \overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu \overrightarrow{b_2}$ respectively, we get	$\frac{1}{2}$

	$\overrightarrow{a_1} = -\hat{\imath} - \hat{\jmath} - \hat{k}$, and $\overrightarrow{b_1} = 7\hat{\imath} - 6\hat{\jmath} + \hat{k}$ $\overrightarrow{a_2} = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}$ and $\overrightarrow{b_2} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ Therefore $\overrightarrow{a_2} - \overrightarrow{a_1} = 4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$ And $\overrightarrow{b_1} \times \overrightarrow{b_2} = (7\hat{\imath} - 6\hat{\jmath} + \hat{k}) \times (\hat{\imath} - 2\hat{\jmath} + \hat{k})$ $= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = -4\hat{\imath} - 6\hat{\jmath} - 8\hat{k}$ $\lvert \overrightarrow{b_1} \times \overrightarrow{b_2} \rvert = \sqrt{16 + 36 + 64} = \sqrt{116}$ Hence, the shortest distance between the given lines is given by $D = \frac{\lvert (\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2}) \rvert}{\lvert \overrightarrow{b_1} \times \overrightarrow{b_2} \rvert} = \frac{\lvert (4\hat{\imath}+6\hat{\jmath}+8\hat{k}).(-4\hat{\imath}-6\hat{\jmath}-8\hat{k}) \rvert}{\sqrt{116}}$ $\frac{\lvert -16-36-64 \rvert}{\sqrt{116}} = \frac{116}{\sqrt{116}} = \sqrt{116} = 2\sqrt{29}$	$\frac{1}{2}$ 1 $\frac{1}{2}$ $1\frac{1}{2}$ 1
OR **Question 33.**	Find the vector equation of the line passing through the point (1, −2,−3) and perpendicular to the two lines : $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{3}$ and $\frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}$.	
Solution:	The vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda \vec{b}$. It is given that, the line passes through (1, −2,−3) So, $\vec{a} = 1\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ Given lines are $\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{3}$ and $\frac{x-2}{2} = \frac{y+1}{1} = \frac{z+1}{2}$ It is also given that, line is perpendicular to both given lines. So we can say that the line is perpendicular to both parallel vectors of two given lines.	1

	We know that, $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ & $\vec{b}$, so let $\vec{b}$ is cross product of parallel vectors of both lines i.e. $\vec{b} = \overrightarrow{b_1} \times \overrightarrow{b_2}$ where $\overrightarrow{b_1} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\overrightarrow{b_2} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and Required Normal $\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 3 \\ 2 & 1 & 2 \end{vmatrix}$ $= \hat{\imath}(-2-3) - \hat{\jmath}(2-6) + \hat{k}(1+2)$ $\vec{b} = -5\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$ Now, by substituting the value of $\vec{a}$ & $\vec{b}$ in the formula $\vec{r} = \vec{a} + \lambda \vec{b}$, we get $\vec{r} = (1\hat{\imath} - 2\hat{\jmath} - 3\hat{k}) + \lambda(-5\hat{\imath} + 4\hat{\jmath} + 3\hat{k})$	**2** **1** **1**
Question 34.	Find the area under the given curve $y = x^2$ and the given lines $x = 1$, $x = 2$ and x-axis.	
Solution:	Equation of the curve is $y = x^2$. It is an upward parabola having vertex at origin and symmetrical about y-axis. $x = 1$ and $x = 2$ are two straight lines parallel to y-axis. $y = x^2$ ....(1) $x = 1$ and $x = 2$ Points of intersections of given curves At $x = 1$, $y = 1$ points are (1, 1) At $x = 2$, $y = 4$ points are (2, 4) ∴ Points in first quadrant A(1, 1) B(2, 4) Points on x- axis with given lines are (1, 0) and (2, 0) Make a rough hand sketch of given curves by taking some corresponding values of x and y.	$\mathbf{\frac{1}{2}}$ $\mathbf{1\frac{1}{2}}$

	 Required area is shaded region ABCD: $\|\int_1^2 y\,dx\| = \|\int_1^2 x^2\,dx\|$ [From equation (1)] $= \left\|\frac{x^3}{3}\right\|_1^2$ $= \frac{1}{3}\|(2^3 - 1^3)\|$ $= \frac{1}{3}\|(8 - 1)\| = \frac{1}{3}(7) = \frac{7}{3}$ sq. units	**1** **2**
OR Question 34.	Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$	
Solution:	Here $\frac{x^2}{4} + \frac{y^2}{9} = 1$ …(1) It is a vertical ellipse having center at origin and is symmetrical about both axes (if we change y to -y or x to -x, equation remain same). Standard equation of an ellipse is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ By comparing, a = 3 and b = 2 From equation (1) $\Rightarrow y^2 = \frac{9}{4}(4 - x^2)$ $\Rightarrow y = \frac{3}{2}\sqrt{4 - x^2}$ …..(2) Points of Intersections of ellipse (1) with x-axis (y = 0) Put y = 0 in equation (1), we have	$\frac{1}{2}$ **1**

$x^2/4 = 1 \Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

Therefore, Intersections of ellipse(1) with x-axis are (0, 2) and (0, -2).

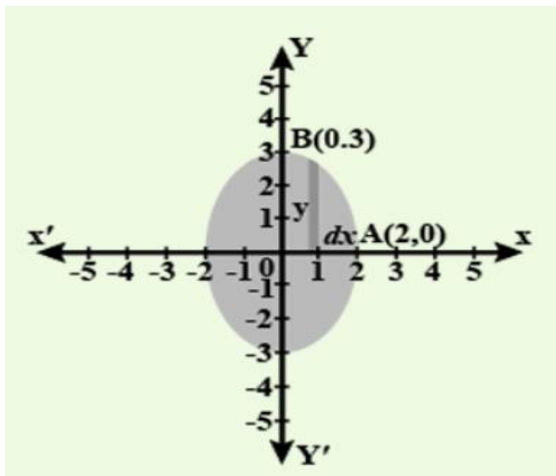
Points of Intersections of ellipse (1) with y-axis (x = 0)

Putting x = 0 in equation (1), $y^2/9 = 1 \Rightarrow y^2 = 9$

$\Rightarrow y = \pm 3$.

Therefore, Intersections of ellipse (1) with y-axis are (0, 3) and (0,-3).

for arc of ellipse in first quadrant. **1**



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Now,

Area of region bounded by ellipse (1)

Total shaded area = 4 x Area OAB of ellipse in first quadrant

$=4\left|\int_0^2 y.\,dx\right|$ [∵ at end B of arc AB of ellipse: x=0 and at end A of arc AB; x=2]

$=4\left|\int_0^2 \frac{3}{2}\sqrt{4-x^2}.\,dx\right| = 6\left|\int_0^2 \sqrt{2^2-x^2}.\,dx\right|$

$= 6\left|\frac{x}{2}\sqrt{2^2-x^2} + \frac{2^2}{2}\sin^{-1}\frac{x}{2}\right|_0^2$ $\left[\because \int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

$= 6[(\frac{2}{2}\sqrt{4-4} + 2\sin^{-1}1) - (0 + 2\sin^{-1}0)] = 6[0 + (2\frac{\pi}{2})]$

$= 6\pi$ sq. Units **$1\frac{1}{2}$**

$\frac{1}{2}$

Question 35.

Solve the following problem graphically:

Minimise and Maximise $Z = 5x + 10y$

Subject to the constraints: $x + 2y \leq 120$

$x + y \geq 60$

$x - 2y \geq 0$

$x \geq 0,\ y \geq 0$

Solution:

$Z = 5x + 10y.$ …(1)

$x + 2y \leq 120$ …(2)

$x + y \geq 60.$ …(3)

$x - 2y \geq 0$ …(4)

$x \geq 0\ ,\ y \geq 0$ …(5)

First of all, let us graph the feasible region of the system of linear inequalities (2) to (5).

Let $Z= 5x + 10y$ ....(1)

Converting inequalities to equalities

$x + 2y = 120$

X	0	120
Y	60	0

Points are (0, 60), (120,0)

Now put (0, 0) in inequation (2),

we find $0 \leq 120$, which is true. $\frac{1}{2}$

Therefore area lies towards the origin from this line.

$x + y = 60$

x	0	60
y	60	0

Points are (0, 60), (60, 0)

Now put (0, 0) in inequation (3),

we find $0 \geq 60$, which is False. $\frac{1}{2}$

Therefore area lies away from the origin from this line.

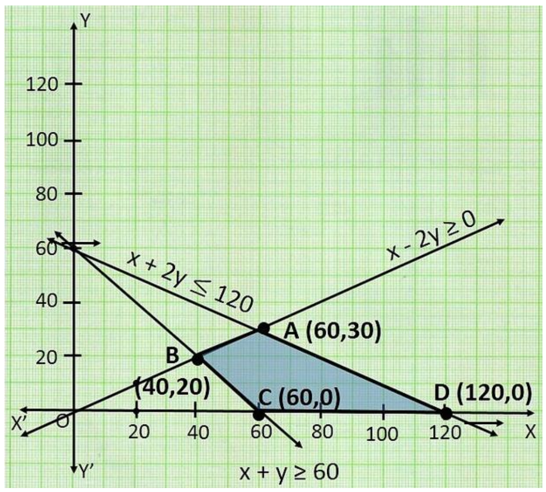
$x - 2y = 0$

X	0	20	40

y	0	10	20

Points are (0,0),(20,10),(40,20)
Now put (1, 0) in inequation (4),
we find $1 \geq 0$, which is true. $\frac{1}{2}$
Therefore area lies towards (1, 0) origin from this line.

Plot the graph for the set of points



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To find maximum and minimum
The feasible region ABCD is shown in the figure. Note that the region is bounded. The coordinates of the corner points A, B, C and D are (60, 30), (40, 20), (60, 0) and (120, 0) respectively. $\frac{1}{2}$

Corner Point	Corresponding Value of $Z = 5x + 10y$
A (60, 30)	**600←Maximum**
B (40, 20)	**400**
C (60, 0)	**300←Minimum**
D (120, 0)	**600←Maximum**
	(Multiple optimal solutions)

$1\frac{1}{2}$

We now find the minimum and maximum value of Z.
From the table, we find that the minimum value of Z is 300 at the point B (60, 0) of the feasible region.

	The maximum value of Z on the feasible region occurs at the two corner points C (60, 30) and D (120, 0) and it is 600 in each case.	$\frac{1}{2}$
	SECTION – E (4Marks × 3Q)	
Question 36.	$P(x) = -6x^2 + 120x + 25000$ (in ₹) is the total profit function of a company, where x denotes the production of the company. ***Based on the above information answer the following:*** (i)Find the profit of the company when the production is 3units. (1) (ii) Find P'(5). (1) (iii) Find the production, when the profit is maximum. (2)	
Solution:	(i) When x = 3 $P(3) = -6(3)^2 + 120(3) + 25000$ $= -54 + 360 + 25000$ $=$ ₹ 25306	1
	(ii) We have, $P(x) = -6x^2 + 120x + 25000$ …(1) Differentiating equation (1) w.r.t. x $P'(x) = -12x + 120$ …(2) $\therefore$ $P'(5) = -12(5) + 120 = 60$	1
	(iii) We have, $P(x) = -6x^2 + 120x + 25000$ …(1) Differentiating equation (1) w.r.t. x $P'(x) = -12x + 120$ …(2) For maximum or minimum value of P(x), P'(x) = 0 we have $-12x + 120 = 0$ $-12x = -120$ i.e. $x = 10$ Differentiating equation (2) w.r.t. x $P''(x) = -12$ Now , At x = 10 $P''(x) = -12 = -ve$ $\Rightarrow$ P(x) has maximum value at x = 10	2
Question 37.	A linear differential equation is of the form $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then such equation is known as linear differential equation. Its solution is given by $y.(IF.) = \int Q(IF.)\, dx + c$, where I.F.(Integrating Factor) = $e^{\int P dx}$ Now, suppose the given equation is $x\frac{dy}{dx} + 2y = x^2$ *Based on the above information, answer the following questions:*	

	(i)What are the values of P and Q respectively? (1) (ii)What is the value of I.F.? (1) (iii)Find the Solution of given equation. (2)	
Solution:	(i) Given equation is $x\frac{dy}{dx} + 2y = x^2$ Dividing on both side by x, we have $\frac{dy}{dx} + \frac{2}{x}y = x$ $\Rightarrow P = \frac{2}{x}$, $Q = x$	1
	(ii) I.F.(Integrating Factor) = $e^{\int Pdx}$ $= e^{\int \frac{2}{x}dx}$ $= e^{2\log x}$ $= x^2$	1
	(iii) Solution of given equation is $y.(IF.) = \int Q(IF.)\, dx + c$ $y(x^2) = \int x(x^2)\, dx + c$ $x^2y = \int x^3dx + c$ $x^2y = \frac{x^4}{4} + c$	2
Question 38.	In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03. *Based on the above information answer the following questions:*	

	(i) The total probability of committing an error in processing the form. (2) (ii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is not processed by Vinay. (2)	
Solution:	(i) Let E_1 = Event of processing form by Vinay. E_2 = Event of processing form by Soniya. E_3 = Event of processing form by Iqbal. $P(E_1) = \frac{50}{100} = \frac{5}{10}$, $P(E_2) = \frac{20}{100} = \frac{2}{10}$, $P(E_3) = \frac{30}{100} = \frac{3}{10}$ Also $P(A/E_1) = 0.06$, $P(A/E_2) = 0.04$, $P(A/E_3) = 0.03$ Required Probability $P(A) = P(E_1).\,P(A/E_1) + P(E_2)\,P(A/E_2) + P(E_3)\,P(A/E_3)$ $= \frac{5}{10}(0.06) + \frac{2}{10}(0.04) + \frac{3}{10}(0.03)$ $= 0.03 + 0.008 + 0.009 = 0.047$	2
	(ii) Probability that the form is not processed by Vinay = $P(\overline{E}_1 \mid A)$ $P(\overline{E}_1 \mid A) = 1 - P(E_1 \mid A)$ By Bayes' Theorem $P(E_1 \mid A) = \frac{P(E_1).P(A\|E_1)}{P(E1).P(A/E1) + P(E2)\,P(A/E2) + P(E3)\,P(A/E3)}$ $P(E_1 \mid A) = \frac{\frac{5}{10}(0.06)}{0.047}$ $P(E_1 \mid A) = \frac{0.03}{0.047} = \frac{30}{47}$ $P(\overline{E}_1 \mid A) = 1 - P(E_1 \mid A)$ $= 1 - \frac{30}{47} = \frac{17}{47}$	2